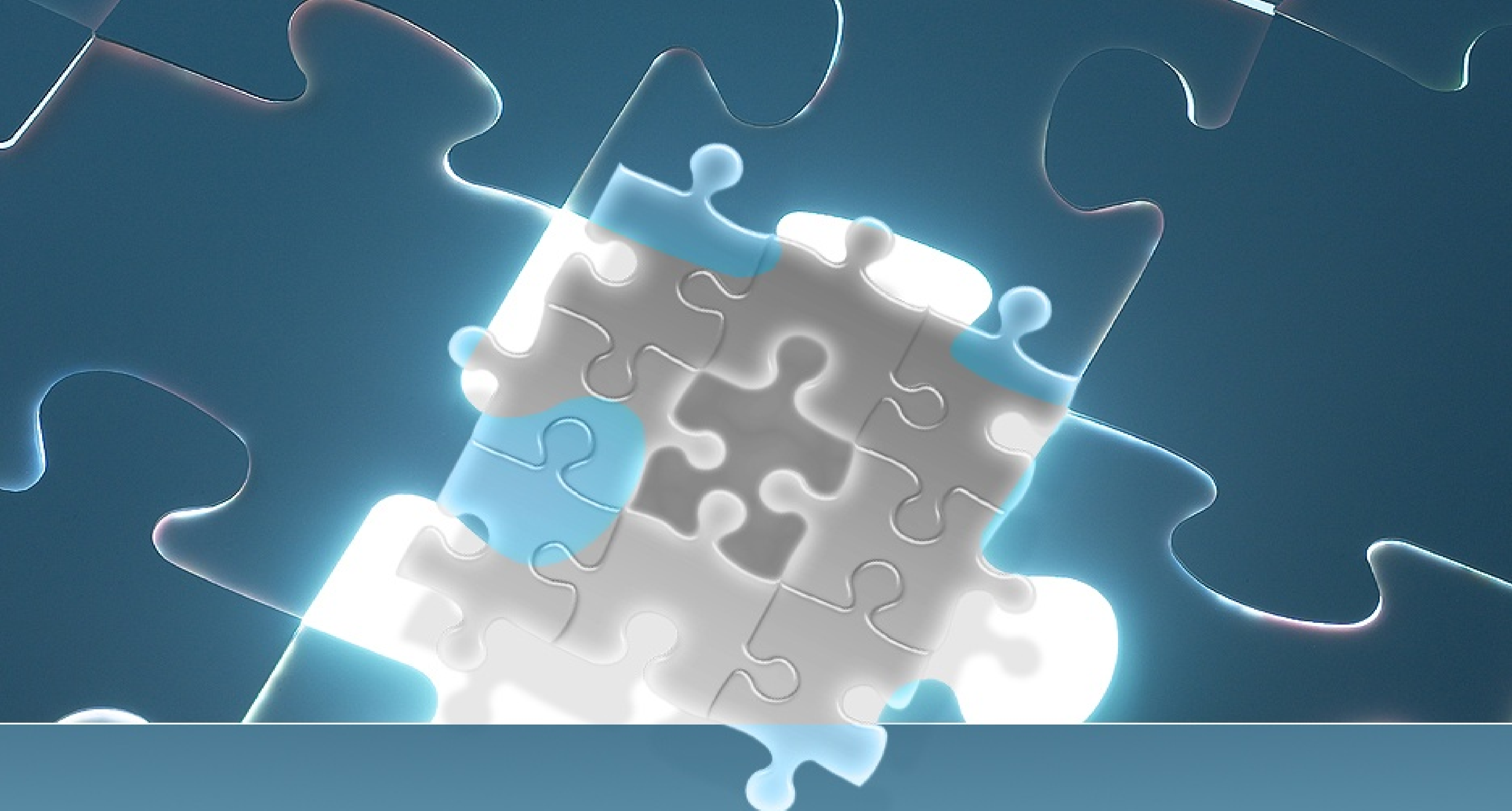


# Introduction to Quantum Teleportation

Zeyuan Zhu 基应61

# A brief retrospection

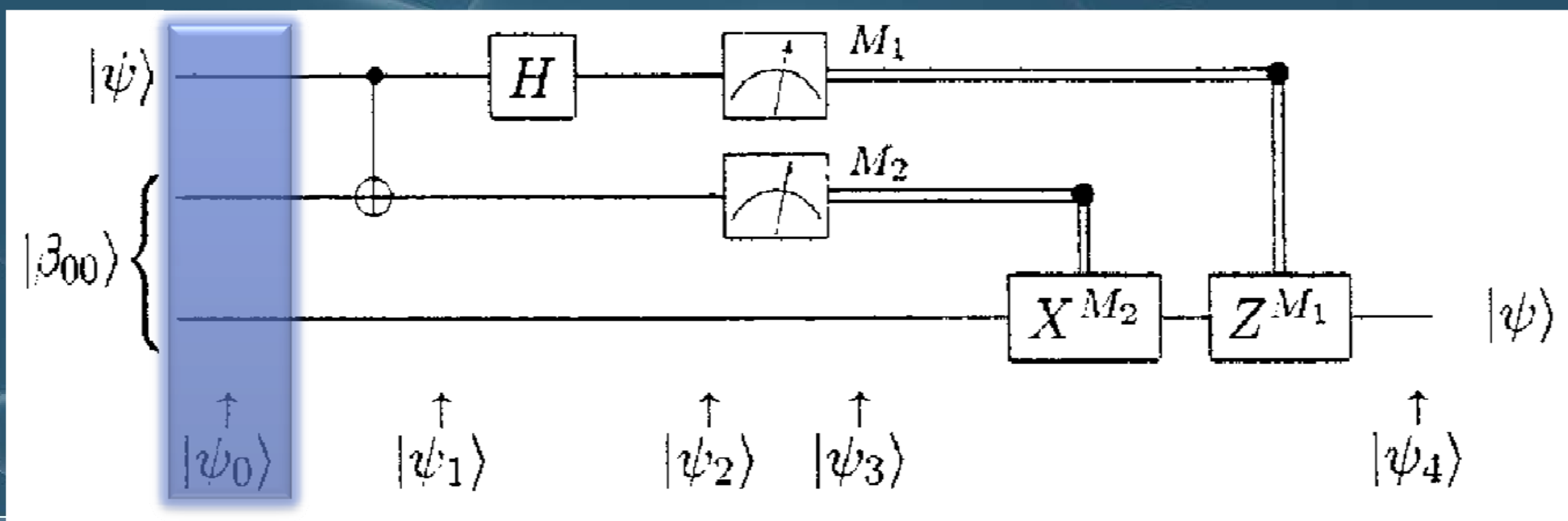
- Direct quantum communication?
  直接量子传态?
- Classical channel 经典信道
- *Non-cloneable 不可克隆定理*
- EPR pair repaired by 3rd-party

