Three Steps to Make the Traffic Circle Go Round

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Summary

With growing traffic, control devices at traffic circles are needed: signals, stop/yield signs, and *orientation signs*—a special sign that we designed.

We create two models—one macroscopic, one microscopic—to simulate transport at traffic circles. The first models the problem as Markov chain, and the second simulates traffic by individual vehicles—a "cellular-automata-like" model.

We introduce a multi-objective function to evaluate the control. We combine saturated capacity, average delay, equity degree, accident rate and device cost. We analyze how best to control the traffic circle, in terms of:

- placement of basic devices, such as lights and signs;
- installation of orientation signs, to lead vehicles into the proper lanes; and
- self-adaptivity, to allow the traffic to auto-adjust according to different traffic demands.

We examine the 6-arm-3-lane Sheriffhall Roundabout in Scotland and give detailed suggestions for control of its traffic: We assign lights with a 68-s period, and we offer a sample orientation sign.

We also test smaller and larger dummy circles to verify strength and sensitivity of our model, as well as emergency cases to judge its flexibility.

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Introduction

We develop two models to simulate traffic flow in a traffic circle. The macroscopic model uses a Markov process to move vehicles between junctions, while the microscopic model concentrates on the behavior of each vehicle, using a modified cellular-automata algorithm. The outcomes of these two approaches show great consistency when applied to a real scenario in Scotland.

We characterize a “good” traffic control method in terms of five main objectives and combine them with an overall measure.

We employ a genetic algorithm to generate the final control method, in particular to determine the green-light period. We also consider the ability to deal with unexpected affairs such as accidents or breakdowns.

General Assumptions

- The geometric design of the traffic circle cannot be changed.
- The traffic circle is a standard one (at grade) with all lanes on the ground, that is, no grade separation structure.
- The flow of incoming vehicles is known.
- People drive on the left (since the example later is from the UK).
- Pedestrians are ignored.
- Motorcycles move freely even in a traffic jam.

Terminology and Basic Analysis

Terminology

- **Junction:** an intersection where vehicles flow in and out of the traffic circle.
- **Lane:** part of the road for the movement of a single line of vehicles. The number of lanes directly affects flow through the circle by limiting entrance and exit of vehicles. However, since both the conventional design and real-time photos suggest that vehicles exit easily, our model ignores restrictions on outward flow.
- l_0: the number of lanes in the traffic circle.
- **Section:** part of the traffic circle between two adjacent arms.
- **Yield/stop signs:** A yield sign asks drivers to slow down and give right of way; a stop sign asks drivers to come to a full stop before merging.

- **Orientation sign:** a sign indicating the lane for vehicles to take according to their destination.
- **Traffic light:** a signaling device using different colors of light to indicate when to stop or move. A traffic light with direction arrows performs much better [Hubacher and Allenbach 2002], so we are inclined to use such a light. However, compared to yield/stop signs, traffic lights slow vehicle movement. At the same time, however, even at a remote motorway traffic circle with few pedestrians, a a traffic-light malfunction will probably lead to an accident [Picken 2008].
- **Cycle period:** the time in which a traffic light experiences exact stages of all three colors. An optimal cycle period is critical whenever traffic lights are employed. The method we use is called the *Webster equation* [Garber and Hoel 2002]. The value that we use in our model is calculated as 68 s.
- **Green-light period:** the time that a traffic light keeps green in one cycle.
- **Timestamps:** a sequence of characters denoting the start/end time of red/green lights.

A Glance at Sheriffhall Roundabout

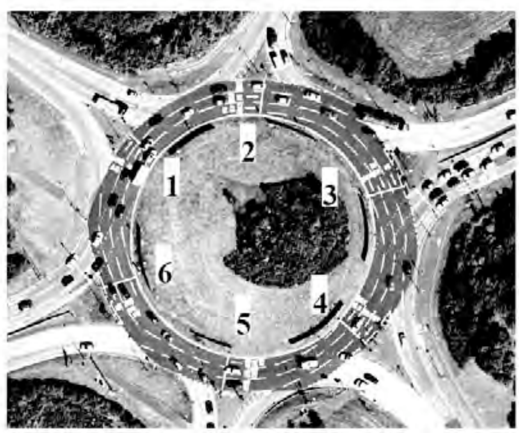


Figure 1. The Sheriffhall Roundabout. Source: Google Earth.

One characteristic of this traffic circle (**Figure 1**) is that the arms in the southwest (6) and northeast (3) directions have larger flow than the others. The arms in the north (2) and south (5) directions have two lanes, while the other four arms and the circle have three lanes. We model the traffic circle as a ring with an inner radius of 38.9 m and an outer radius of 50.4 m.

We use the origin-destination flow (**Table 1**) given by Yang et al. [Maher 2008]. Since the traffic demand is far from saturated, we experiment on different scalings of this inflow matrix, specifically, multiples by 1.2, 1.4, 1.6, and 1.8.

Table 1.
Origin-destination flows (vehicles/hr) at Sheriffhall Roundabout.
Source: Wang et al. in Maher [2008].

