Mechanism Design with Approximate Valuations

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Mechanism Design

generate good outcomes for data you don’t have

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generate good outcomes for data you don’t have
by leveraging the players’
KNOWLEDGE and **RATIONALITY**

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classically

YES, if player’s self-knowledge is **EXACT**

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classically

YES, if player’s self-knowledge is **EXACT**

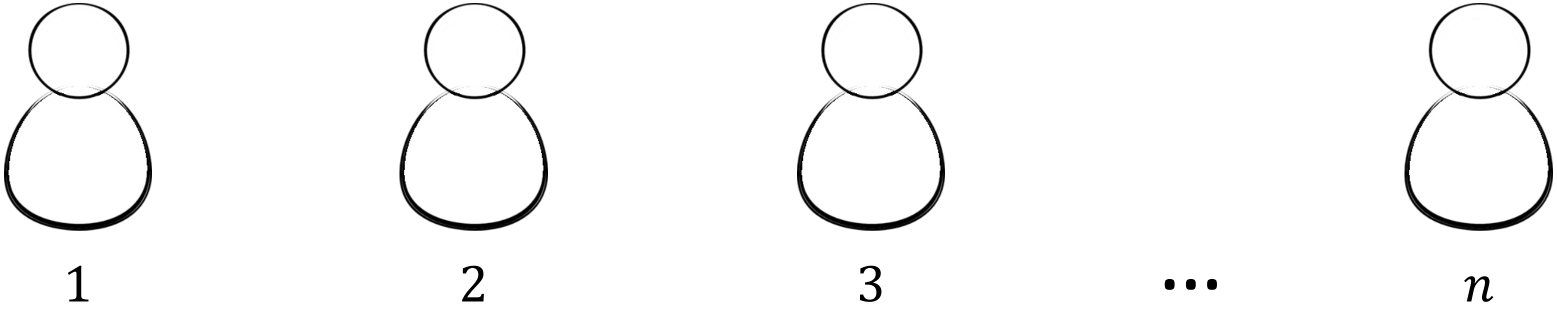
TODAY

YES, if player’s self-knowledge is **APPROXIMATE**
(in single-good auctions)

Rolex Auction

GOAL

give Rolex to player who values it the most (max. *social welfare*)



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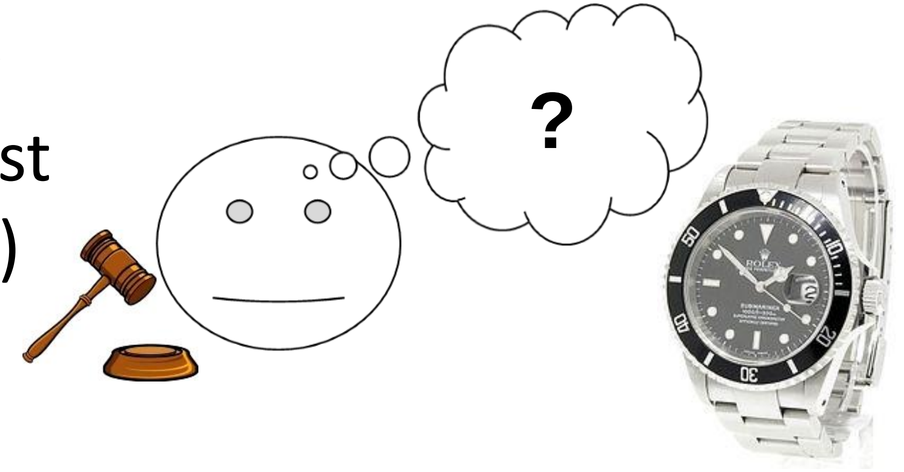

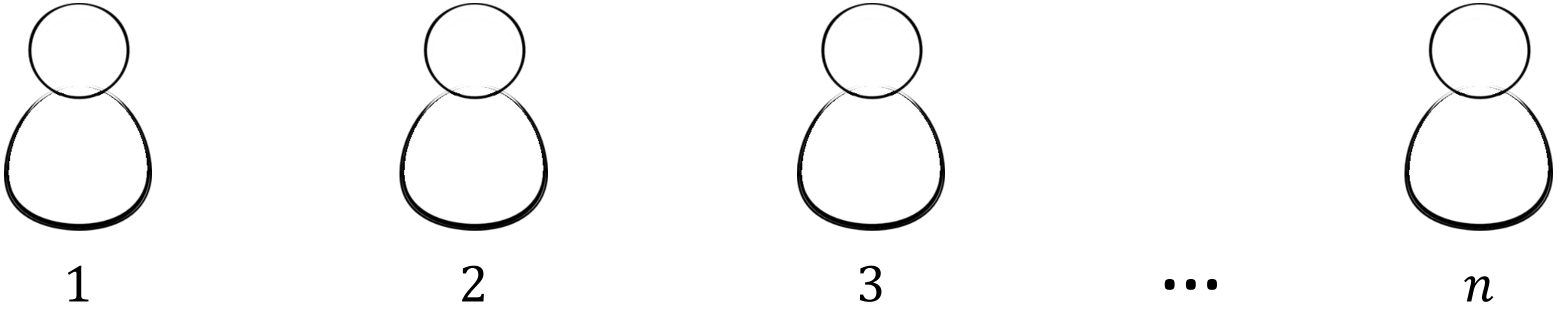

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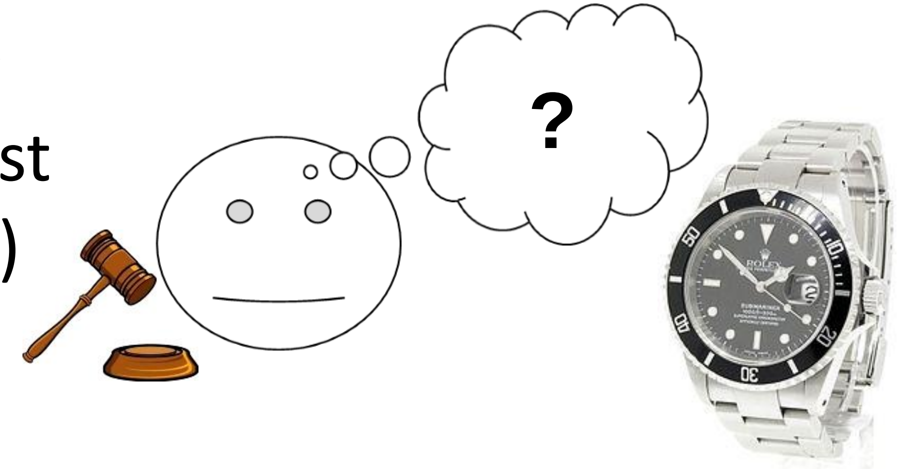
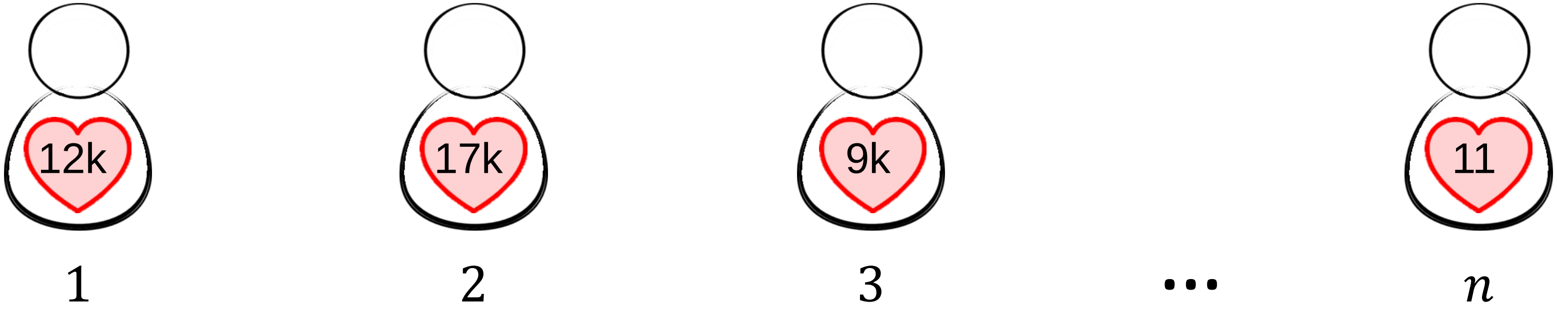
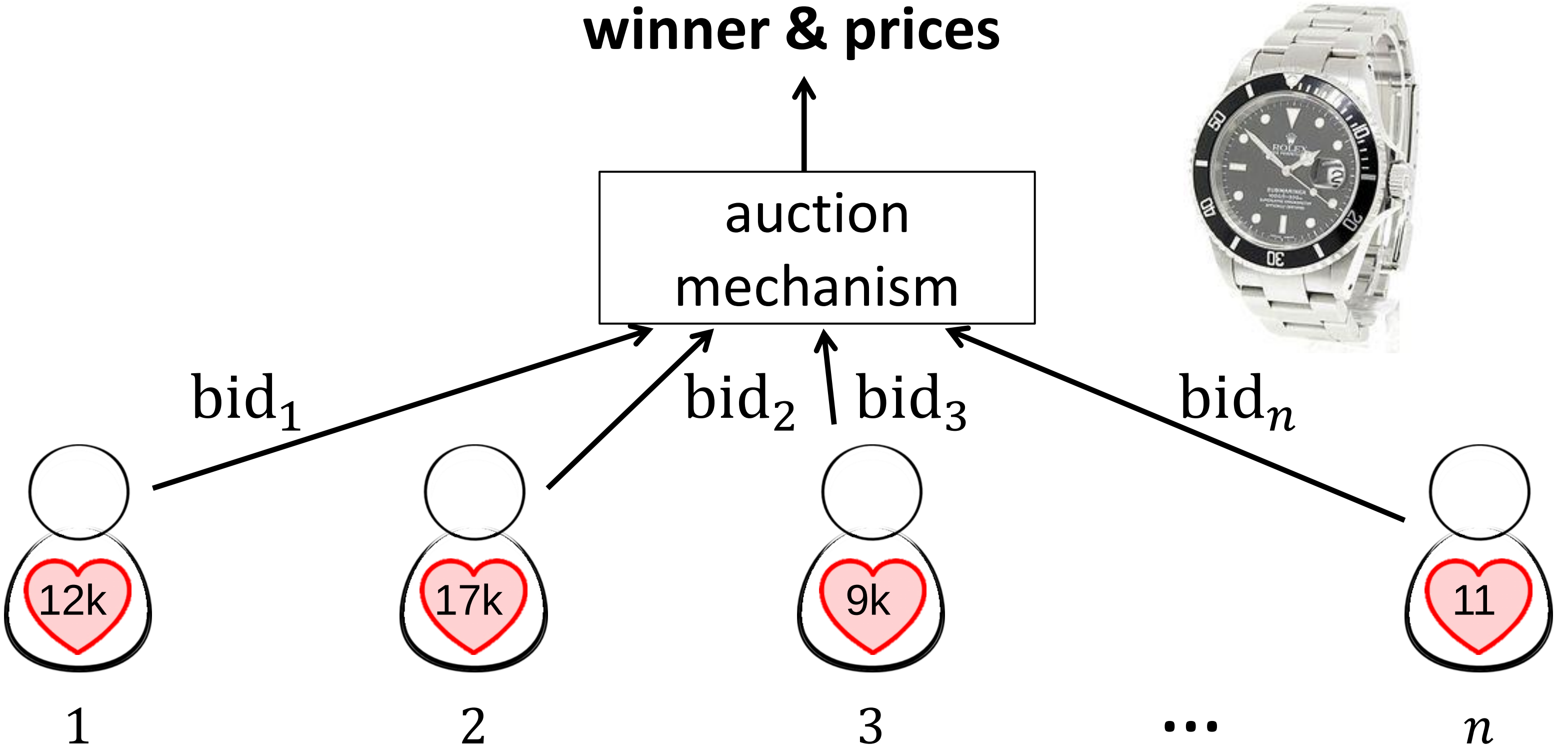