# In the language of state vectors.

$$|\psi\rangle = \alpha|0\rangle + \beta|1\rangle$$

$$|\Psi_0\rangle = (\alpha|0\rangle + \beta|1\rangle) \otimes \frac{(|00\rangle + |11\rangle)}{\sqrt{2}}$$

$$= \frac{1}{\sqrt{2}}[\alpha|0\rangle(|00\rangle + |11\rangle) + \beta|1\rangle(|00\rangle + |11\rangle)]$$

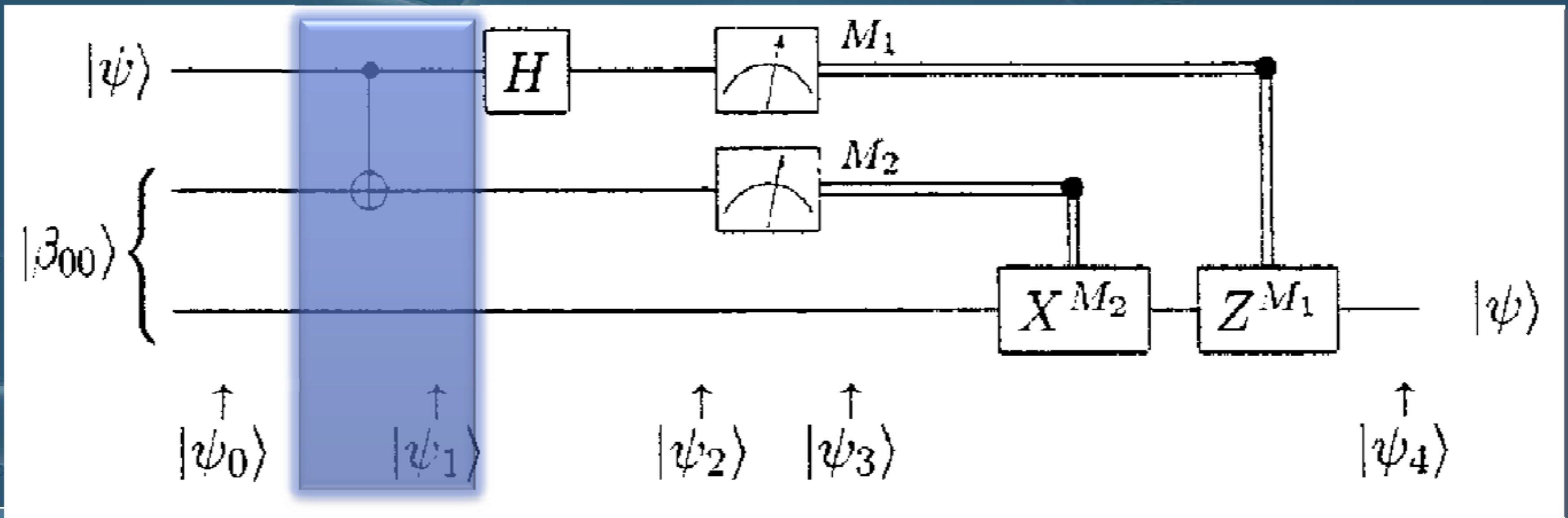


# In the language of state vectors.

CNOT

$$|\Psi_0\rangle = \frac{1}{\sqrt{2}}[\alpha|0\rangle(|00\rangle + |11\rangle) + \beta|1\rangle(|00\rangle + |11\rangle)]$$

$$|\Psi_1\rangle = \frac{1}{\sqrt{2}}[\alpha|0\rangle(|00\rangle + |11\rangle) + \beta|1\rangle(|10\rangle + |01\rangle)]$$



# In the language of state vectors.

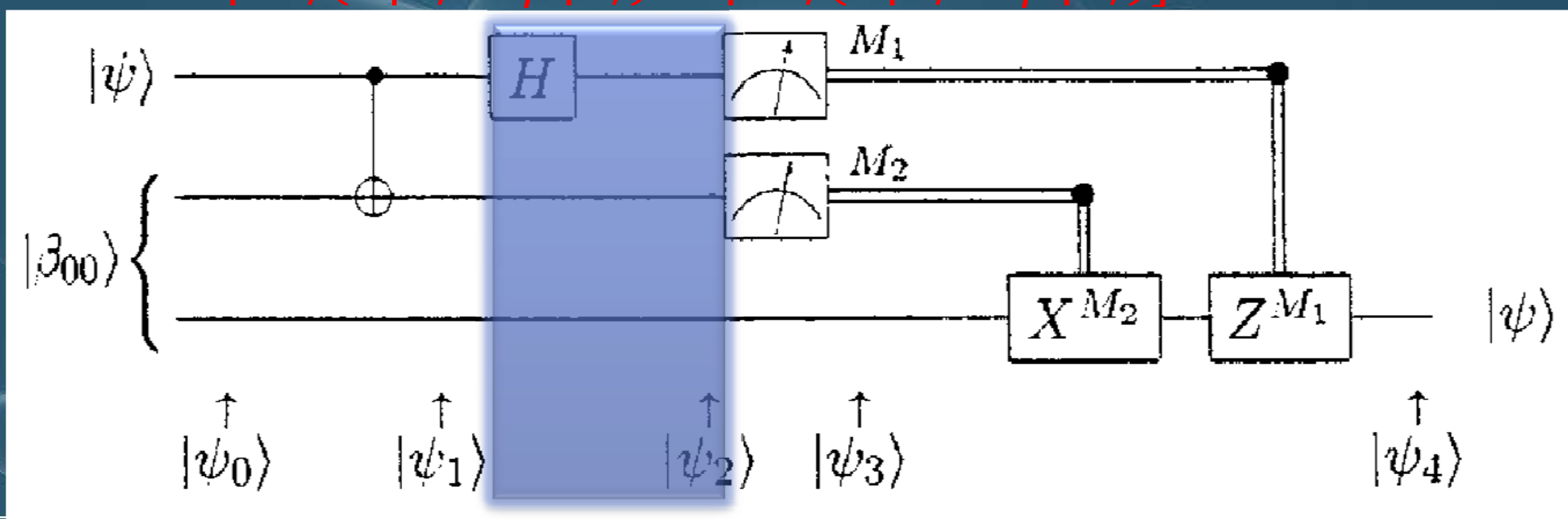
$$|\Psi_1\rangle = \frac{1}{\sqrt{2}}[\alpha|0\rangle(|00\rangle + |11\rangle) + \beta|1\rangle(|10\rangle + |01\rangle)]$$

Hadamard

$$|\Psi_2\rangle = \frac{1}{2}[\alpha(|0\rangle + |1\rangle)(|00\rangle + |11\rangle) + \beta(|0\rangle - |1\rangle)(|10\rangle + |01\rangle)]$$

$$= \frac{1}{2}[|00\rangle(\alpha|0\rangle + \beta|1\rangle) + |01\rangle(\alpha|1\rangle + \beta|0\rangle)$$

$$+ |10\rangle(\alpha|0\rangle - \beta|1\rangle) + |11\rangle(\alpha|1\rangle - \beta|0\rangle)]$$