From\To	1	2	3	4	5	6
1	-	0	0	188	77	67
2	0	-	8	37	41	95
3	2	0	-	119	79	1007
4	338	129	63	-	0	208
5	116	124	142	0	-	16
6	90	172	988	236	10	-

Simulation Models

Model I: The Macroscopic Simulation

Usually, we do not know where each vehicle enters and exits the circle; we know only the numbers of vehicles coming in and out of each arm, so we adopt a macroscopic simulation.

We first combine the lanes in the sections and arms together and regard them as one-lane roads. We then explain how the multilane simulation works.

Assumptions

- Vehicles in the same section of the circle are distributed uniformly in the section.
- The arrival rate at each arm is constant in the period that we simulate.
- For simplicity, we consider an ideal round traffic circle (**Figure 2**). The macroscopic simulation itself does not depend on the shape of the circle.

Sections and Arms

We divide the traffic area into sections and take vehicles in the same section as a whole. We label the sections and the arms as in **Figure 2**. Associated to section i are the quantities:

1. Number num_i^t of vehicles in the section at time t.
2. Number arm_i^t of vehicles waiting to enter through one arm at time t.
3. The maximum number cap_i^t of vehicles that can enter the traffic section through one arm per unit time.

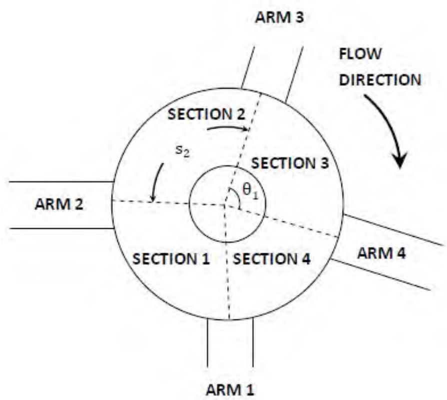


Figure 2. Sample traffic circle.

A Markov Process

The traffic state at time $t + 1$ depends only on the traffic state at time t, so traffic is a Markov process. To describe the state of the whole system, only the quantities num_i^t and arm_i^t are needed. To implement the simulation, we must determine num_i^{t+1} and arm_i^{t+1}, for $i = 1, 2, \ldots, n$.

In principle, we can calculate the transition probability matrix; but not in our problem. For a traffic circle with four arms/sections and each holding up to 10 vehicles, the number of traffic states is 10^8.

Considering this sobering fact, we use the expectations $\overline{\text{num}}_i^t$ and $\overline{\text{arm}}_i^t$, instead of the actual distribution of cars, to denote a state.

The Simulation Process

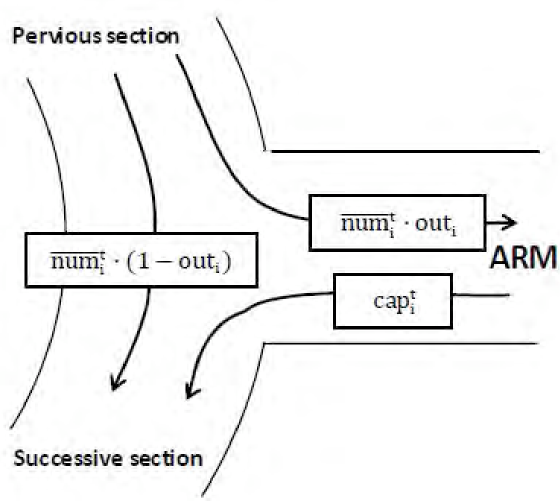


Figure 3. Flows at a junction.

- $\overline{\text{num}}_i^t \times \text{out}_i^t$ vehicles leave the circle from section i. The ratio out_i^t drops when $\overline{\text{num}}_i^t$ approaches its capacity.
- To deal with the junction, there are two streams $\overline{\text{num}}_i^t \cdot (1 - \text{out}_i)$ and cap_i^t trying to flow into the next section. If there is a traffic light, only one of them is allowed. If stop/yield sign is used (at the arm side, for example), then only a small fraction of cap_i^t can flow in. This fraction is denoted by the *disobey rate* α_{stop} or α_{yield}.
- An inflow of in_i newly-arrived vehicles runs into arm i.

Multilane Traffic Circle

We assume that vehicles do not change lanes within arms or sections, which means that they can change lanes only at junctions.

To treat lanes differently, we need to know what proportion of vehicles pass through each lane. At each junction, the outflow for a given lane is distributed into successive lanes according to their popularity.

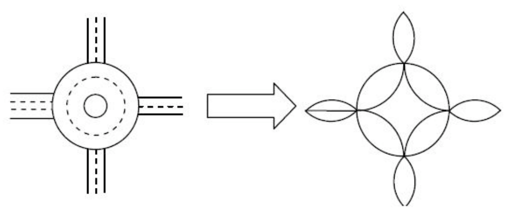


Figure 4. A two-lane circle divided into lanes. Each arc on the right denotes a single lane.