Rolex Auction



winner & prices

auction mechanism

bid_1 bid_2 bid_3 bid_n

12k 17k 9k 11

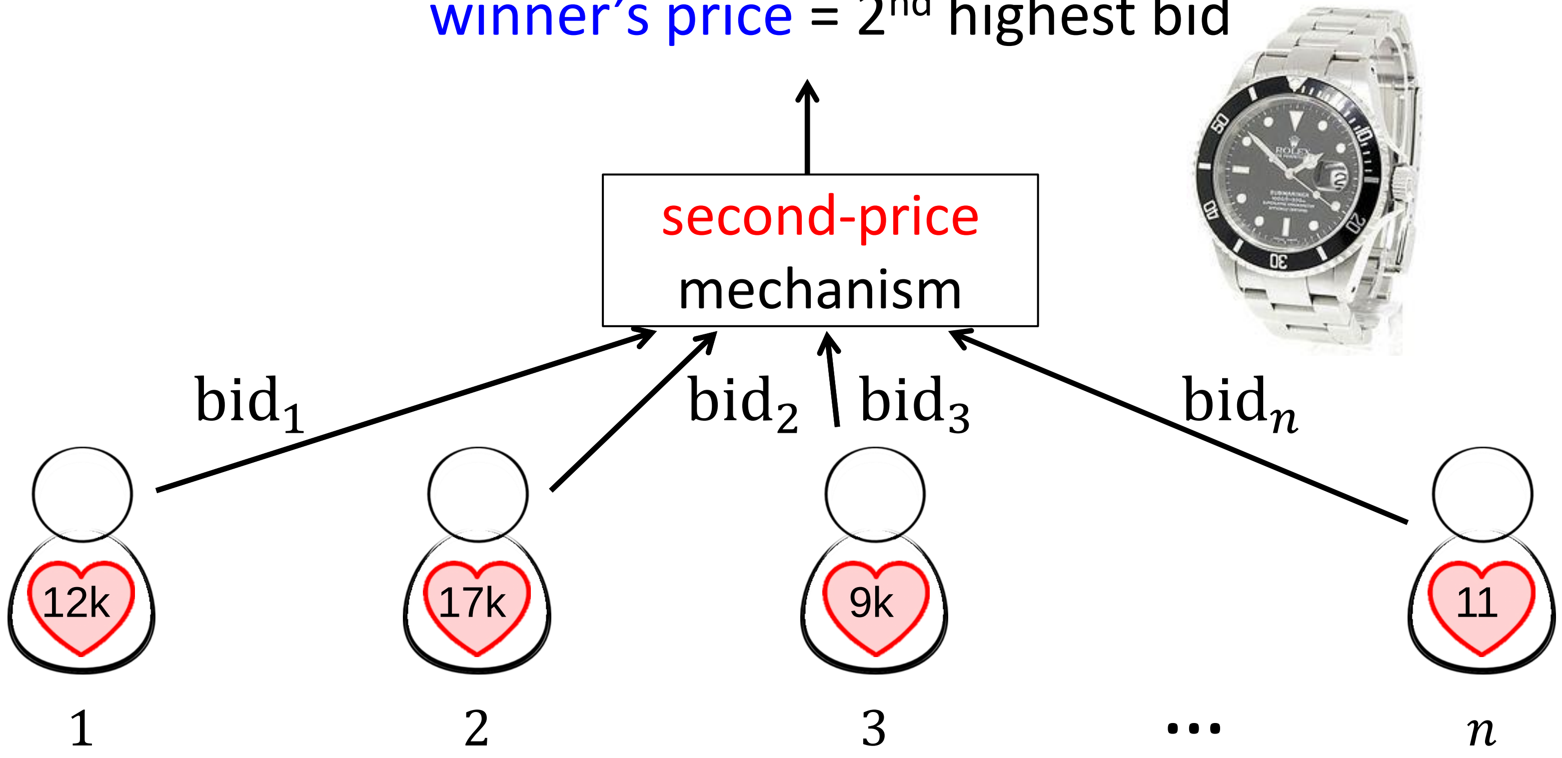
1 2 3 … n

use auction mechanism to
extract information from the players

Rolex Auction

winner = player with max bid

winner's price = 2^{nd} highest bid



second-price mechanism

bid_1 bid_2 bid_3 bid_n

Rolex Auction

winner = player with max bid

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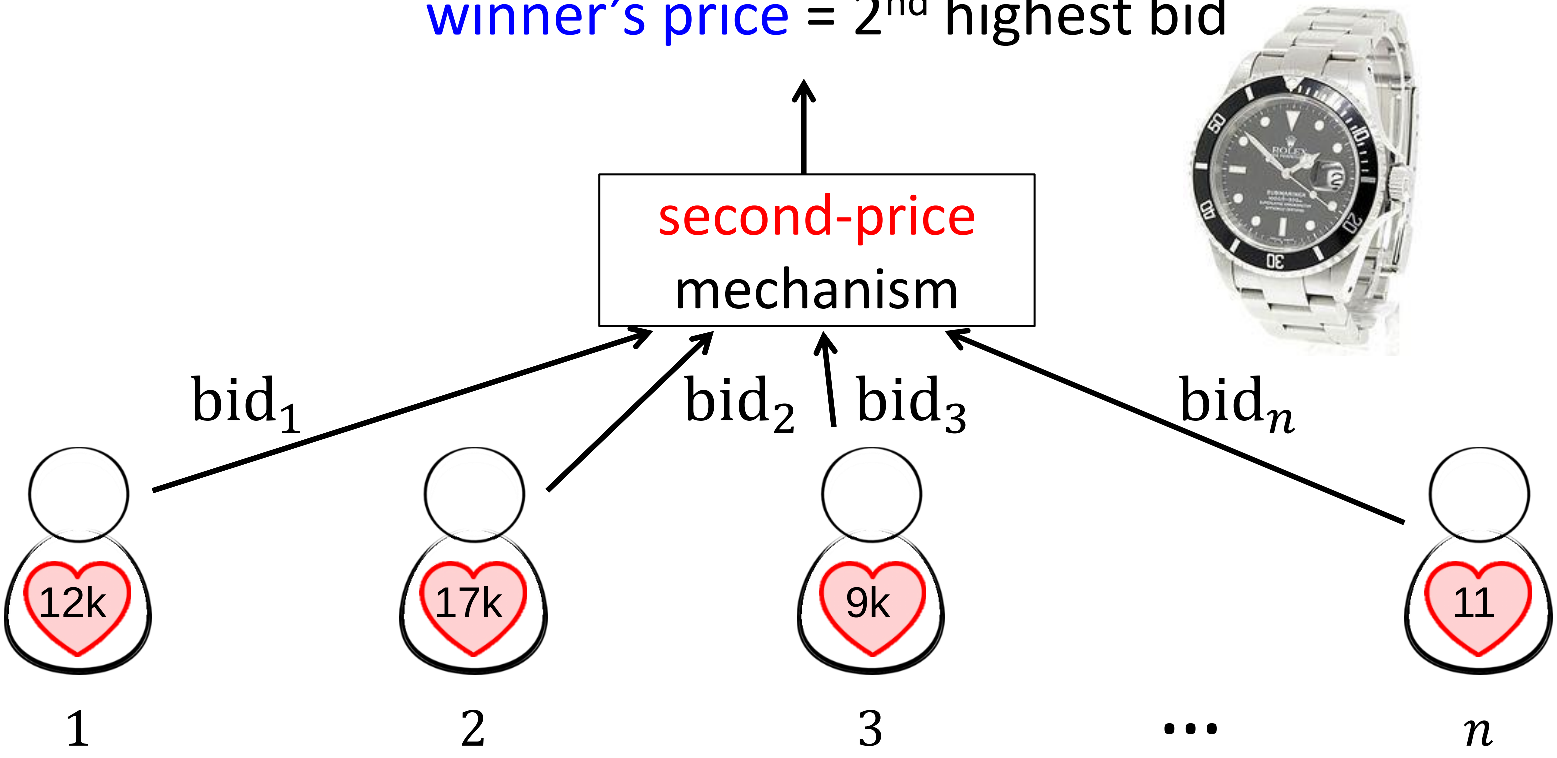
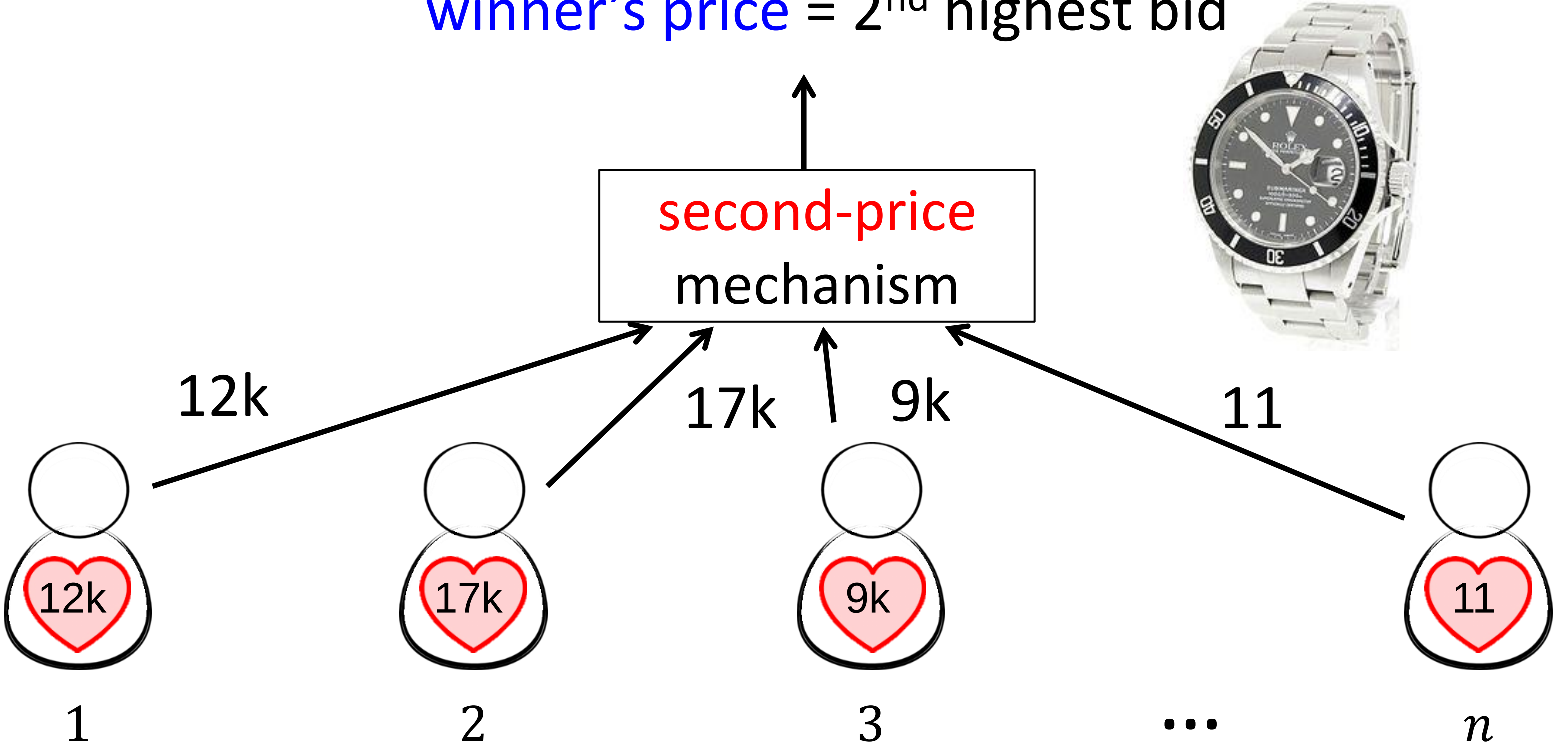


second-price mechanism

bidding true valuation is a (very weakly) *dominant strategy*

Rolex Auction

winner = player with max bid
winner's price = 2nd highest bid



bidding true valuation is a (very weakly) *dominant strategy*

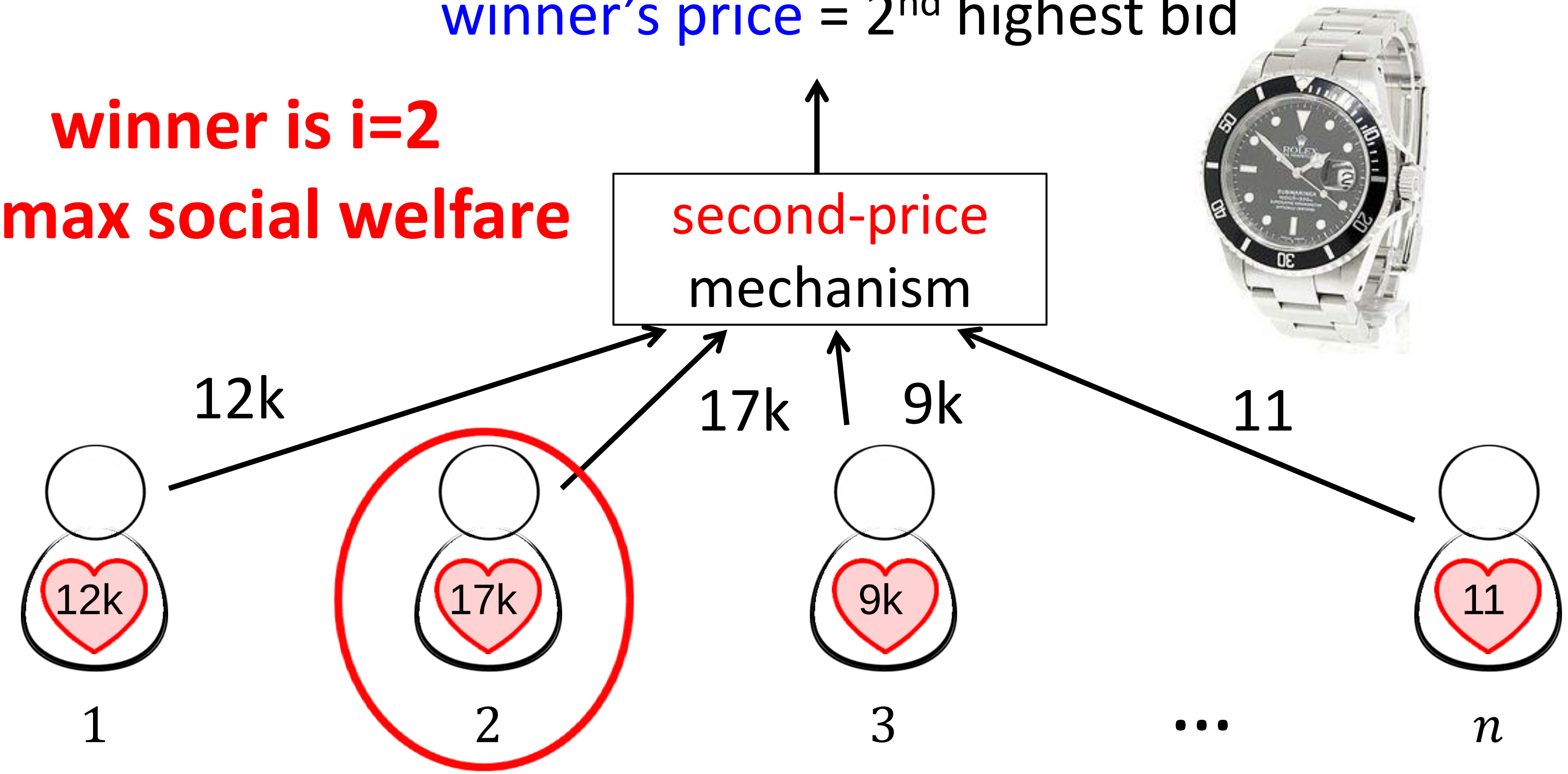
Rolex Auction

winner = player with max bid

winner's price = 2nd highest bid

winner is i=2

→ max social welfare



bidding true valuation is a (very weakly) *dominant strategy*

WARNING!