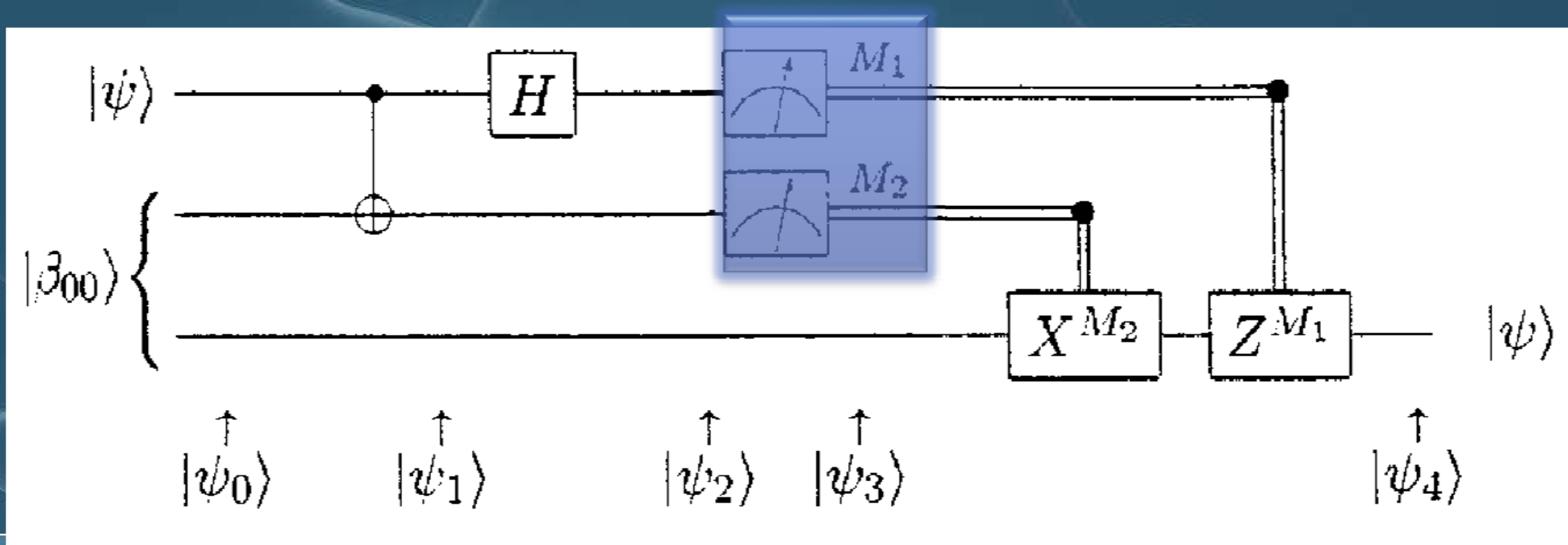
# In the language of state vectors.

$$|\Psi_2\rangle = \frac{1}{2}[\alpha(|0\rangle + |1\rangle)(|00\rangle + |11\rangle) + \beta(|0\rangle - |1\rangle)(|10\rangle + |01\rangle)]$$

$$= \frac{1}{2}[|00\rangle(\alpha|0\rangle + \beta|1\rangle) + |01\rangle(\alpha|1\rangle + \beta|0\rangle)$$

$$+ |10\rangle(\alpha|0\rangle - \beta|1\rangle) + |11\rangle(\alpha|1\rangle - \beta|0\rangle)]$$

Measure



# A brief retrospection

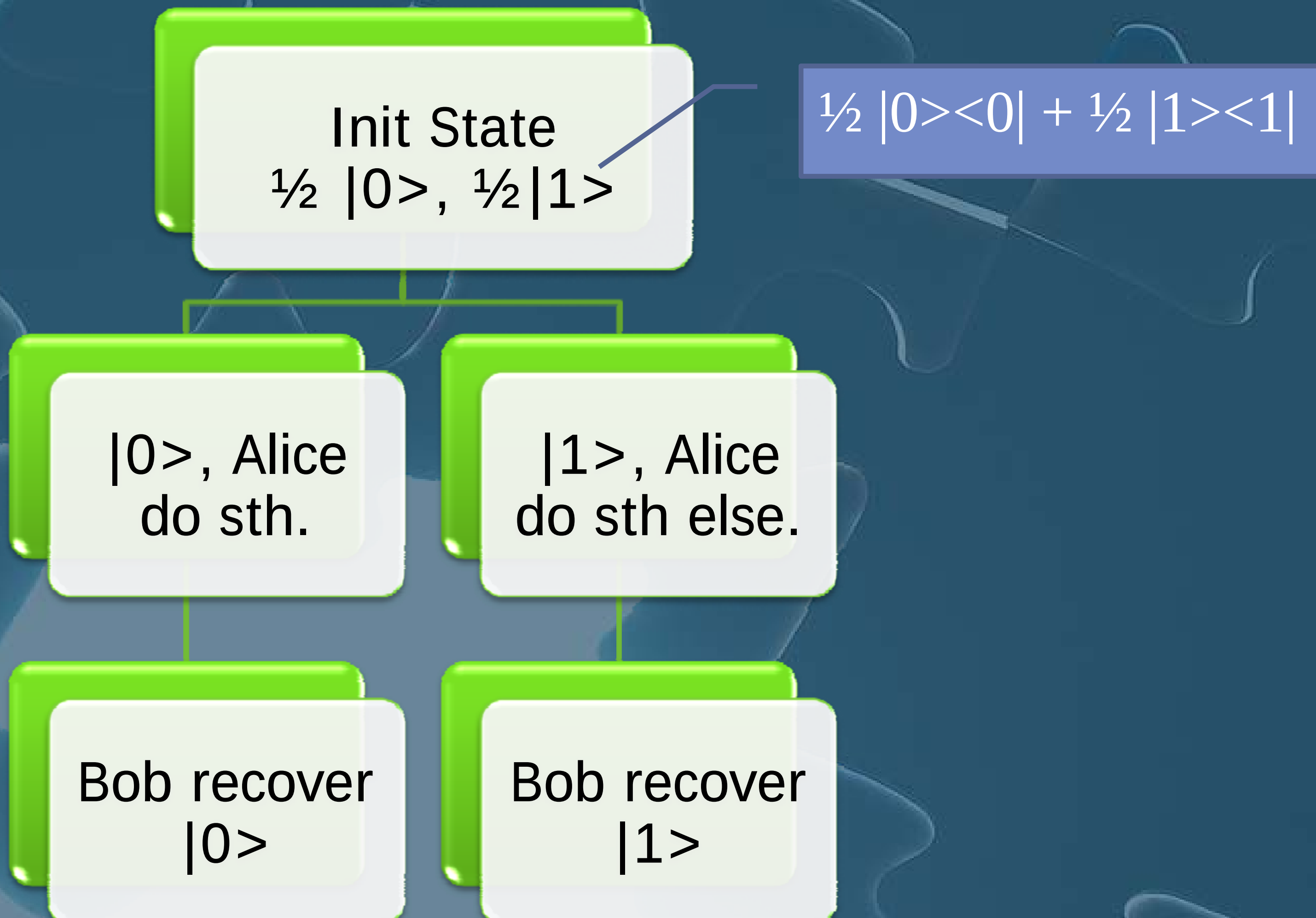
- Classical channel 经典信道
  - 00, 01, 10, 11 - 2bits
- *Non-cloneable 不可克隆定理*
  - |00>, |01>, |10>, |11>
- EPR pair repaired by 3rd-party