Model II: The Microscopic Simulation

Partially inspired by sequential cellular automata, we adopt a microscopic model. The traffic circle is divided into l_0 lanes. Vehicles are points with polar coordinates but with discrete radius values. We model the behavior of each individual vehicle, with the help of some general principles:

- **Traffic coming in:** As described in **Table 1**, the number of vehicles per hour is given in a matrix $(a_{i,j})_{n\times n}$. We use a Poisson distribution with mean $a_{i,j}/T$ to describe the incoming vehicles from arm i to arm j.
- **Lane choosing and changing:** For a specific vehicle from arm i to arm j, the driver has a desired ideal lane to be in. The hidden principle is [SetupWeasel 1999]: The more sections the vehicle has to pass before its exit, the more likely the driver will wish to take an inner lane, both in the arm and in the circle. We adopt this rule.

- **Vehicle speed:** We define a maximum speed and a maximum acceleration for vehicles, and record the speed individually. The principles for a vehicle to accelerate or decelerate are:
  - When a vehicle faces a red light or other vehicles, its speed decreases to zero.
  - When a vehicle changes lanes, it decelerates.
  - Otherwise, a vehicle attempts to accelerate up to maximum speed.
- **The function of a yield sign:** When a vehicle faces a yield sign, it checks whether the lane is empty enough for it to enter the junction. If not, the vehicle waits until it is empty enough—but with a disobey rate α_{yield}, it ignores the sign and scrambles. Naturally, this reaction affects the accident rate.
- **The function of a stop sign.** When a vehicle faces a stop sign, it should stop instantaneously. At the next time step, it functions as if at a yield sign; the only difference is that it will accelerate from a zero speed. The disobey rate is α_{stop}.
- **The effect of traffic lights:** Just like normal.

We discretize time and follow the rules above for each vehicle after it comes to the circle. We calculate the average traversal time for a vehicle, as well as the accident rate (by the total number of touches of vehicles). A vivid view of the simulation result is presented in **Figure 5**.

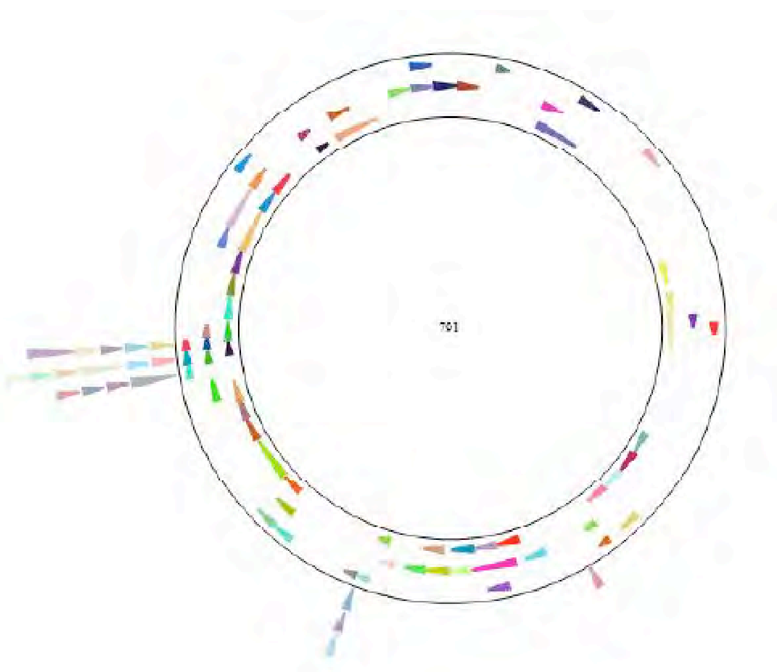


Figure 5. The vehicles around the traffic circle.

Comparison and Sensitivity Analysis

Results

We use the two different models to simulate a real traffic circle: The Sheriffhall Roundabout in Scotland. We use the traffic-light configuration in Maher [2008]. For simplicity, we consider only the average time needed for a vehicle to traverse the traffic circle. This value is 42.7 s for Model I and 41.6 s for Model II. The two results are close, so we can believe that the actual traversal time is around 42 s.

Sensitivity

We analyze sensitivity by running the program with modified parameters (see **Table 2**).

Table 2.

Sensitivity test of simulation mode.

Parameter	variation	model I	model II
v_{max}	+10%	-2.6%	-8.5%
	-10%	10.5%	11.1%
l_0	+1	-19.6%	-16.4
	-1	121%	65.2%
r_{out}	+10%	-7.3%	-3.9%
	-10%	1.1%	3.3%
Traffic flow	+10%	10.6%	7.0%
	-10%	-3.0%	-6.7%

The two models give similar sensitivity results. The average passing time is relatively insensitive to all the parameters except l_0. This is reasonable, since the number of traffic lanes in the circle affects the passing time significantly.

Model II is a random simulation, which enables us to calculate the standard deviation of the traversal time, which is no larger than 3% of the mean.

Complexity

The time complexity of the algorithms for the two models is proportional to the maximum number of vehicles that the circle can hold and the number of iterations. In practice, 1,000 iterations suffice.

Model I is a little simpler than Model II, since we do not need to trace each individual. Conversely, Model II needs more a priori information than

Model I. Since the two models are consistent and give similar results, we adopt Model II for further study.