optimal performance from an

ASSUMPTION:

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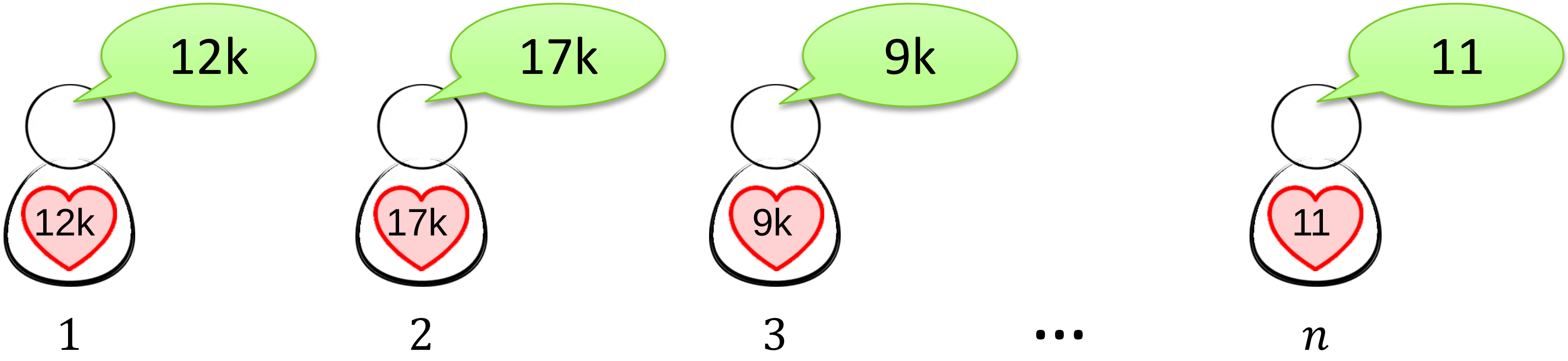
each player knows his own valuation exactly



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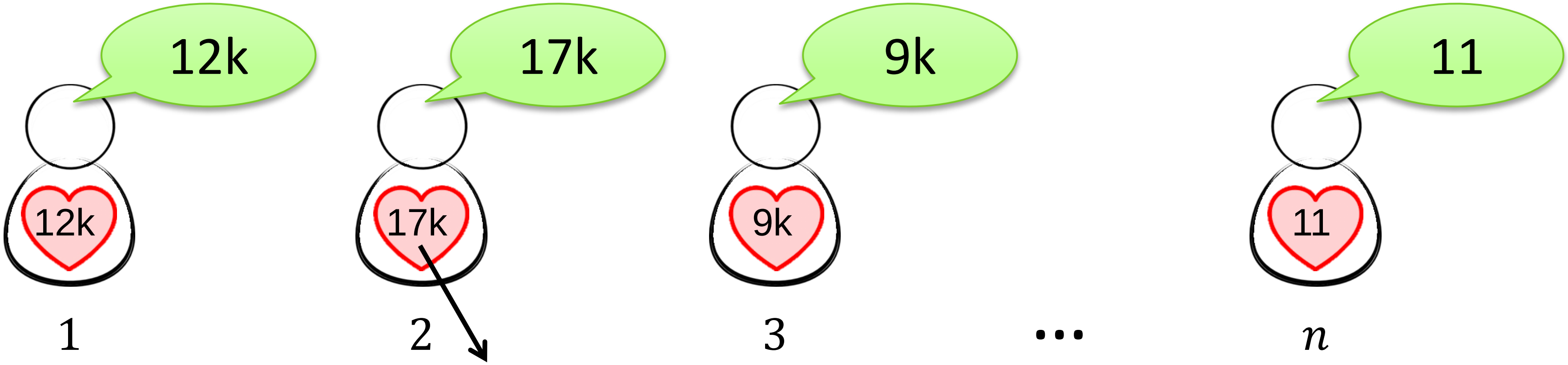

17k. Or 17.01k?

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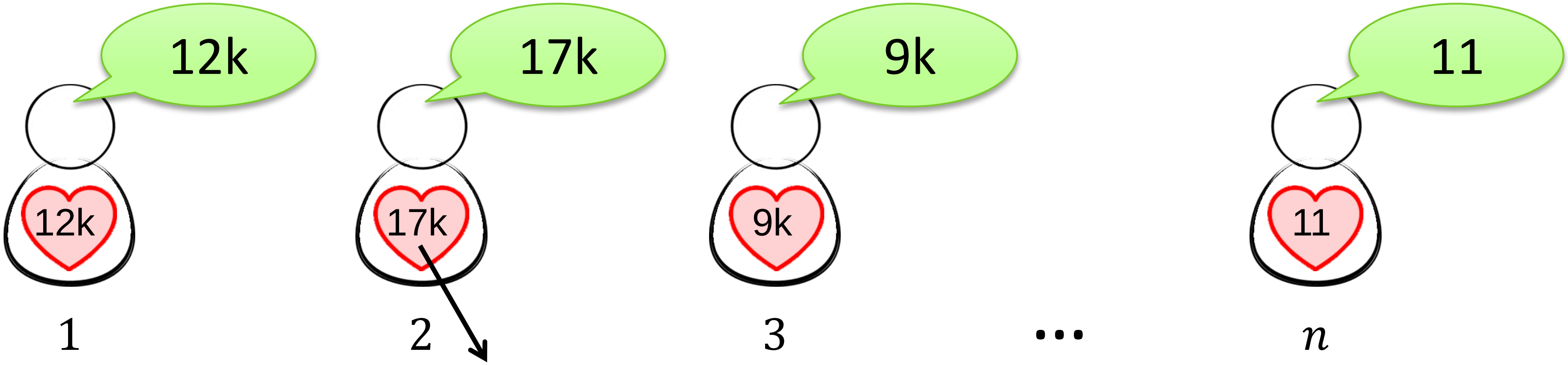

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Either:
(a) it does not make any difference
(b) exact knowledge is VERY strong assumption

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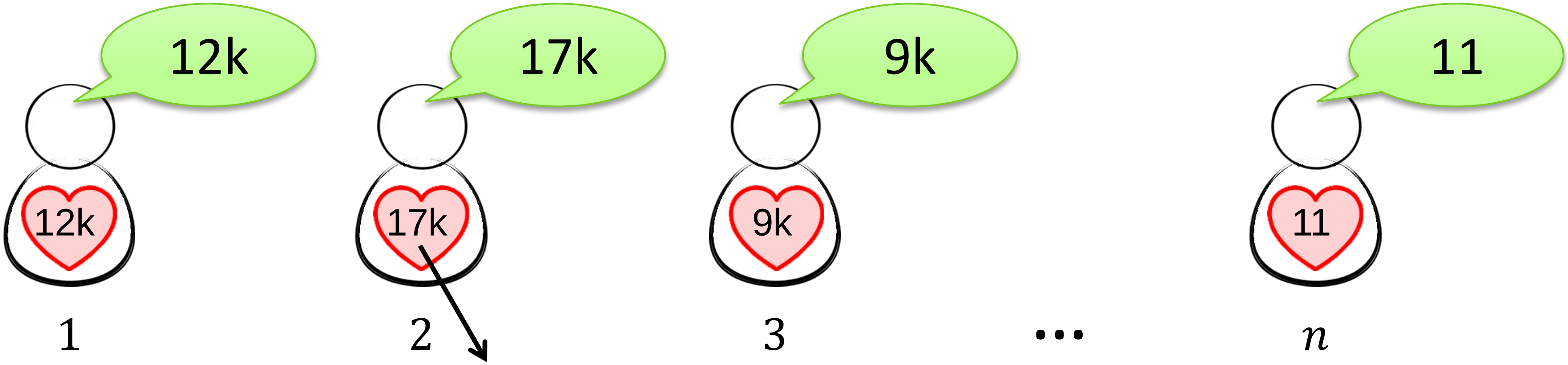

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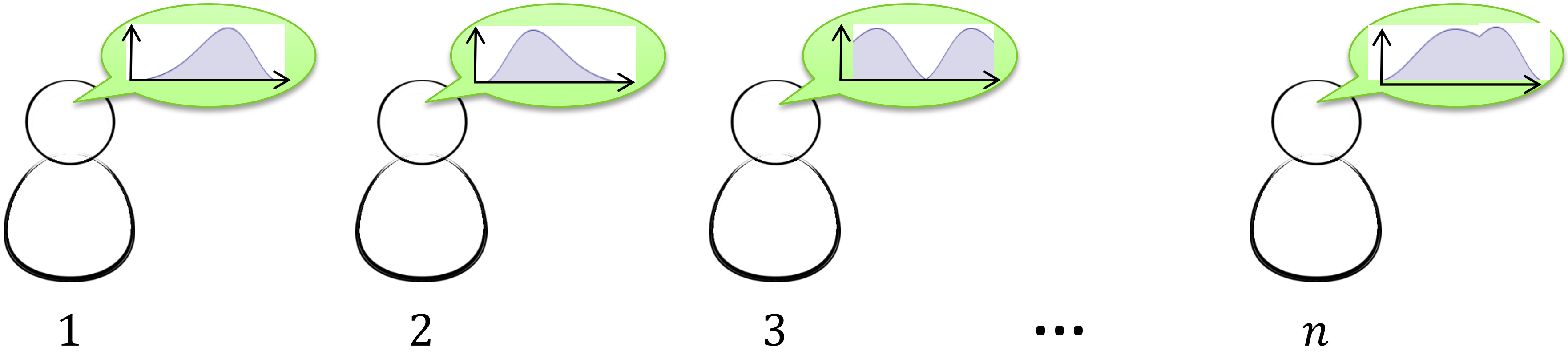
WEAKER ASSUMPTION: Bayesian?

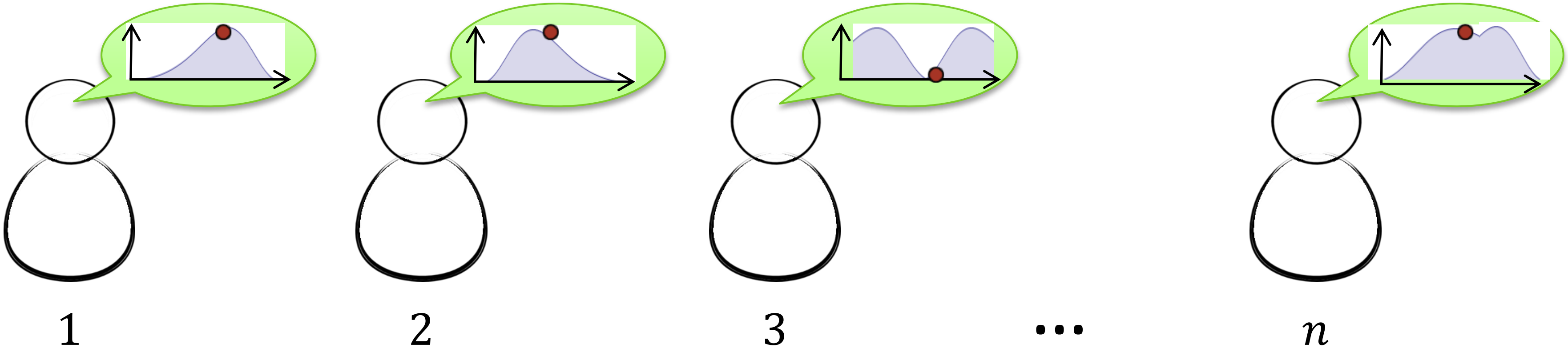
each player knows his own individual Bayesian



WEAKER ASSUMPTION: Bayesian?

each player knows his own individual Bayesian

same second-price mechanism:
just truthfully bid your expected value



WEAKER ASSUMPTION: Bayesian?
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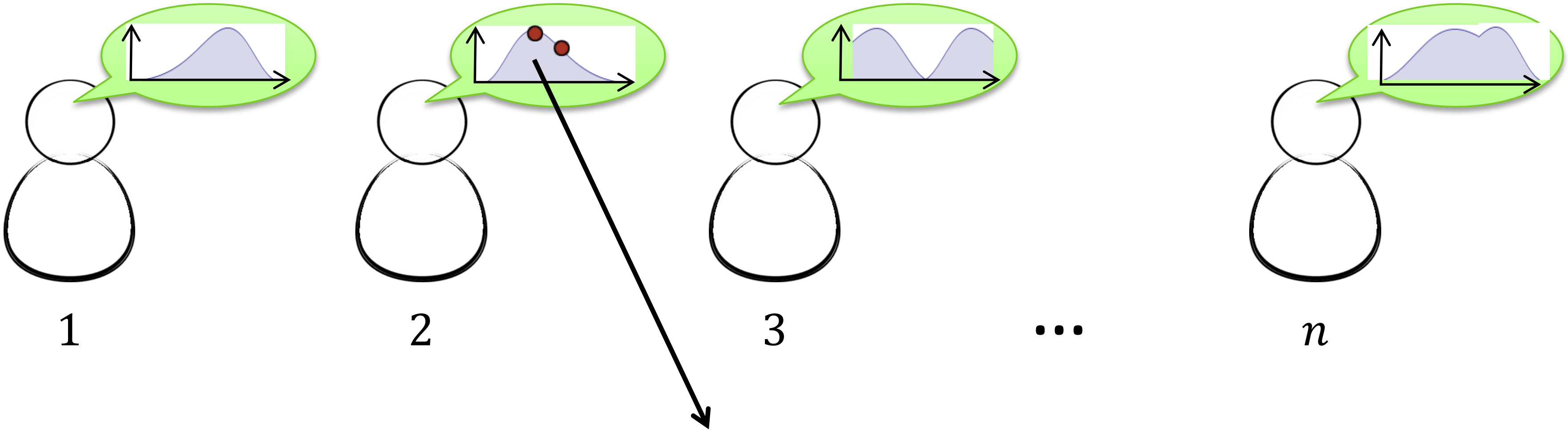


Does player 2 really know that Pr(16k) = 1.5 Pr(16.6k)?