## How about for a mixed state?

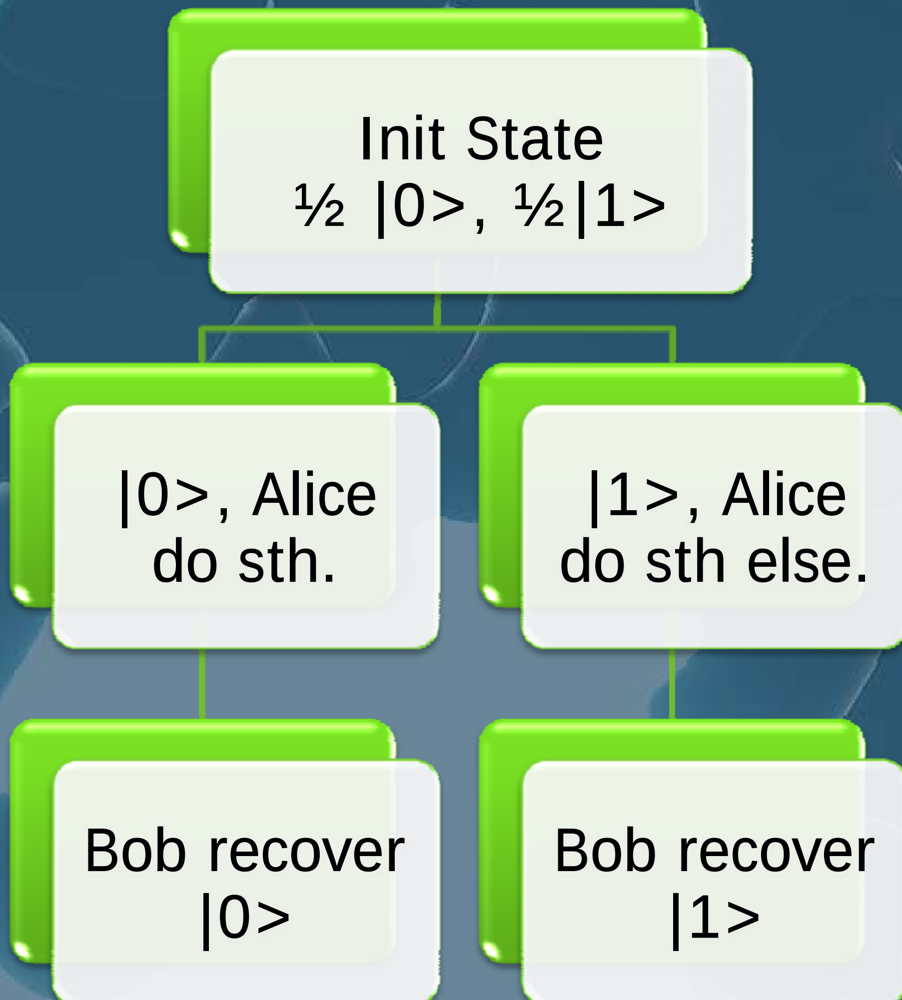
If all qubits of mixed state can be transported, ?

Large number of quibits!

# Negative


Init State
½ |0>, ½|1>
½ |0><0| + ½ |1><1|
|0>, Alice do sth.
|1>, Alice do sth else.
Bob recover |0>
Bob recover |1>

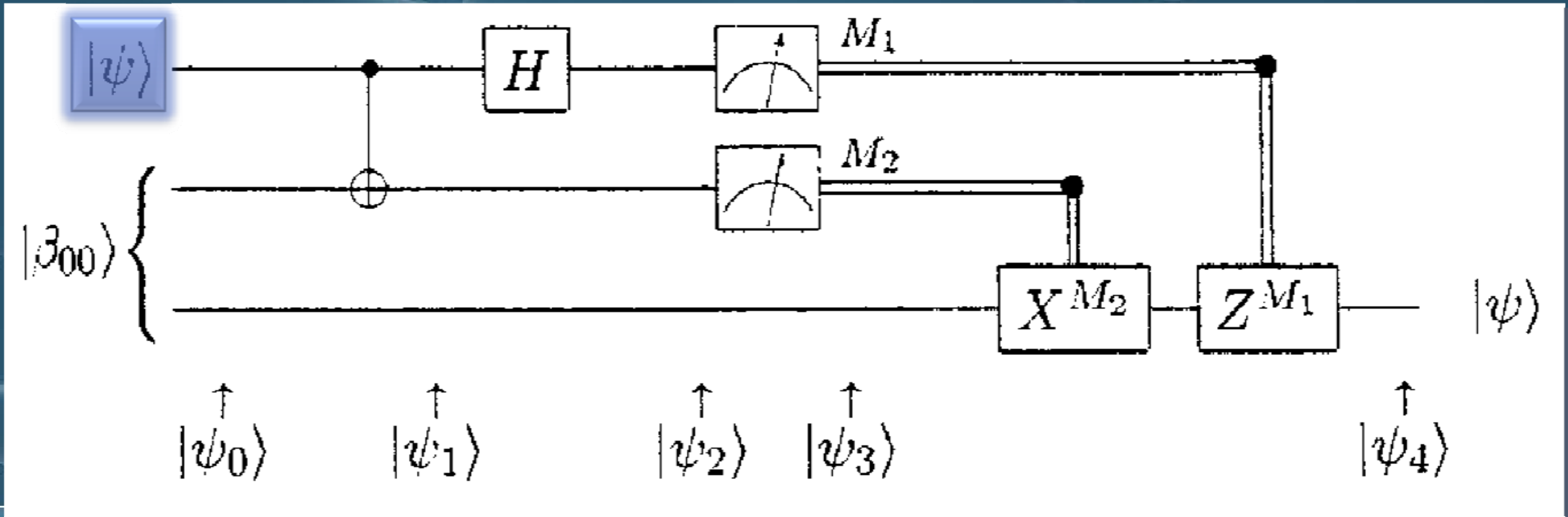
# ~~Negative~~ Positive!