The Multi-Objective Function

Basic Standards

We want to include both subjective evaluations (such as the feelings of drivers) and objective measures (such as the expense of devices). Also, the standards should be calculated from available data. We choose five evaluation standards:

- **Saturated flow capacity:** The threshold flux to avoid backing up traffic on the arms.
- **Average delay:** The difference between the average time to traverse the traffic circle and the time to traverse an empty one.
- **Equity degree:** A multi-arm traffic circle may distribute the incoming flow inequitably, to the annoyance of drivers. The relative difference in average delay is the equity degree.
- **Accident expectation:** The average number of accidents per vehicle.
- **Device cost:** The total expense of traffic signs and lights.

How the Objectives Are Affected

Saturated Flow Capacity

A yield sign is likely to work effectively, since it seldom causes unnecessary stops for vehicles. A stop sign, however, at least adds the acceleration/deceleration delay to every vehicle rushing inside. The efficiency of a traffic light is highly related to its green-light period. Fixed-period lights sometimes block vehicles from entering an empty circle, while adaptive ones can work according to conditions.

In fact, a traffic circle with yield signs at all junctions bears the heaviest traffic in the simulations above, and traffic lights are left with great potential to improve in optimization.

Average Delay

The average delay is controlled by the incoming flow. The delay time will increase rapidly when traffic starts to congest. In our model, the delay time of a vehicle is calculated when it exits the traffic circle. When this delay time is considered in the overall objective, there should be penalties on congestion, which is calculated from the current flow and the saturated flow capacity.

Equity Degree

Equity degree is calculated directly from the delay time distribution. Not only the flow distribution but also the total flux contributes to the equity degree, since high flux may lead to unexpected distribution failures.

Accident Expectation

We assume that each kind of signal reduces accidents by a specific percentage; we use data from Hubacher and Allenbach [2002], Transport for London... [2005], and Fitzpatrick [2000].

Device Cost

This expense is based on the numbers of each kind of signal.

The Combined Objective: The Money Lost

Now we come to a combined objective, the *combined expense (CE)*, that takes into account expense and economic loss, which we attempt to minimize.

The prices of traffic-control devices are easy to find [Traffic Light Wizard n.d.; TuffRhino.com n.d.]. Apart from the expense of maintenance and operation, we calculate the average operating cost per hour for each kind of device. Since traffic lights consume much electricity, we ignore the money spent on other types of devices. A traffic light is expected to cost \$0.23/hr [Wang 2005; Ye 2001].

For accident expense losses, we take data from an annual report of a local traffic office on average loss per accident [Hangzhou Public Security Bureau... 2006] and set

$$\text{Accident loss} = \$630 \times \text{Flux}.$$

The average delay time must be accompanied by a cost of delay. According to the Federal Highway Administration [2008], about \$1.20 per vehicle is lost in a delay of 1 hr:

$$\text{Delay expense} = \$1.20 \times \text{Flux} \times \text{Average delay time}.$$

The unused part of saturated capacity takes care of any extra incoming traffic; we set its value as

$$\text{Capacity bonus} = 5\% \times \$1.2 \times (\text{Saturated capacity} - \text{flux}) \times (\text{Average delay time}),$$

in which 5% is the probability of an unexpected vehicle coming.

Equity degree (ED) is a tricky component in the determination.The most annoying situation is to keep two "main arms" open to traffic by sacrificing all other arms. Equity degree is estimated to be a function of the number of arms n:

$$\text{Reference equity degree (RED)} = \sqrt{\frac{n(n-2)}{2(n-1}}.$$

The equity degree will be normalized by this reference and appear in a penalty on delay expense:

$$\text{Corrected delay expense} = \text{Delay expense} \times \left(1 + \frac{\text{ED}}{\text{RED}}\right).$$

The combined index is then calculated as

$$\text{CE} = \text{Corrected delay expense} - \text{Capacity bonus} + \text{Accident loss} + \text{Device cost},$$

which serves as the final objective function that we use in the following optimization.

Application: Evaluate Typical Arrangements

We take a glance at three general control methods: pure traffic light, stop sign only, or yield sign only.

We first normalize the five objectives, converting values to an interval between 0 and 1, from worst to best. A superficial look at the radar chart of **Figure 6** raises doubt about the expensive traffic lights. However, traffic lights are superior in controlling the accident rate, while the two signs may be hazardous by accelerating the flow. The convoluted relationship is clear when we compare their CE values, in **Table 3**.

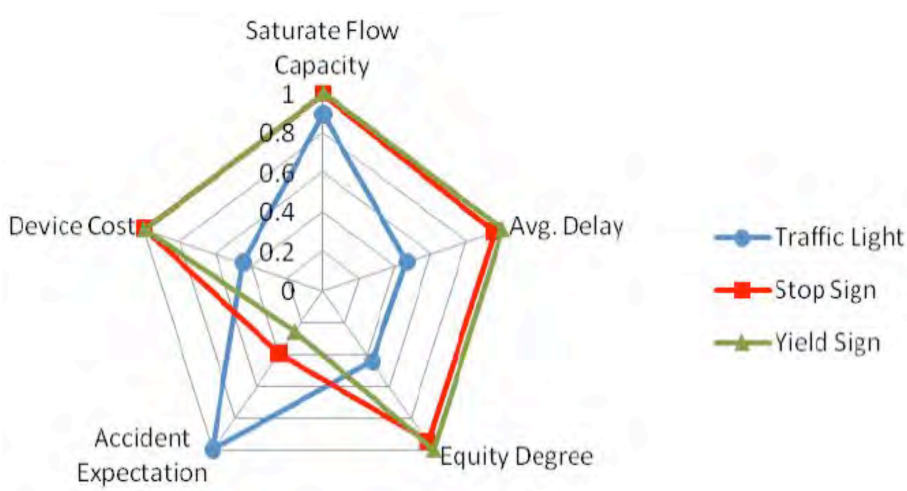


Figure 6. A view of 5 objectives of 3 general control methods.