If no, and it matters, still very strong!

NEED

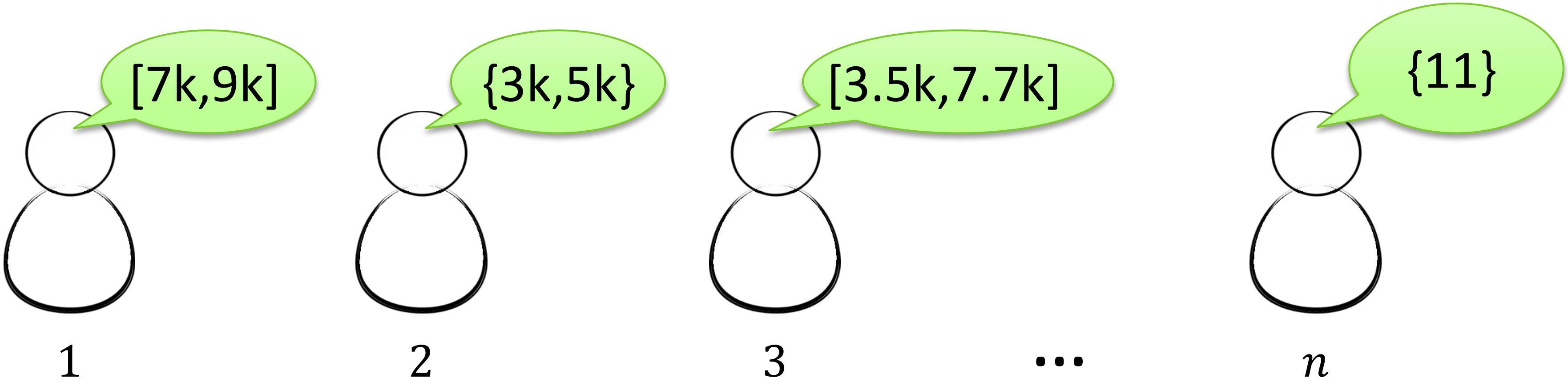
A

MORE CONSERVATIVE MODEL

OUR APPROXIMATE KNOWLEDGE MODEL

OUR APPROXIMATE KNOWLEDGE MODEL

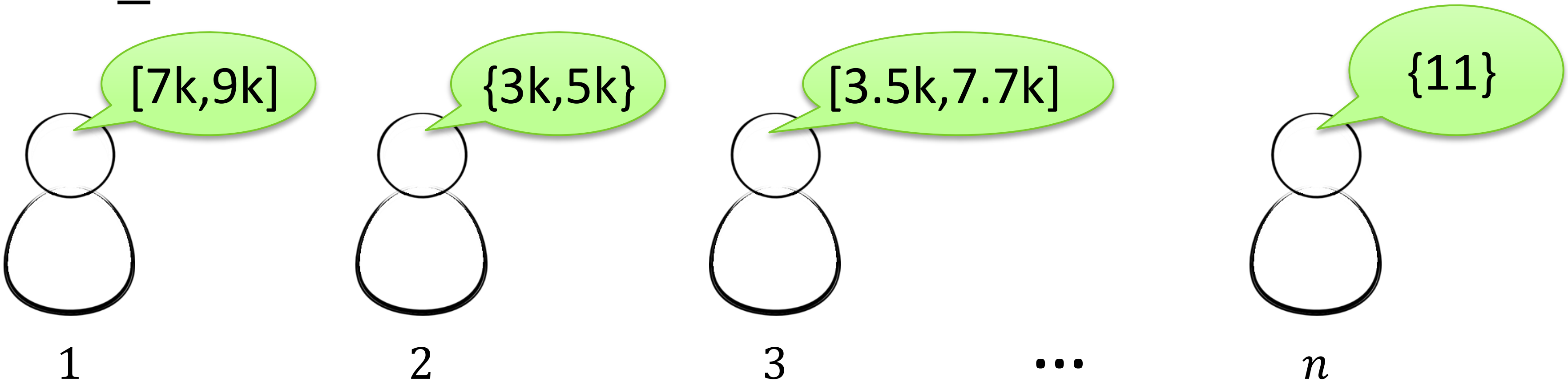
each player approximately knows his valuation



OUR APPROXIMATE KNOWLEDGE MODEL

each player approximately knows his valuation

$8\text{k} \pm 12.5\% \cdot 8\text{k}$

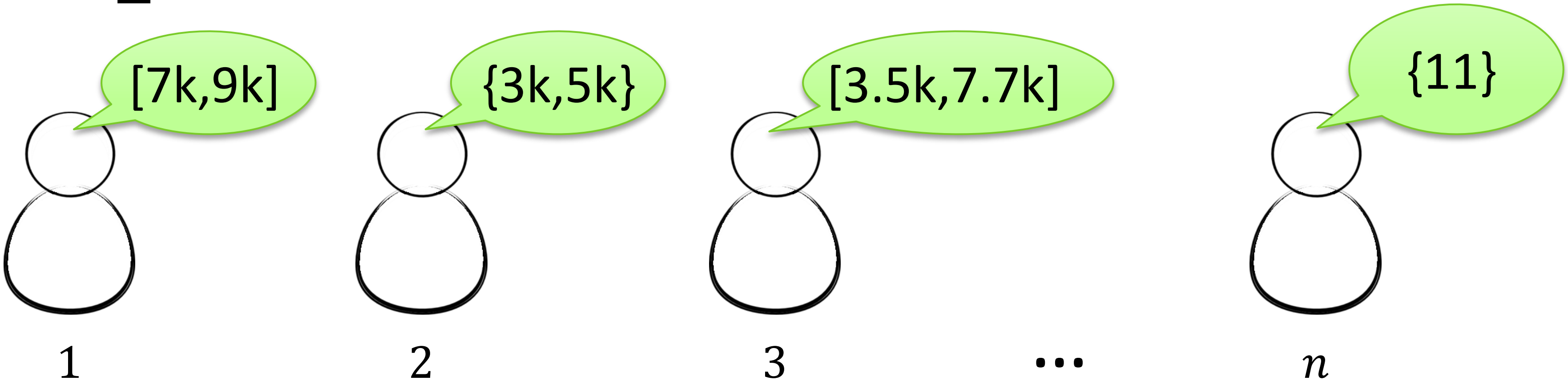


OUR APPROXIMATE KNOWLEDGE MODEL

each player approximately knows his valuation

subset of

$8\text{k} \pm 12.5\% \cdot 8\text{k}$ $4\text{k} \pm 25\% \cdot 4\text{k}$



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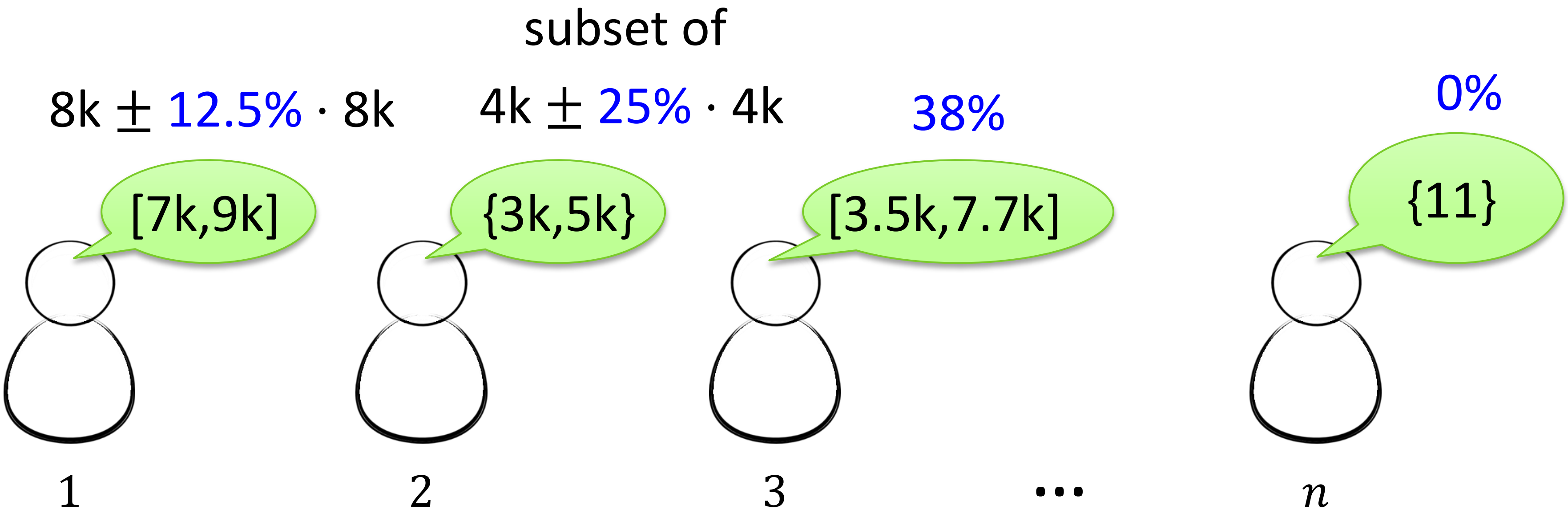

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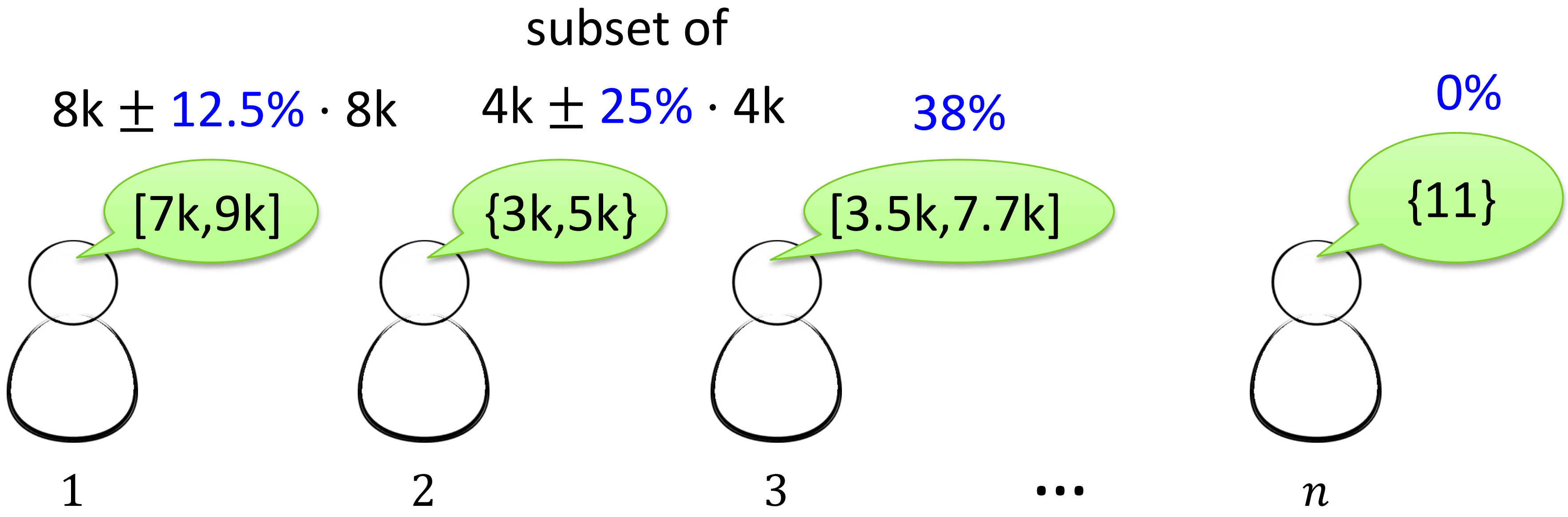
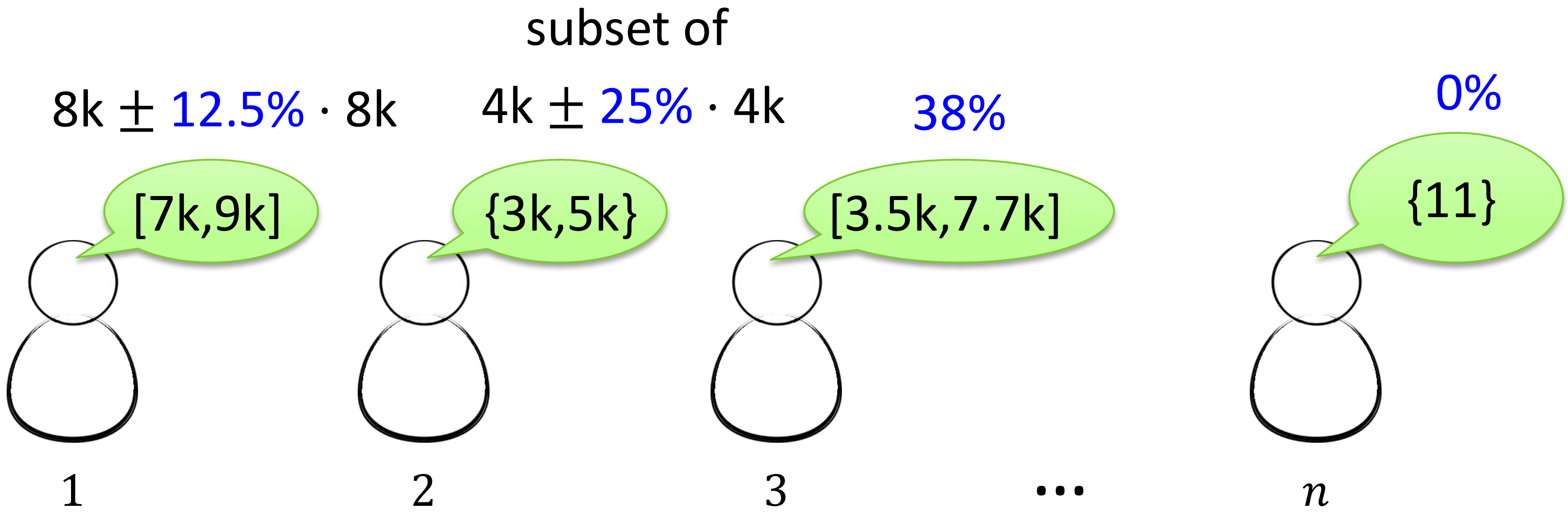


approximate knowledge induces *inaccuracy param.* δ

(above, all players within inaccuracy e.g. $\delta = 40\%$)