# In the language of density operator

$$\rho_{init} = \begin{pmatrix} \alpha & \beta \\ \beta^* & 1-\alpha \end{pmatrix}$$

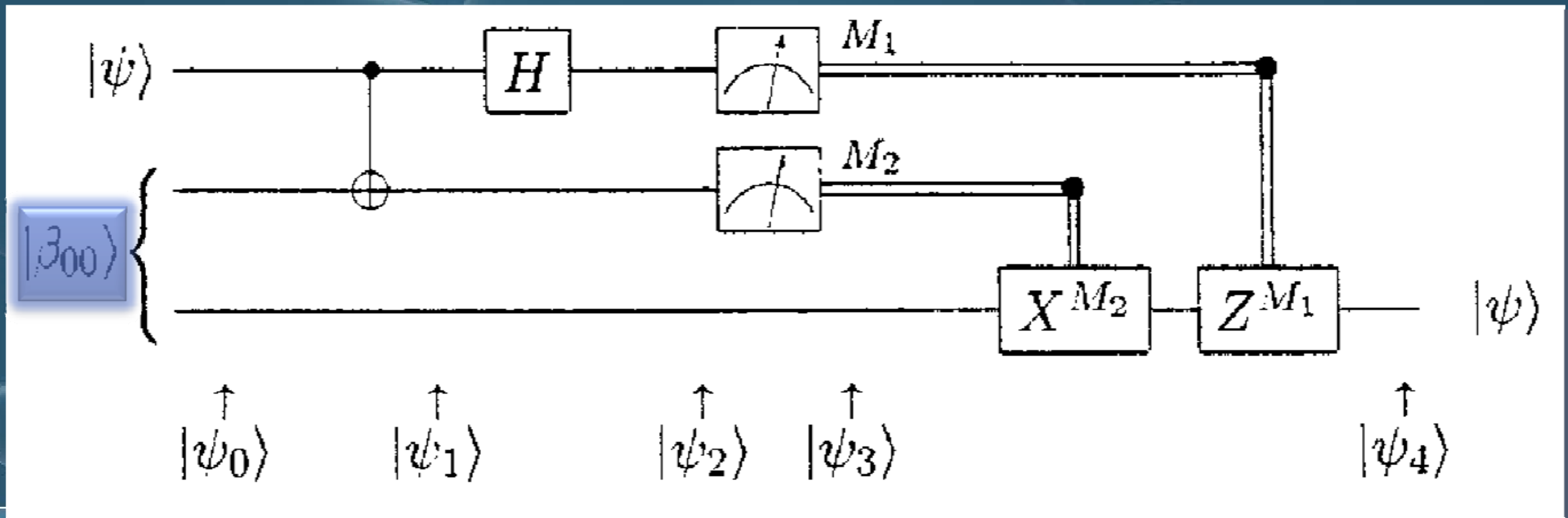
$$= \alpha|0\rangle\langle 0| + \beta|0\rangle\langle 1| + \beta^*|1\rangle\langle 0| + (1-\alpha)|1\rangle\langle 1|$$



# In the language of density operator

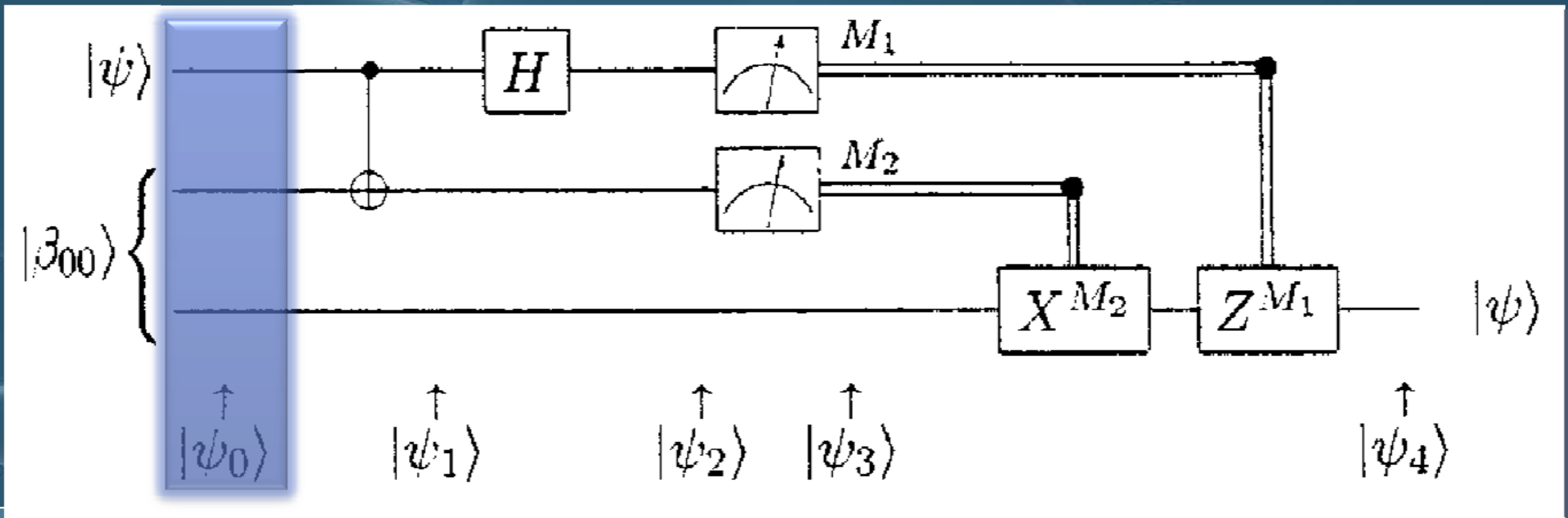
$$\rho_{|\beta_{00}\rangle} = \begin{pmatrix} 1 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 1 \end{pmatrix} \Big/ 2$$

$$= \frac{1}{2}\left(|00\rangle\langle 00| + |00\rangle\langle 11| + |11\rangle\langle 00| + |11\rangle\langle 11|\right)$$



# In the language of density operator

$$\rho_0 = \begin{pmatrix} \alpha & \beta \\ \beta & 1-\alpha \end{pmatrix} \otimes \begin{pmatrix} 1 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 1 \end{pmatrix} / 2$$