The results above suggest that traffic lights are worthwhile for heavy traffic. Optimization, however, needs more insight.

Table 3.
Combined expense for 3 typical control methods.

Control Method	The Combine Expense (US$/hour)
Traffic light	66.76
Stop sign	103.29
Yield sign	116.61

Optimization Model

The All-Purpose Solution

Because the objective function is calculated in our simulation model, an analytical form for it is difficult to obtain. In such a situation, a quasi-optimal solution is welcome, and approximation algorithms become candidates.

In this problem, a normal approximation algorithm can fall into local maxima. However, some high-level technique can be used such as simulated annealing or—what we use—a genetic algorithm. Specifically, the traffic controls in different junctions are used as genes. The configuration of a traffic circle is a vector of genes, containing all the devices used in different junctions. **Table 4** gives details.

Table 4.
Explanation of the genetic algorithm used for optimization.

Process	Explain
Breeding	Combine the traffic control methods of two different configurations.
Mutation	Randomly mutate the traffic control in a single junction.
Evolution	Locally adjust the traffic controls in all junctions, and seek for better solution.

We consider three kinds of traffic control devices: 1) traffic lights, 2) yield/stop signs, and 3) *orientation signs*, a special kind of traffic sign that we designed ourselves. We call the first two basic devices.

Step 1: Basic Device and Timestamp Choice

A traffic junction can be equipped with any one of the following five devices: 1) traffic light, 2) yield sign in the circle, 3) yield sign at the entrance, 4) stop sign in the circle, and 5) stop sign at the entrance. Besides, the timestamps of red/green lights for traffic lights are also changeable.

Sheriffhall Roundabout

Considering all potential variables above, we run our program against the Sheriffhall Roundabout, using the origin-destination flow data in **Table 1**, assuming that this flow matrix remains fixed over a one-hour period. The solution of our program shows that *traffic lights should be used rather than stop/yield signs*; otherwise, the accident rate will be dramatically higher.

In **Figure 7**, green (light) represents right of way for vehicles from the incoming road, and red (dark) indicates right of way for vehicles in the circle. The optimal configuration creates a long period of red light for all junctions and allows digestion of vehicles quickly during the interval. This configuration accelerates the flows but has a lower saturated flow capacity, as **Table 5** summarizes.

Arm	G	R
1	0	14
2	7	14
3	38	18
4	3	19
5	3	13
6	38	14

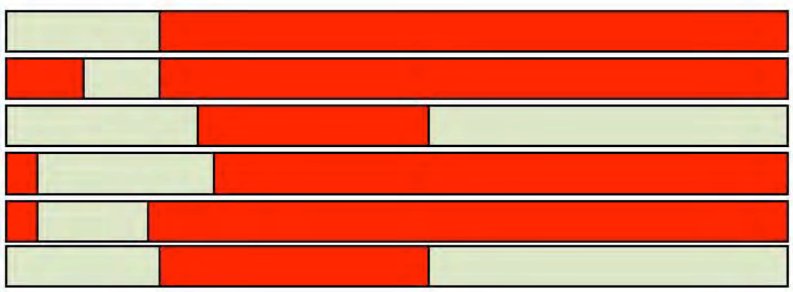
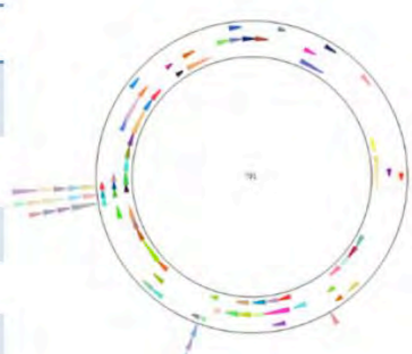


Figure 7. The traffic light timestamps in 6 junctions (green (light) vs. red (dark)). Period = 68 s (calculated in assumption). Original flow information is used.

Table 5.

The multi-objectives of the optimal configuration of Sheriffhall, original flow.

Objective	Value
Saturated Flow Capacity	6904 vehicles / hour
Average Delay	42.763 seconds / vehicle · hour = 62.04$ / hour
Equity Degree	0.3187
Accident Expectation	4.63$ / hour
Device Cost	1.38$ / hour
Combined expense	**78.98$ / hour**



Sheriffhall Roundabout with 1.8 × Original Inflow

When the incoming flow density increases to 1.8 times as much, the optimal configuration shows a significant difference—see **Figure 8**.