OUR APPROXIMATE KNOWLEDGE MODEL

each player approximately knows his valuation

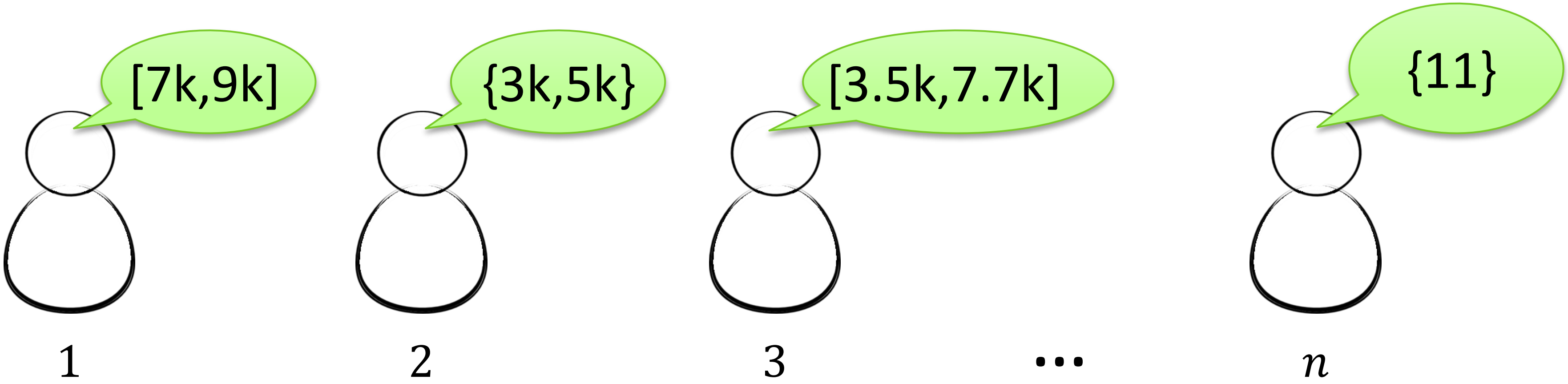


Q:

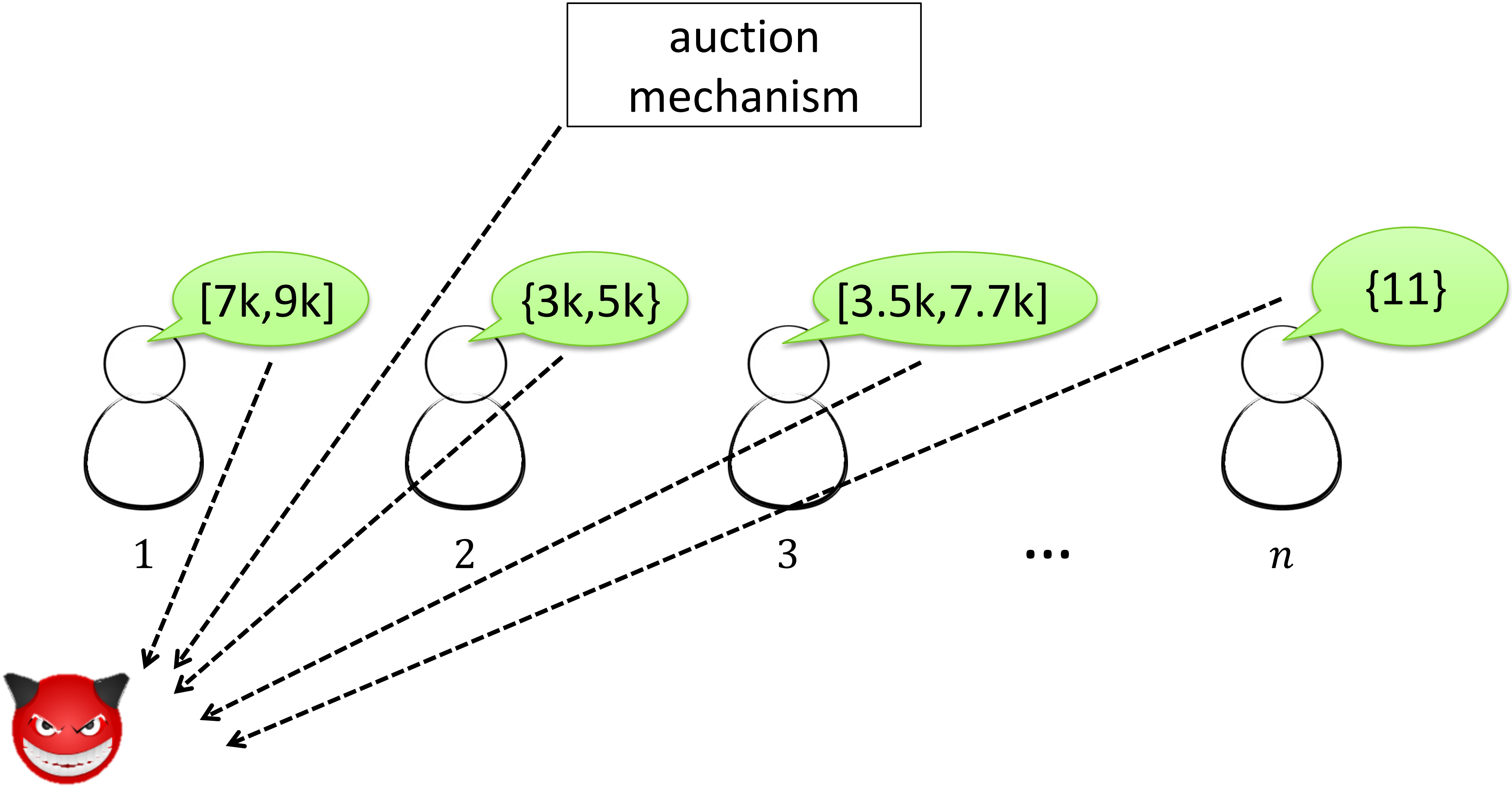
How well can we leverage approximate knowledge?

HOW TO MEASURE PERFORMANCE?

auction
mechanism



ADVERSARIAL PERFORMANCE MEASURE



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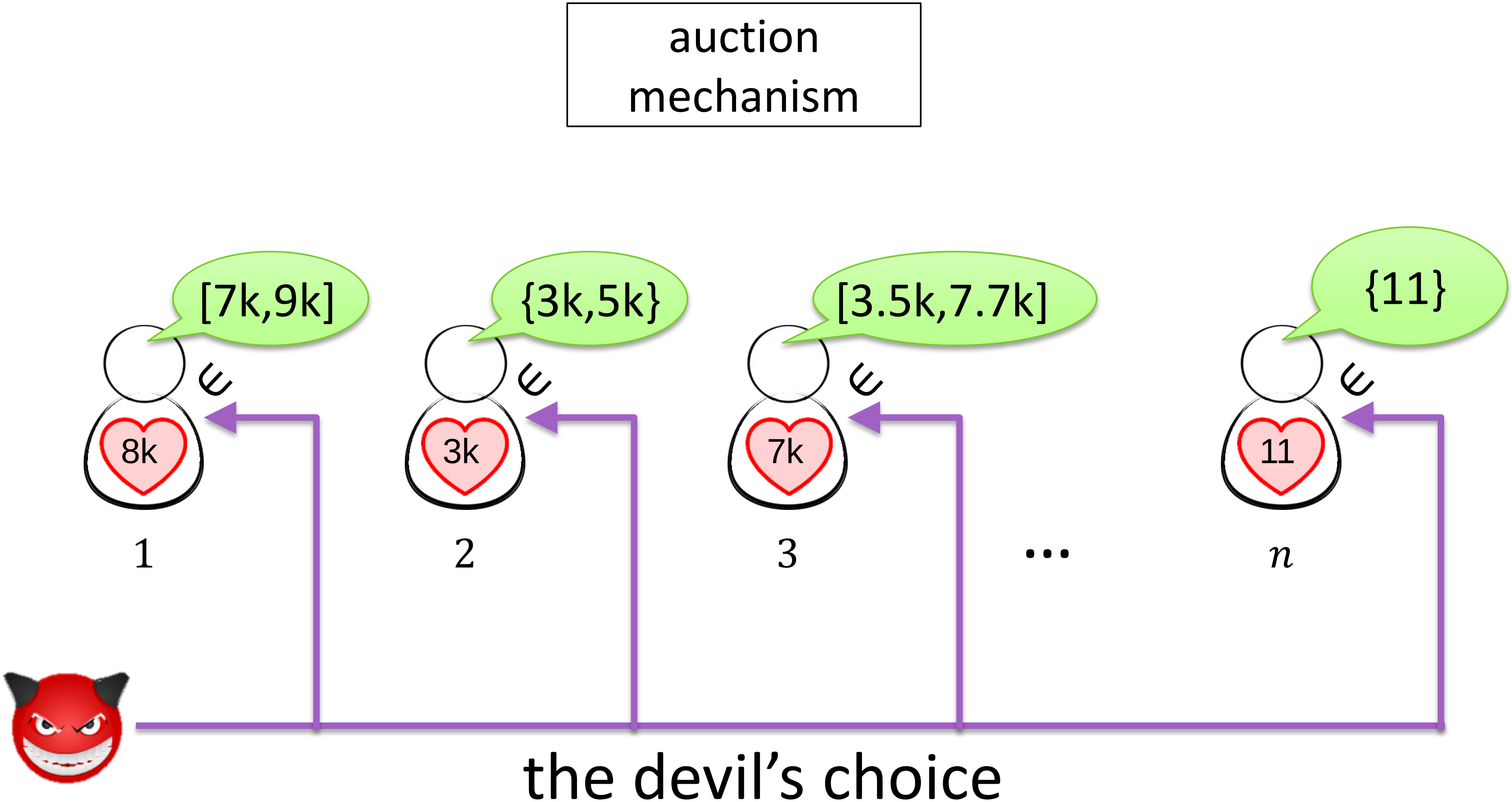
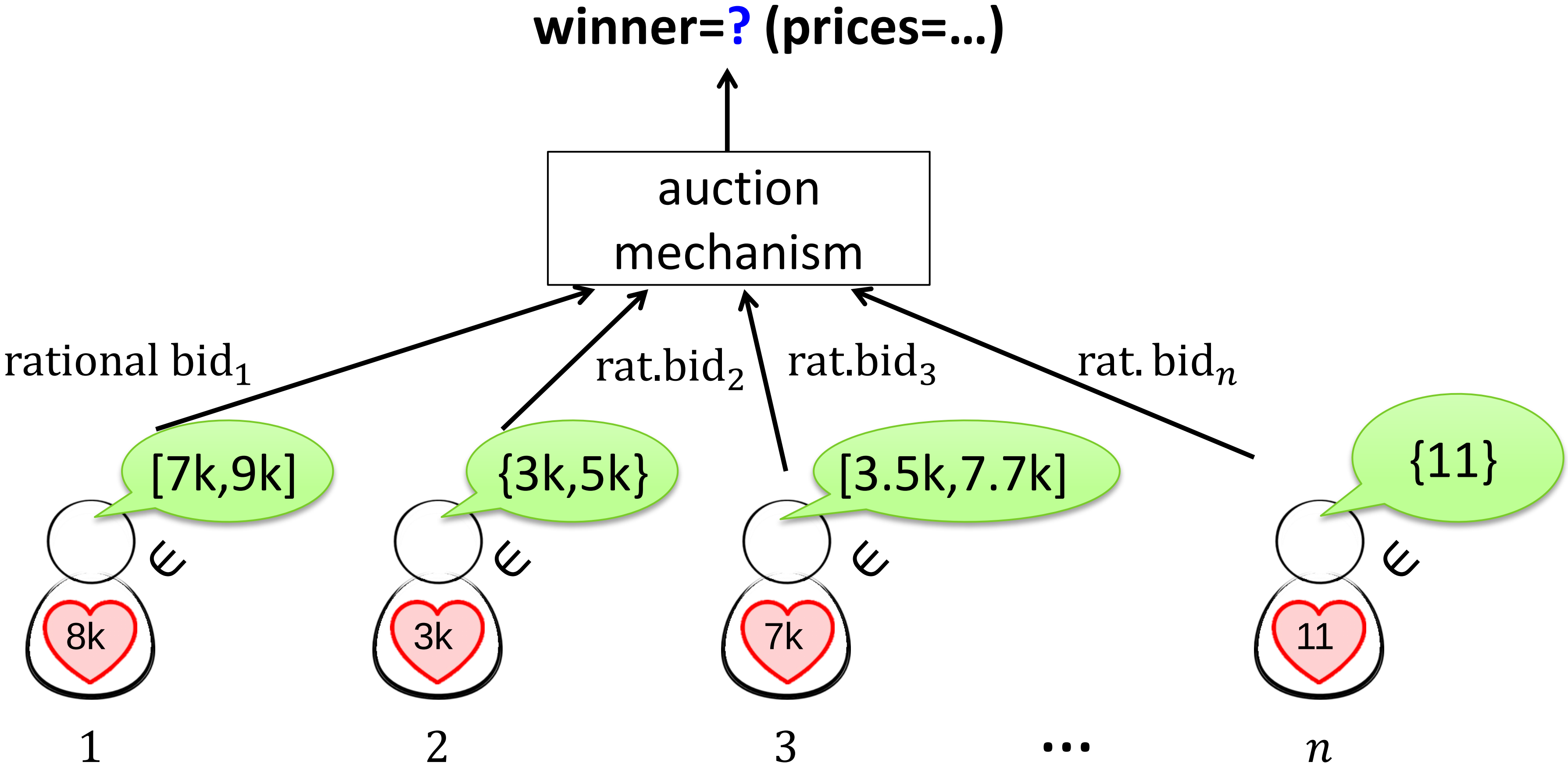


the devil's choice

ADVERSARIAL PERFORMANCE MEASURE



winner=? (prices=...)