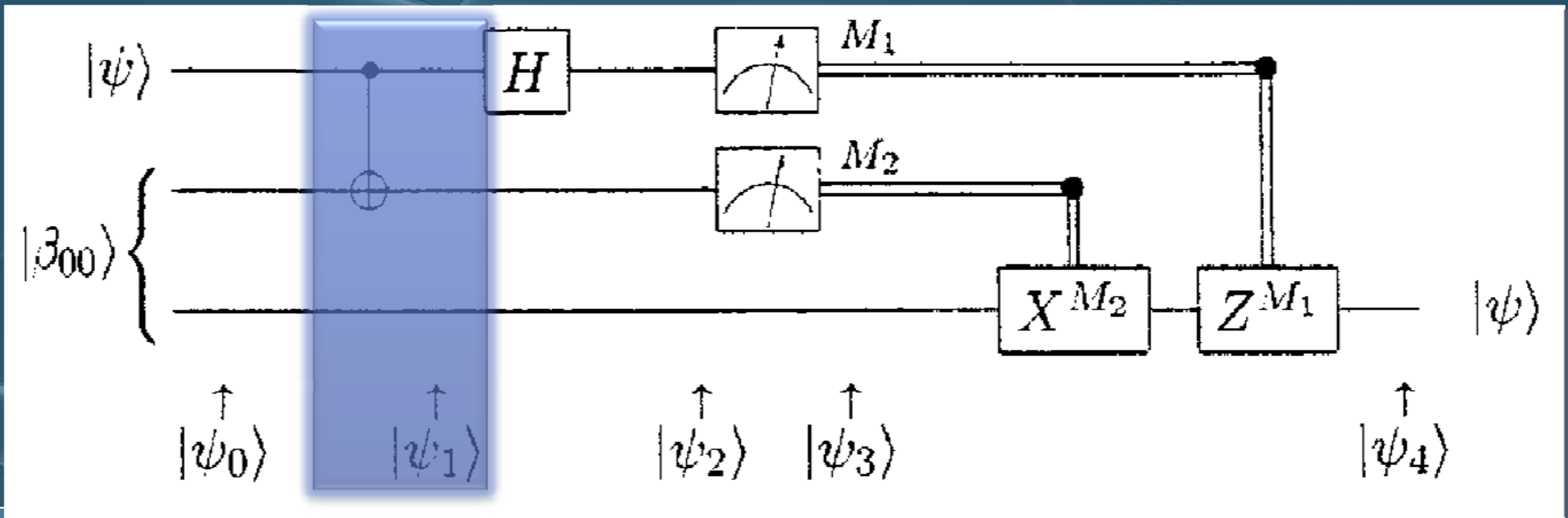
# In the language of density operator

CNOT

$$U_{CNOT} = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 \end{pmatrix} \otimes \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$$

Remark: $|00\rangle\langle 00| + |01\rangle\langle 01| + |11\rangle\langle 10| + |10\rangle\langle 11|$

$$\rho_1 = U_{CNOT}^{\dagger} \cdot \rho_0 \cdot U_{CNOT}$$

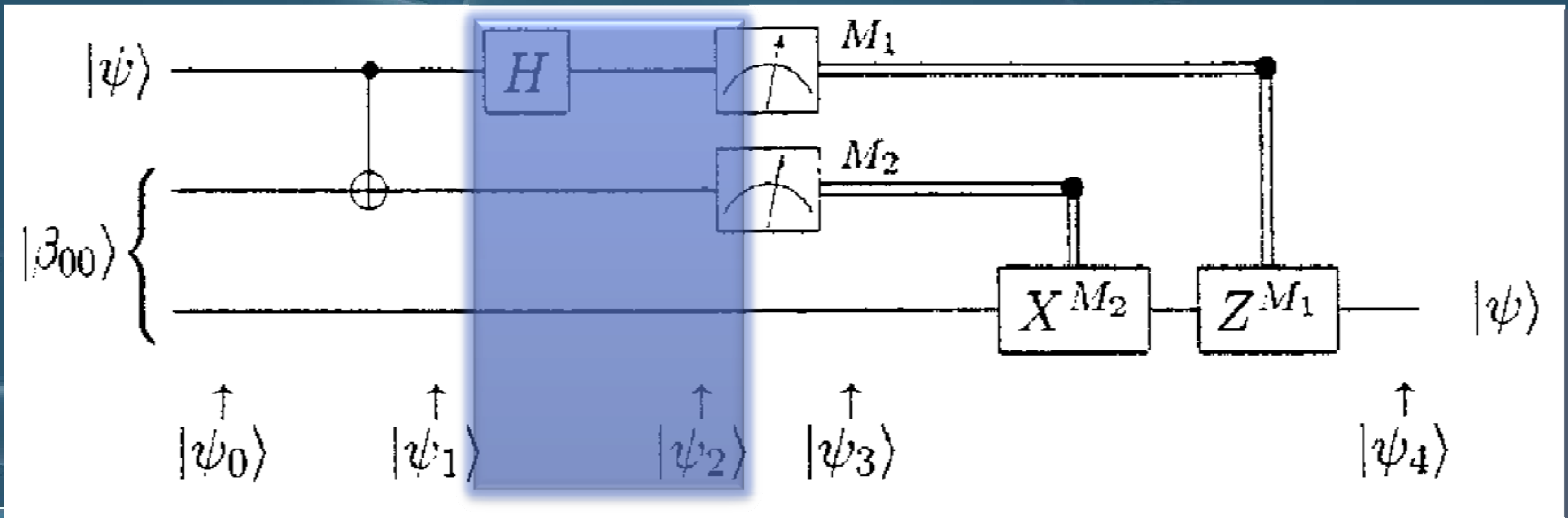


# In the language of density operator

Hadamard

$$U_H = \begin{pmatrix} 1 & 1 \\ 1 & -1 \end{pmatrix} / \sqrt{2} \otimes \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

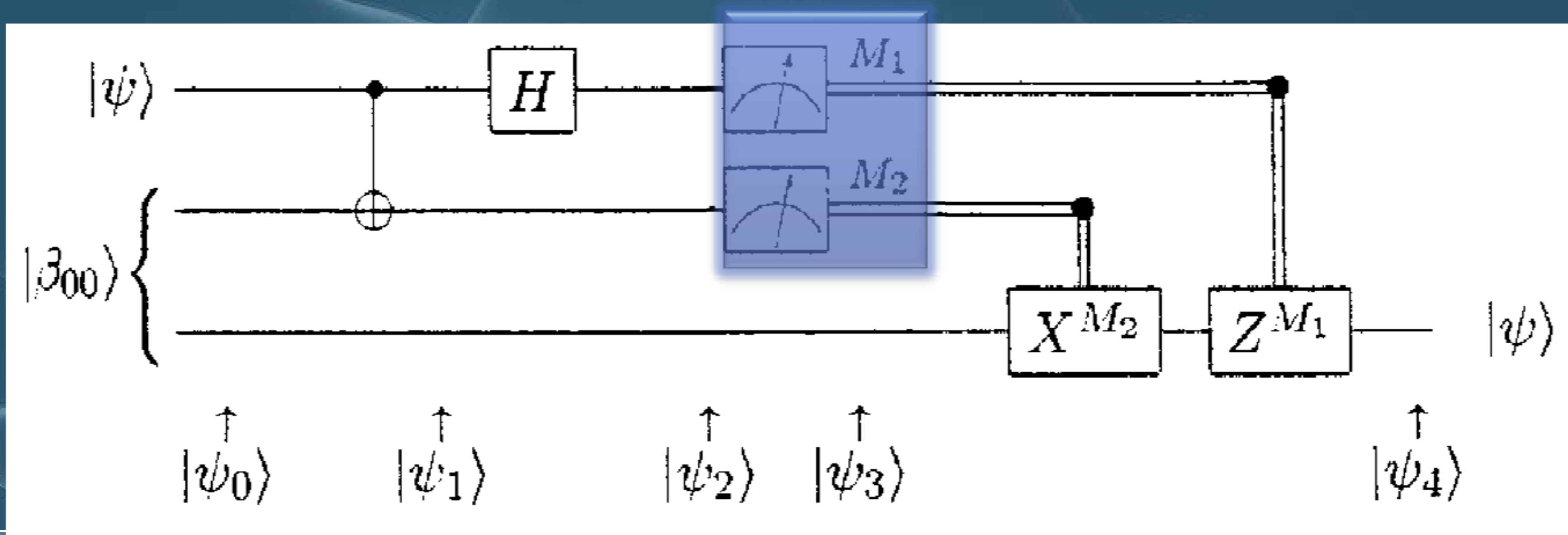
$$\rho_2 = U_H^{\dagger} \cdot \rho_1 \cdot U_H$$