Arm	G	R
1	0	12
2	10	18
3	49	15
4	38	57
5	41	52
6	10	45

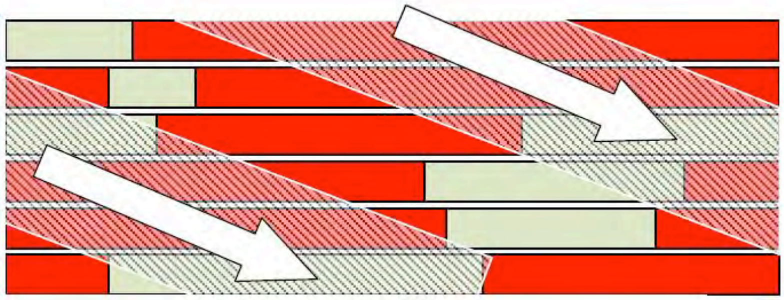
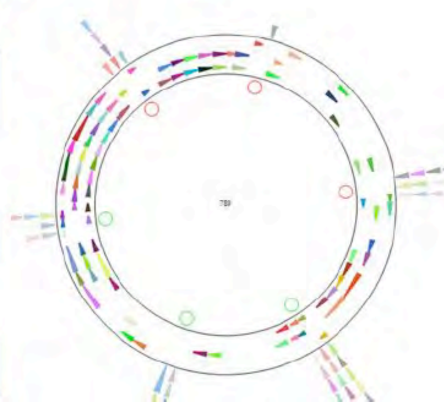


Figure 8. The traffic light timestamps in 6 junctions. Period = 68 s (calculated in assumption). Original flow × 1.8 is used.

In **Figure 8**, the green-light periods for all junctions are shortened to let the circle digest the greater number of incoming vehicles. There is no longer a long period with all junctions having a red light. As an alternative, there is free passage between Junction 3 and Junction 6 (shadowed stripe), which greatly increases the saturated flow capacity but reduces the traversal speed (see **Table 6**). To see why, one needs to look at the origin-destination flow **Table 1**, in which the flow between Junction 3 and 6 constitutes a significant portion of all the inflows. The white stripe in **Figure 8** actually gives a good opportunity for vehicles to travel between them.

Table 6.
The multi-objectives of the optimal configuration of Sheriffhall, original flow × 1.8.

Objective	Value
Saturated Flow Capacity	8354 vehicles / hour
Average Delay	81.278 seconds / vehicle· hour = 117.91$ / hour
Equity Degree	0.3042
Accident Expectation	5.41$ / hour
Device Cost	1.38$ / hour



Step 2: Orientation-Sign Placement

Normally, the number of lanes in a traffic circle and the number of junctions are not equal. In some countries, a hidden rule [SetupWeasel 1999] is: *the vehicle nearer its exit should stay left* (Remark: we are driving on the left!) We refine this rule.

Let there be n arms. Suppose that a vehicle is at Junction a $(1 \le a \le n)$, and its destination is b $(1 \le b < n)$ junctions farther on. We manage two variables lower_a^b and upper_a^b so that such a vehicle is suggested to stay in the range $[\text{lower}_a^b, \text{upper}_a^b]$. Our aim is to distribute vehicles into lanes to minimize congestion. To optimize these intervals $[\text{lower}_a^b, \text{upper}_a^b]$, we use a genetic algorithm again.

Figure 10 demonstrates the effect of the orientation sign of **Figure 9** in reducing the average delay, for different amounts of inflow. As the number of incoming vehicles increases, the positive effect of our orientation sign becomes evident. The configuration without orientation sign has saturated flow capacity 8354 (**Table 6**), and this number has increased to 8812 with the help of this newly-introduced sign. In short, *the very last potential capacity has been extracted in our model.*

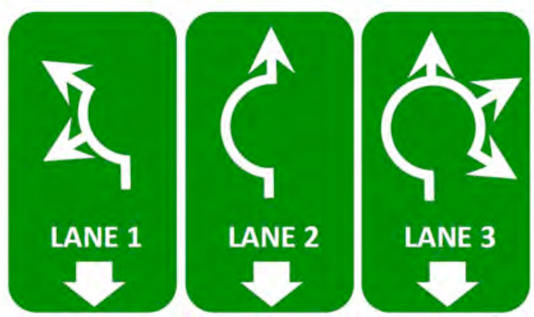


Figure 9. The orientation sign over the junction entrance. (At junction 3, with 1.8 $\times$ original inflow.)

Step 3: Time Variance and Self-Adaptivity

Origin-destination flows vary from morning to evening. The easiest way to handle this is to run our previous program with different traffic demand information for different time periods. Actually, we can go further, and make the traffic control self-adaptive by using traffic lights.

Given the traffic light timestamps calculated in Step 1, and assume that in the following hour the traffic demands change to new values. We select the original configuration as our seed, and carry out the genetic algorithm to gain a similar but better solution. **Figure 11** gives an example.