auction mechanism

rational bid_1

rat.bid_2

rat.bid_3

rat. bid_n

[7k,9k]

{3k,5k}

[3.5k,7.7k]

{11}

$\in$

8k

3k

7k

11

1

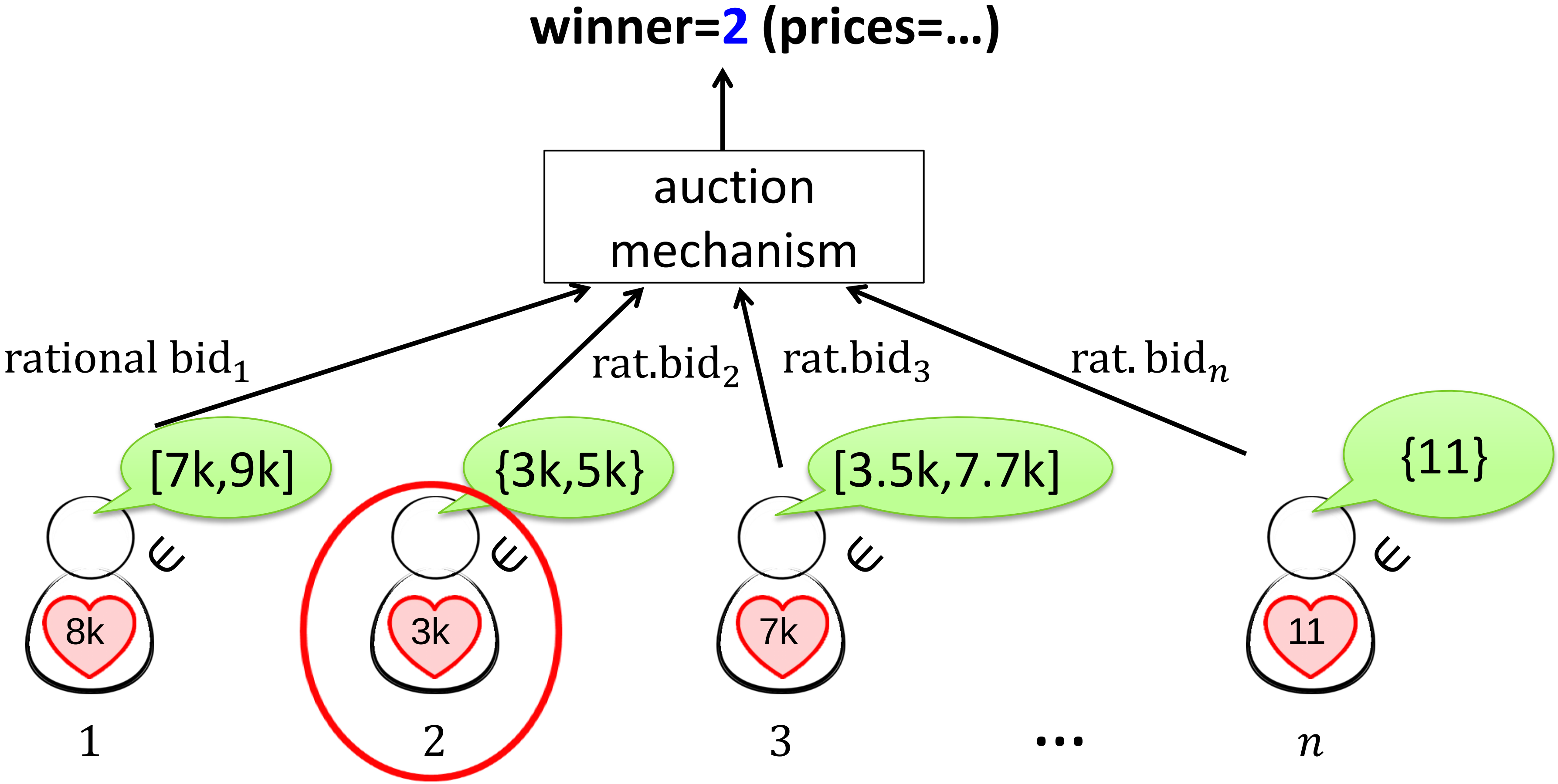
2

3

…

n

ADVERSARIAL PERFORMANCE MEASURE



winner=2 (prices=…)

auction mechanism

rational bid_1 rat.bid_2 rat.bid_3 rat. bid_n

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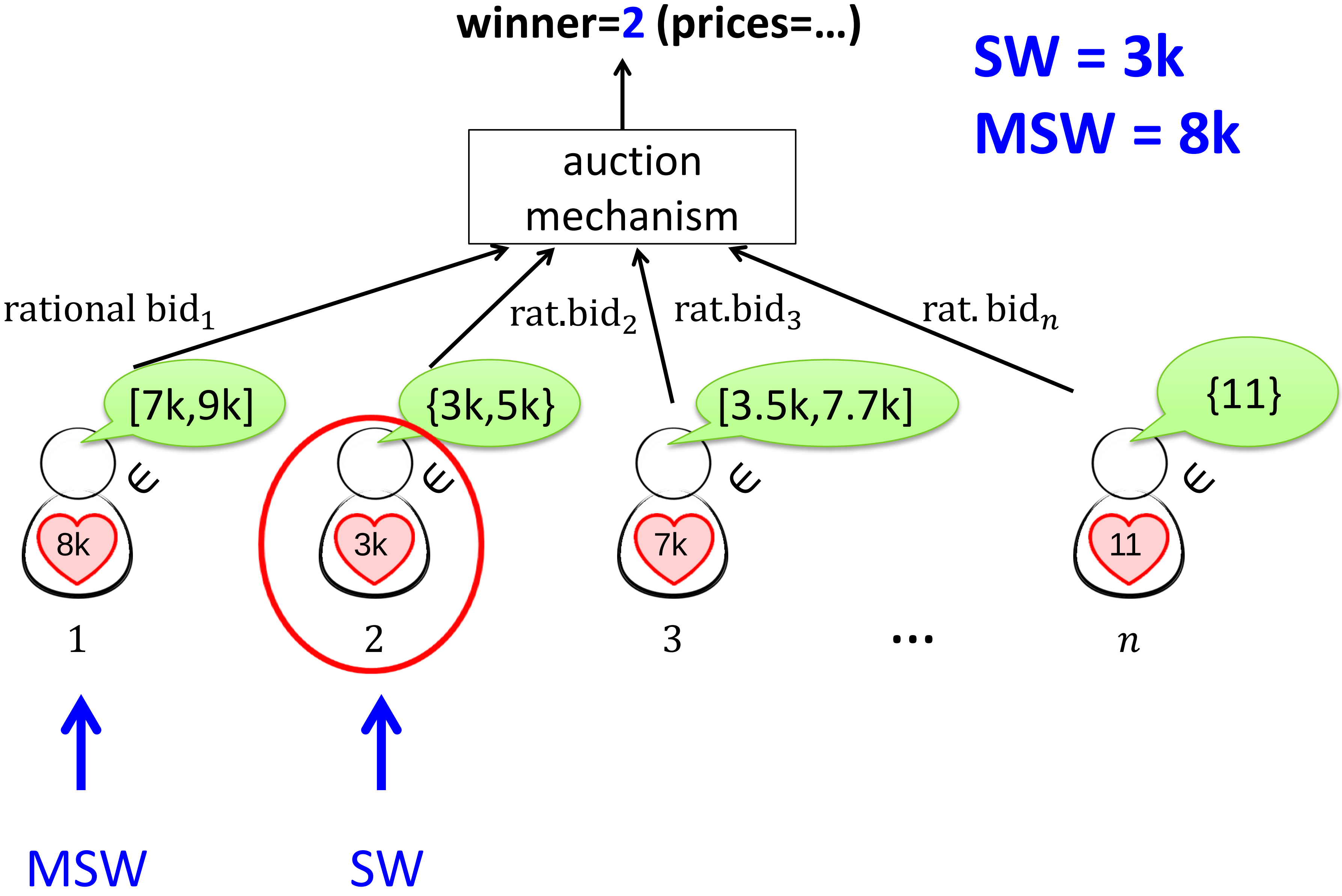


winner=2 (prices=…)

SW = 3k

MSW = 8k

auction mechanism

rational bid_1

rat.bid_2

rat.bid_3

rat. bid_n

[7k,9k]

{3k,5k}

[3.5k,7.7k]