# In the language of density operator

$$\{M_1, M_2, M_3, M_4\} = \left\{ \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix} \otimes I_2, \begin{pmatrix} 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix} \otimes I_2, \begin{pmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix} \otimes I_2, \begin{pmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix} \otimes I_2 \right\}$$

**Measure**



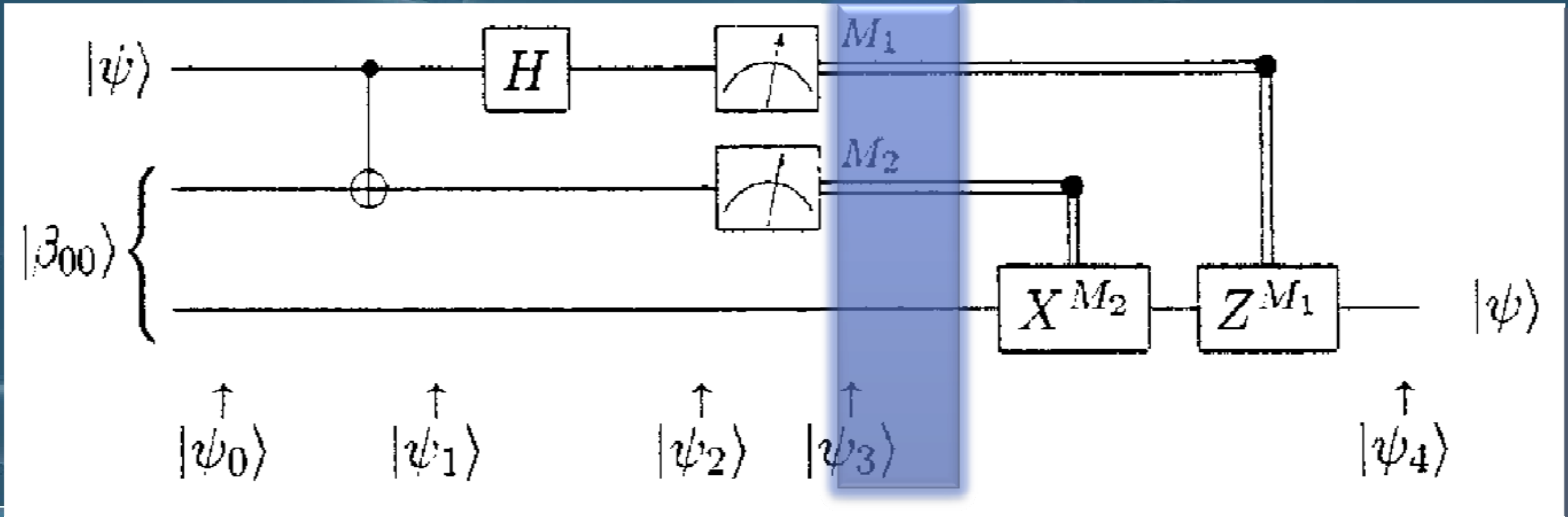
# In the language of density operator

$$\rho_{00} = |00\rangle\langle 00| \otimes \begin{pmatrix} \alpha & \beta \\ \beta^* & 1-\alpha \end{pmatrix}, \quad \rho_{01} = |01\rangle\langle 01| \otimes \begin{pmatrix} 1-\alpha & \beta^* \\ \beta & \alpha \end{pmatrix}$$

$$\rho_{10} = |10\rangle\langle 10| \otimes \begin{pmatrix} \alpha & -\beta \\ -\beta^* & 1-\alpha \end{pmatrix}, \quad \rho_{11} = |11\rangle\langle 11| \otimes \begin{pmatrix} 1-\alpha & -\beta^* \\ -\beta & \alpha \end{pmatrix}$$

Outcomes

Remark: $\dfrac{M_i^\dagger \rho_2 M_i}{\mathrm{Tr}\left(M_i^\dagger \rho_2 M_i\right)}$



# In the language of density operator

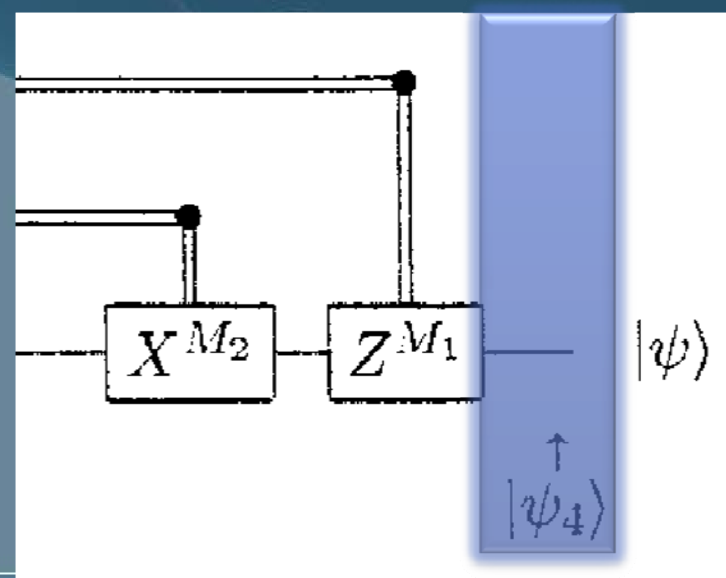
Outcomes

$$\begin{pmatrix} \alpha & \beta \\ \beta^* & 1-\alpha \end{pmatrix},$$

$$U_X^\dagger \begin{pmatrix} 1-\alpha & \beta^* \\ \beta & \alpha \end{pmatrix} U_X = \begin{pmatrix} \alpha & \beta \\ \beta^* & 1-\alpha \end{pmatrix}$$

$$U_Z^\dagger \begin{pmatrix} \alpha & -\beta \\ -\beta^* & 1-\alpha \end{pmatrix} U_Z = \begin{pmatrix} \alpha & \beta \\ \beta^* & 1-\alpha \end{pmatrix},$$