One may find that the timestamps change little and hence will not significantly affect vehicles already in the circle. As night falls, traffic demands

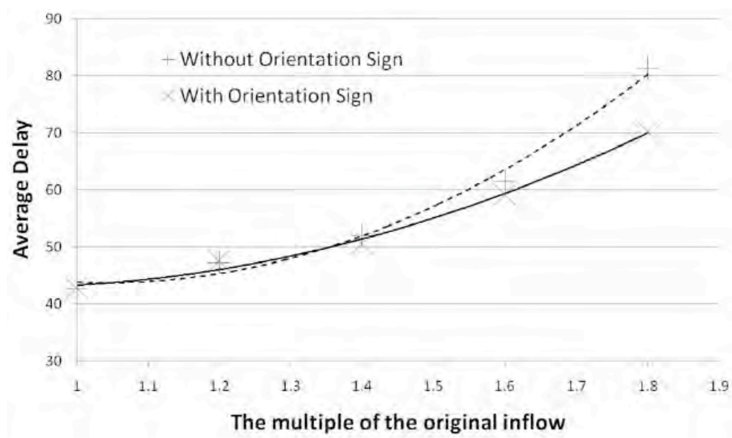


Figure 10. The magic effect of the orientation sign.

Arm	G	R
1	0	12
2	10	18
3	49	15
4	38	57
5	41	52
6	10	45

Arm	G	R
1	0	14
2	11	18
3	51	15
4	21	46
5	41	53
6	12	45

Figure 11. Self-adaptivity as inflow drops in 1 hr from 1.8 to 1.4 $\times$ original inflow.

fall off, and the traffic lights could be replaced in effect by yield signs by switching the lights to flashing yellow, an international signal [Wikipedia 2009] to remind drivers to be careful.

Verification of the Optimization Model

The Circle at Work

Figure 12 shows that when the inflow is 1.8 times as high as in **Table 1**, the traffic circle still works.

Accuracy

As a follow-up study to verify the optimization model, we need to test it on different traffic circles. For lack of data, we create our own dummy

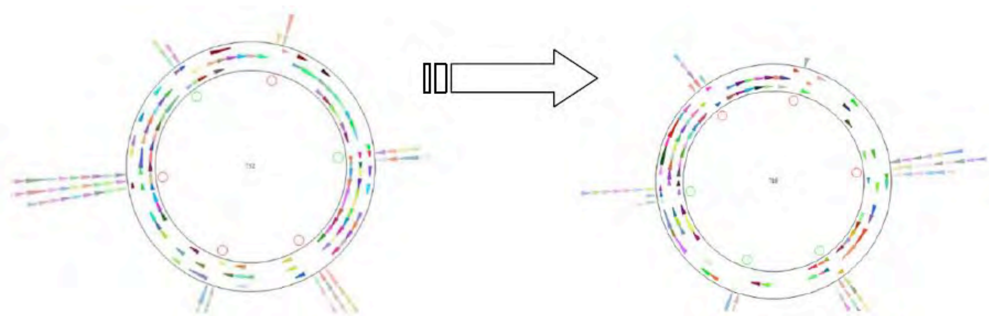


Figure 12. 39 seconds later, most of the vehicles waiting at Junction 6 move in.

traffic circles. In particular, we test a large traffic circle with 12 arms and 6 lanes, and the result shows that our model can deal with such large cases.

Testing on a dummy suburban circle with 4 arms and relatively lower traffic demand, we find as optimal solutions either in-circle stop signs or else a mixture of stop signs and traffic lights (**Figure 13**). In this example, the origin-destination flow between Junction 1 to Junction 3 is remarkably greater than all other pairwise flows.

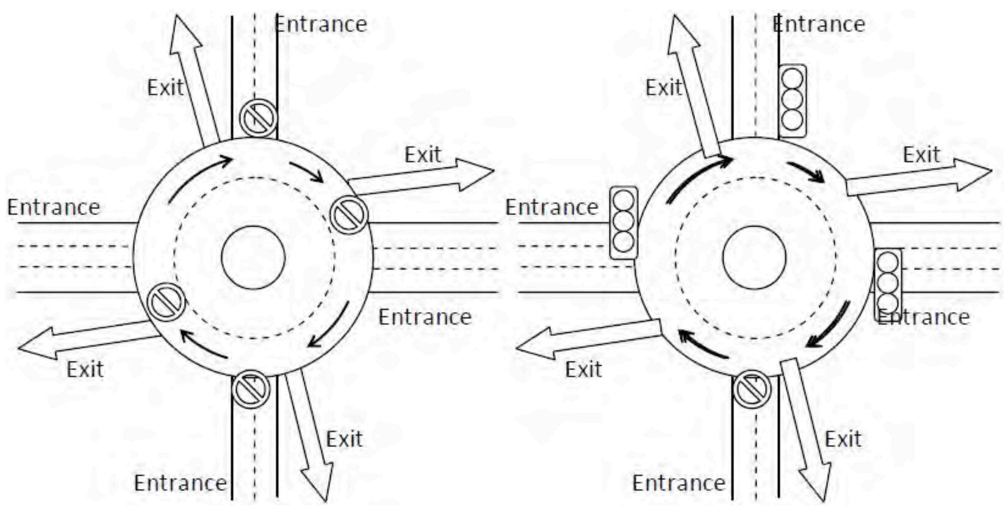


Figure 13. Two intuitive configurations generated by our model. The one on the left has two in-circle stop signs and guarantees fast pass from left to right; the one on the right has a mixture of traffic lights and stop signs.