{11}

∈ ∈ ∈ ∈

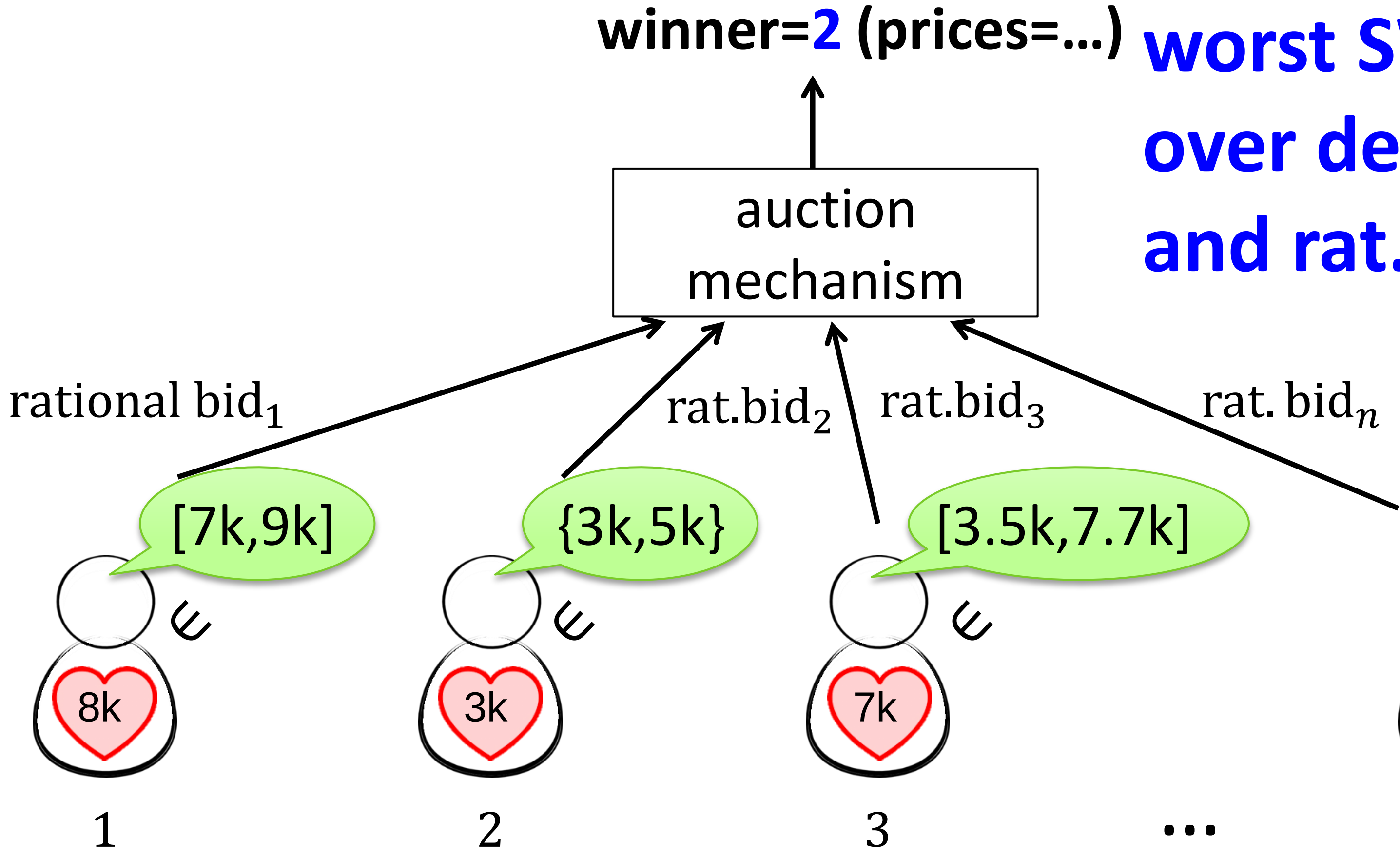
8k 3k 7k 11

1 2 3 … n

MSW

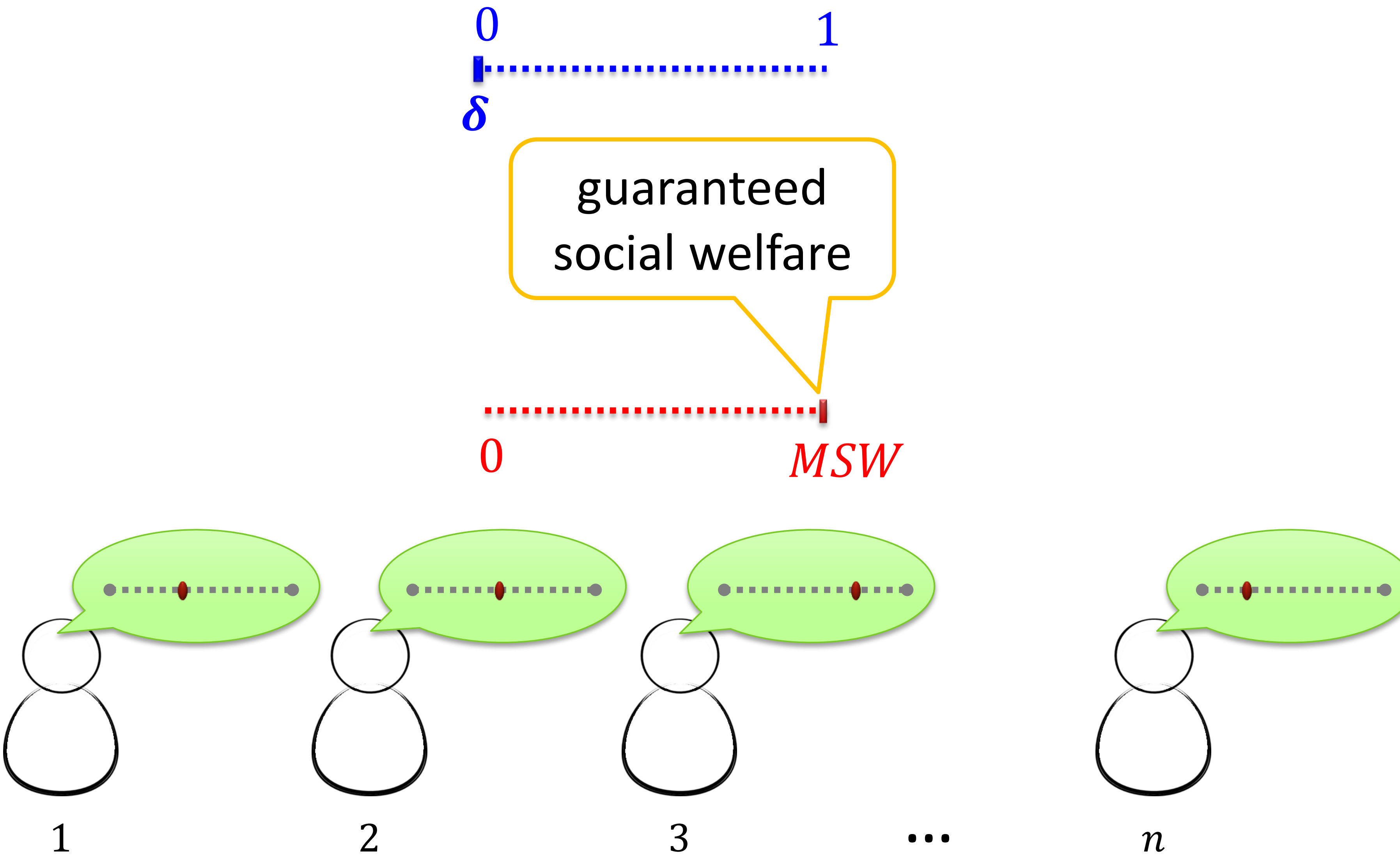
SW

ADVERSARIAL PERFORMANCE MEASURE



worst SW/MSW over devil choices and rat. bids

How Much SW Can We Guarantee?



guaranteed social welfare

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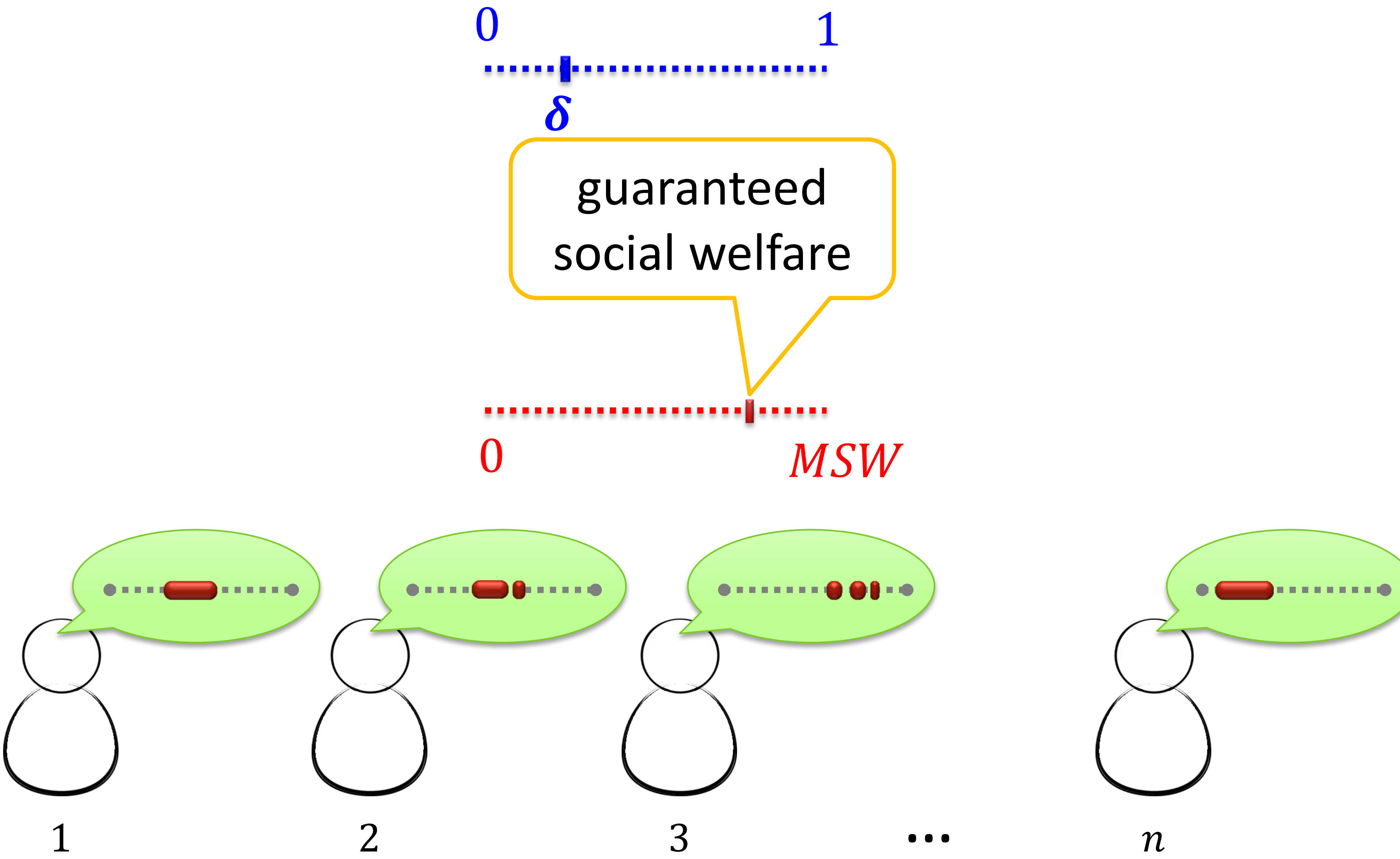

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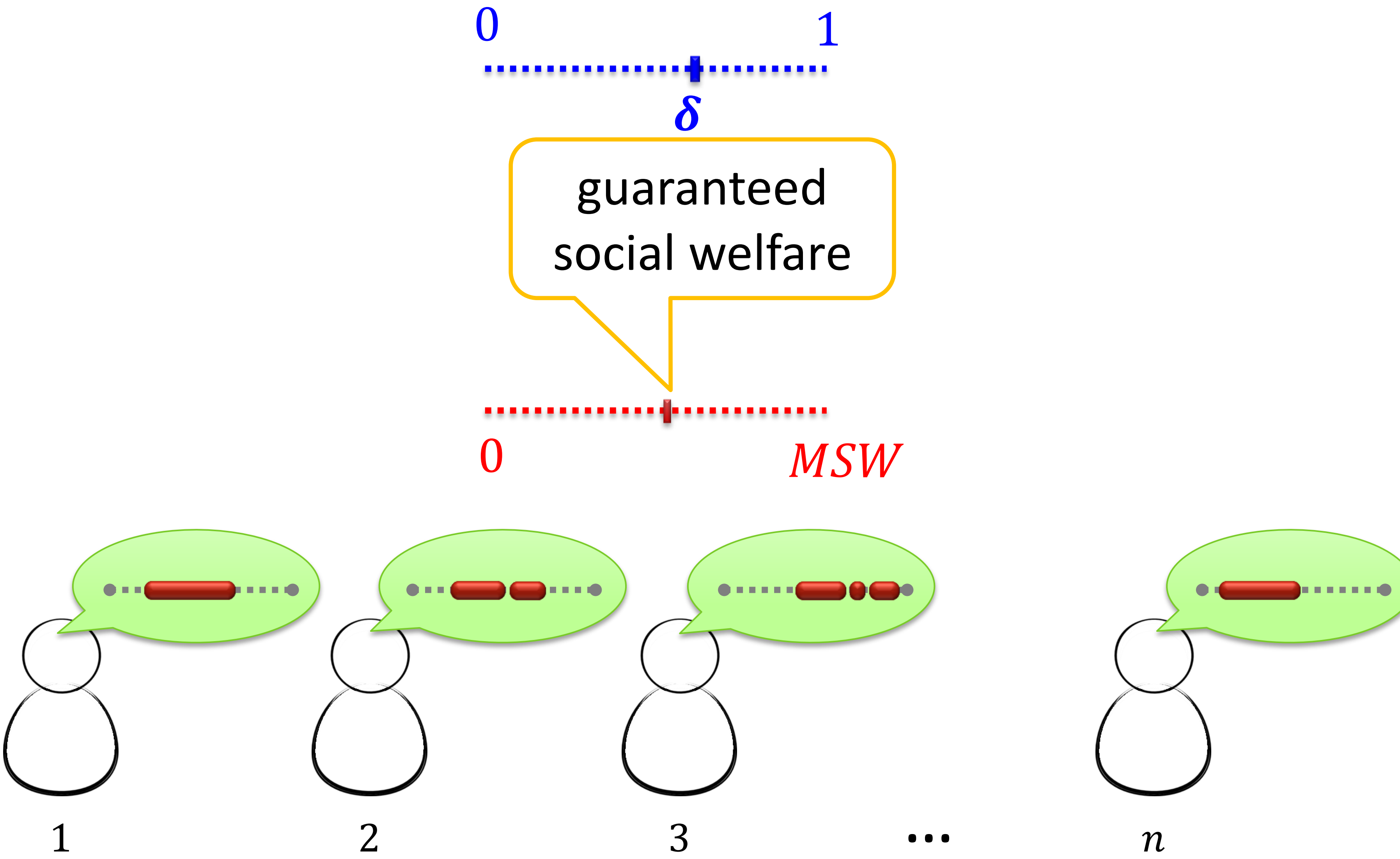