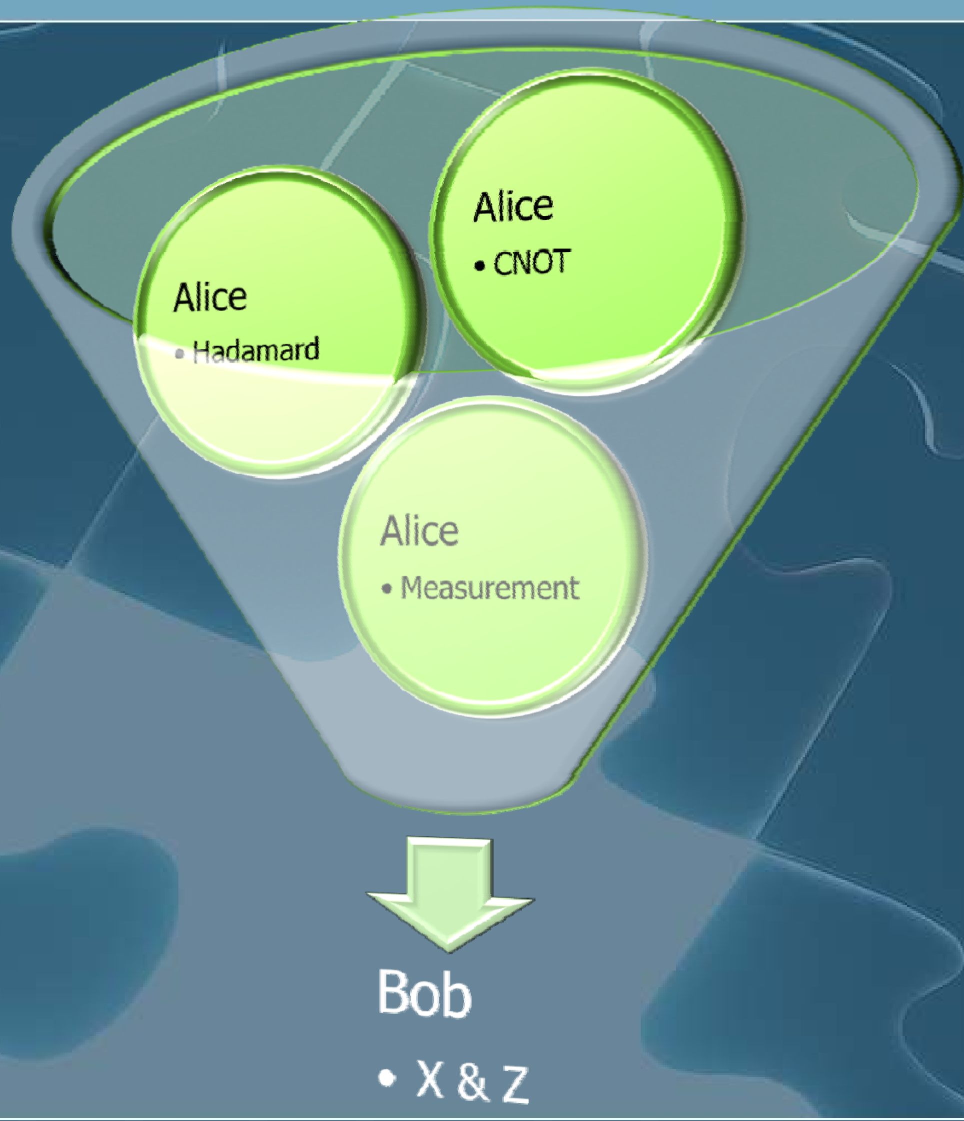
$$U_Z^\dagger U_X^\dagger \begin{pmatrix} 1-\alpha & -\beta^* \\ -\beta & \alpha \end{pmatrix} U_X U_Z = \begin{pmatrix} \alpha & \beta \\ \beta^* & 1-\alpha \end{pmatrix}$$

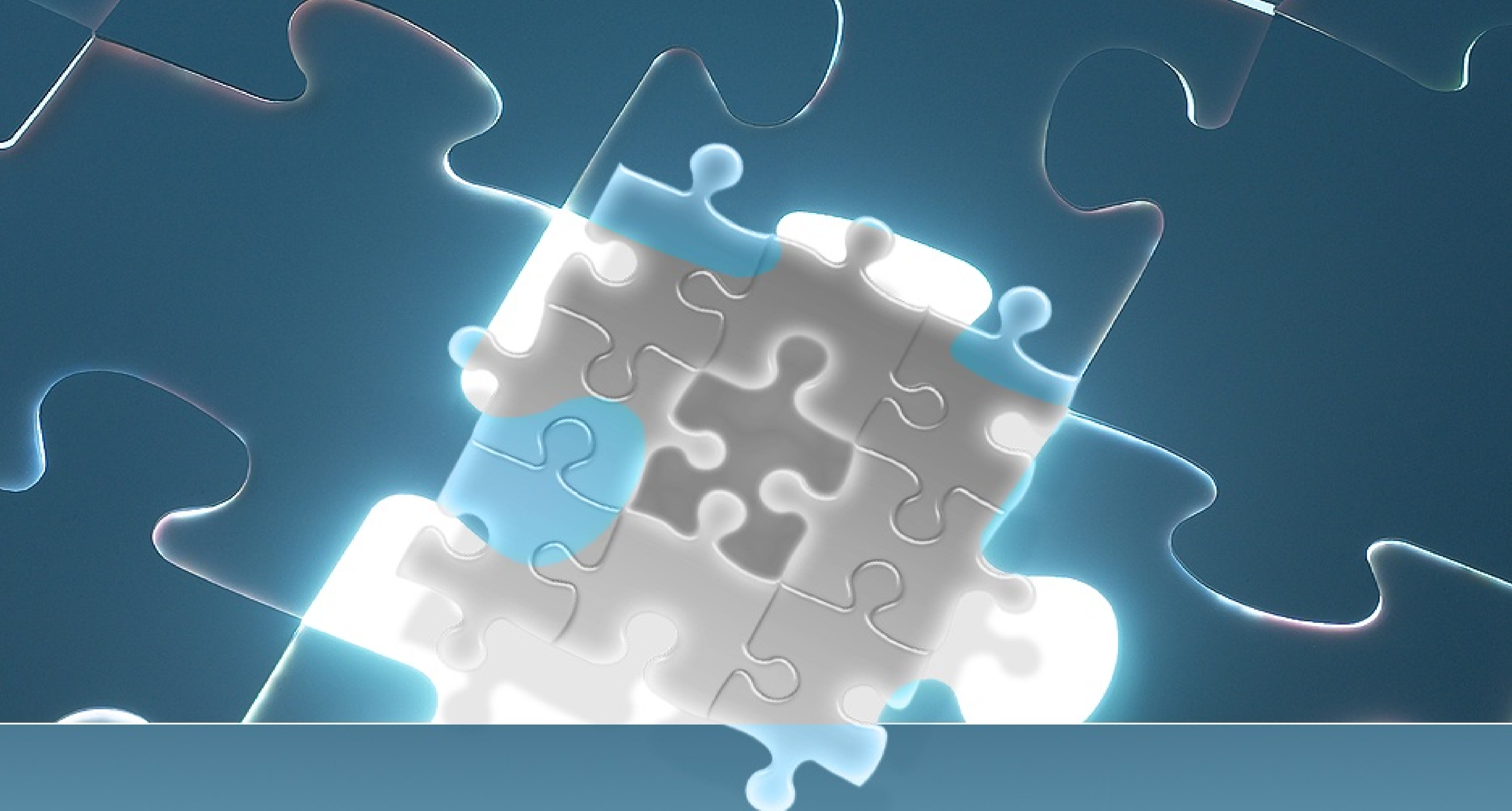


# Data leakage?

- If the classical channel has been spied?
  - Does the information reveals anything about the input?
  - 00, 01, 10, 11 with equal probability of ¼!
  - Safe!
- Without Alice's information?
  - If Bob's qubit has been stolen?
  - Safe!
- Classical channel + Bob's qubit = Bob 2#

# Summary





# Thank you

zhuzeyuan@hotmail.com