Sensitivity

We tested sensitivity of our model by running it 50 times. **Table 7** shows the mean and standard deviations for runs against various level of inflow.

Table 7.
Sensitivity test of the optimization model.

The multiple of income flow	Average Delay	Standard Deviation
1.0 times	42.76 seconds / vehicle · hour	0.95 seconds / vehicle · hour
1.2 times	47.22 seconds / vehicle · hour	1.56seconds / vehicle· hour
1.4 times	51.99 seconds / vehicle · hour	2.54 seconds / vehicle · hour
1.6 times	61.54 seconds / vehicle · hour	3.81 seconds / vehicle · hour
1.8 times	81.28 seconds / vehicle · hour	8.30 seconds / vehicle · hour

Emergency Case

Our model can simulate an emergency. In **Figure 14**, one of the cars breaks down and block an entire lane. However, the traffic circle still works, but the average delay time has increased by 10 s.

The self-adaptivity of our model lets us adjust the light timestamps and reduce the traffic jam in an emergency case. However, because of limited time, we cannot describe the adaptivity here.

Figure 14. Breakdown of a vehicle slows the traffic, but traffic still circulates.

Conclusion

To estimate the overall performance of a traffic circle with a specific vehicle flow, we develop two simulation models. The first uses a Markov process to consider the entire flow, the second devotes its attention to the individual behavior of each vehicle.

We choose five objectives to evaluate the control method and convert them to a combined expense. We apply this standard to a real-life traffic circle with typical traffic control device setups.

We offer an optimization model to select traffic devices and determine the green-light period when traffic lights are used. In addition, we introduce orientation signs as a thoroughly new measure to bring efficiency. The flexibility of these solutions is proved when confronted with accidents.

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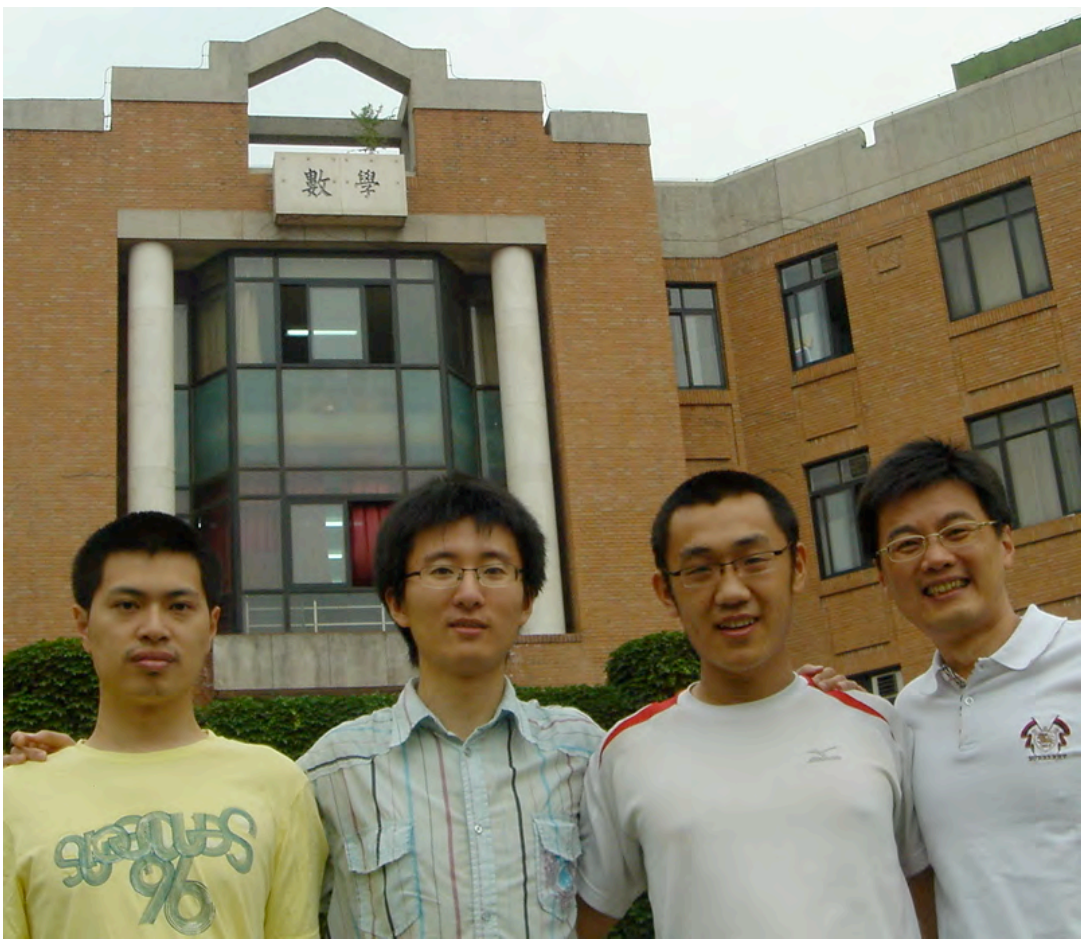

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