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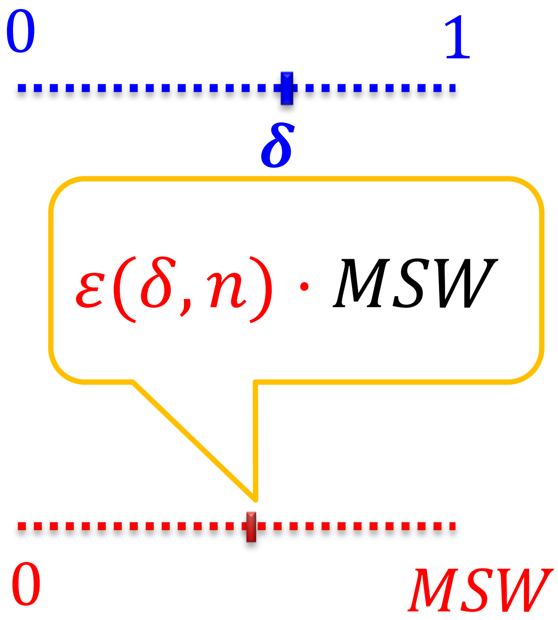
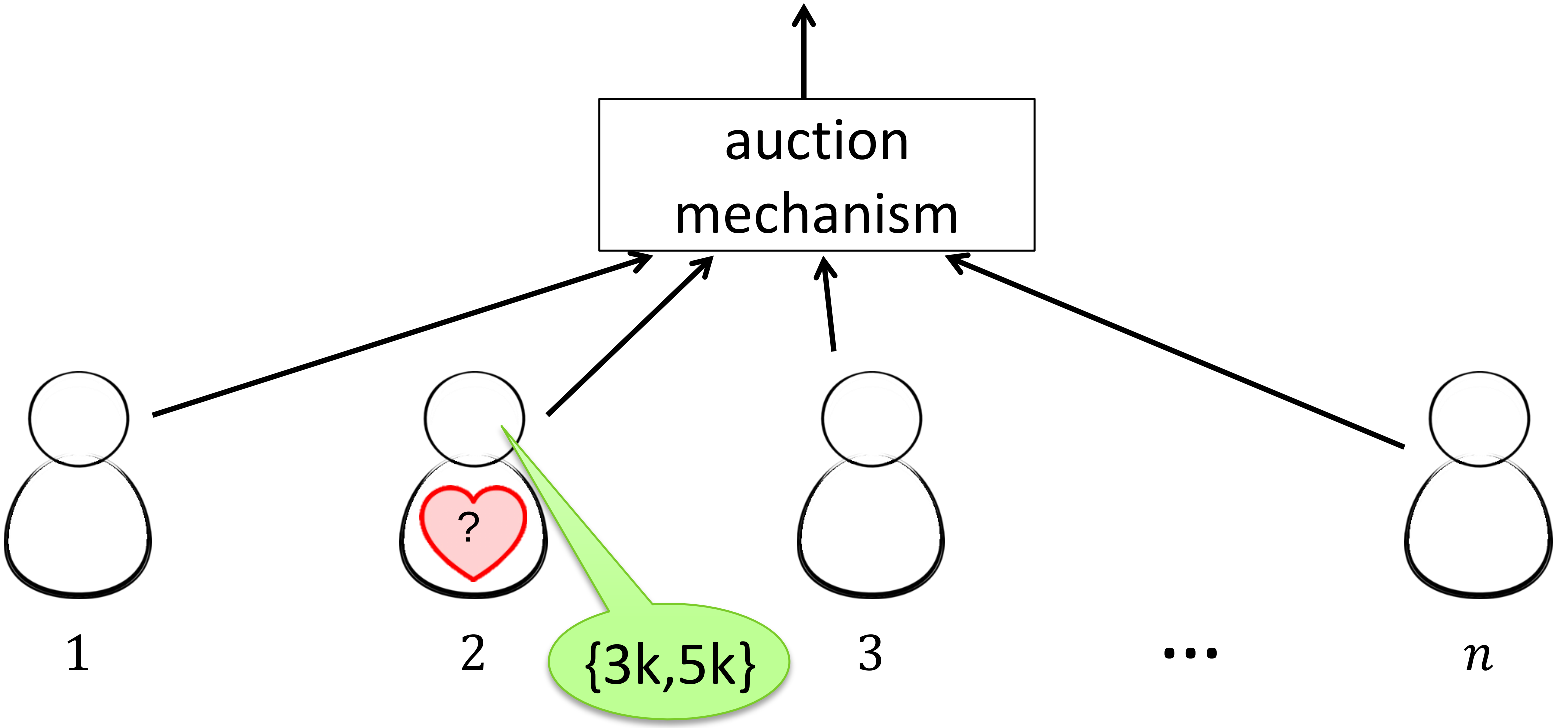


QUESTION (now more precise)

What is the max $\varepsilon(\delta, n)$ that we can guarantee?

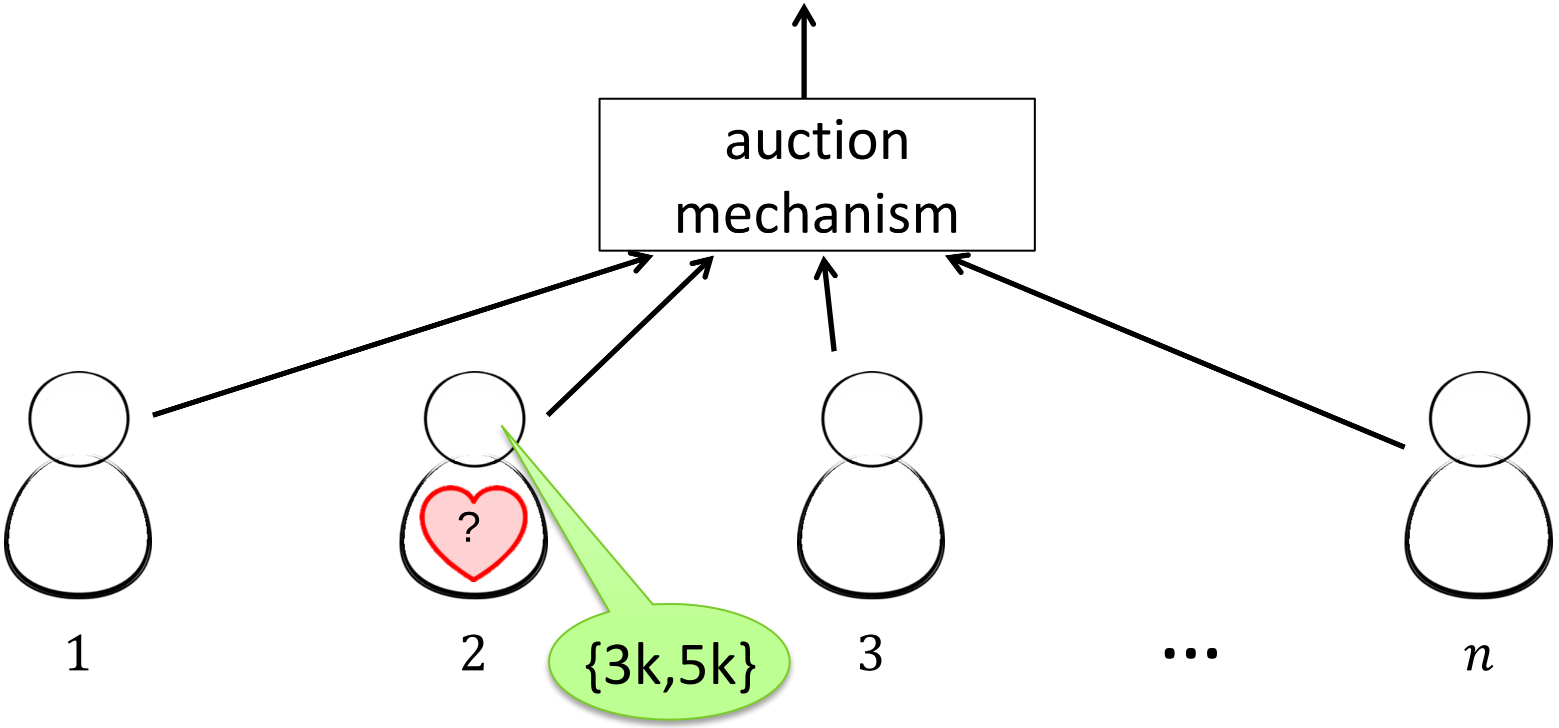
Our Results

How about dominant strategies?



Should player 2 bid 3k or 5k?

How about dominant strategies?



Should player 2 bid 3k or 5k?

What if he can report a set?

(if reporting the ``true’’ set is dominant, we may be all set…)

Old Theorem

if $\boldsymbol{\delta = 0}$ $\exists$ dominant-strategy mechanism guaranteeing $\boldsymbol{100\% \cdot MSW}$

New Theorem

if $\boldsymbol{\delta > 0}$ ∃ dominant-strategy mechanism guaranteeing

$$\boldsymbol{(1-\delta)} \cdot \boldsymbol{MSW}\ \mathbf{?}$$

New Theorem

if $\delta > 0$ $\exists$ dominant-strategy mechanism guaranteeing

$$(1-\delta)^2 \cdot MSW \,\mathbf{?}$$

New Theorem

~~if $\delta > 0$ $\exists$ dominant-strategy mechanism guaranteeing $(1-\delta)^2 \cdot MSW$?~~

Theorem 1

$\forall\ \delta > 0$, every dominant-strategy mechanism guarantees at most $\frac{1}{n} \cdot MSW$

New Theorem

~~**if $\delta > 0$ $\exists$ dominant-strategy mechanism guaranteeing**~~

~~$$(1-\delta)^2 \cdot MSW \,?$$~~

Theorem 1

$\forall\ \delta > 0$, every dominant-strategy mechanism guarantees at most $\frac{1}{n} \cdot MSW$

Remark 1: dominant-strategy mechanism exist

Remark 2: they perform terribly!

a random assignment trivially guarantees $\frac{1}{n}$

New Theorem

~~if $\delta > 0$ $\exists$ dominant-strategy mechanism guaranteeing~~

~~$(1-\delta)^2 \cdot MSW$?~~

Theorem 1

$\forall\, \delta > 0$, every dominant-strategy mechanism guarantees at most $\frac{1}{n} \cdot MSW$

Interpretation

dominant strategy useful

iff

exact knowledge or Bayesian

New Theorem

if $\delta > 0$ $\exists$ ~~dominant-strategy mechanism guaranteeing $(1-\delta)^2 \cdot MSW$?~~

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$70(1 \pm 0.1)?$

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~~$70(1 \pm 0.1)$~~

$70(1 \pm 0.01)$?

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Theorem 1

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~~$70(1 \pm 0.1)$~~

~~$70(1 \pm 0.01)$~~

$70(1 \pm 0.001)$?

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~~$70(1 \pm 0.1)$~~

~~$70(1 \pm 0.01)$~~

~~$70(1 \pm 0.001)$~~

70

Interpretation

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iff

exact knowledge or Bayesian

A New World

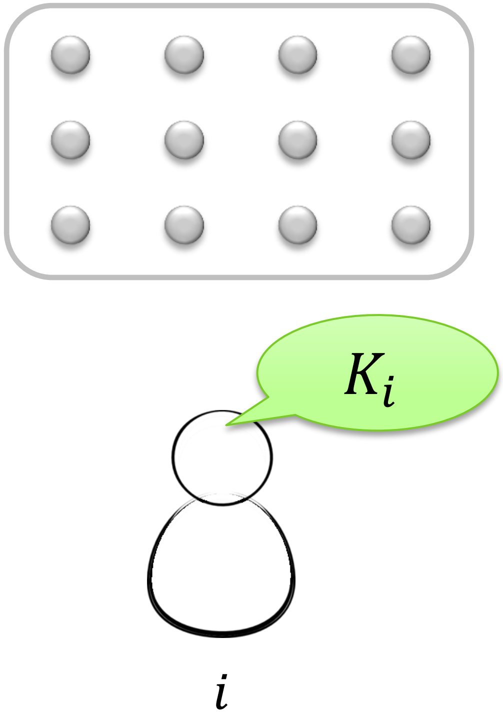
Dominant strategies not useful…

What other solution concepts could make sense?

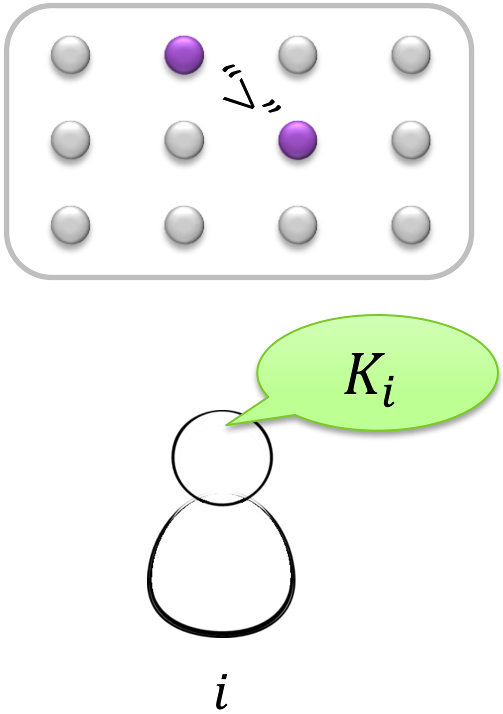
undominated strategies [Jackson, BLP]

Undominated Strategies

Undominated Strategies



Undominated Strategies



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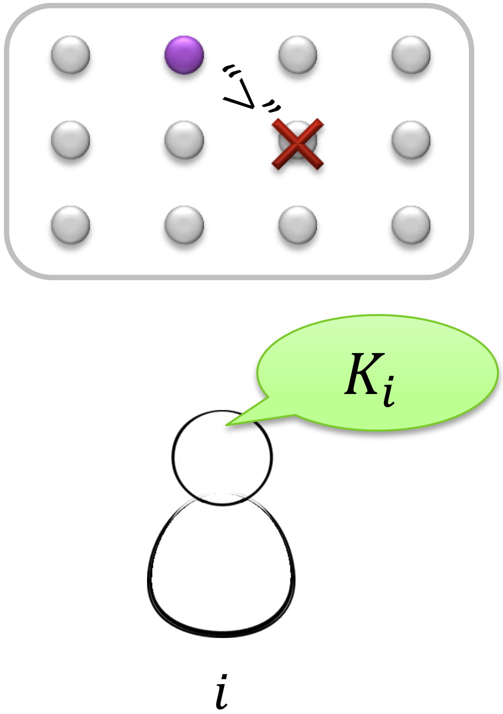

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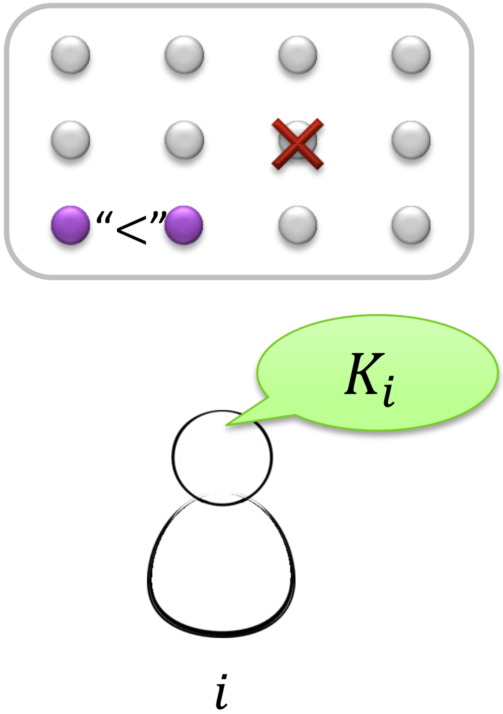

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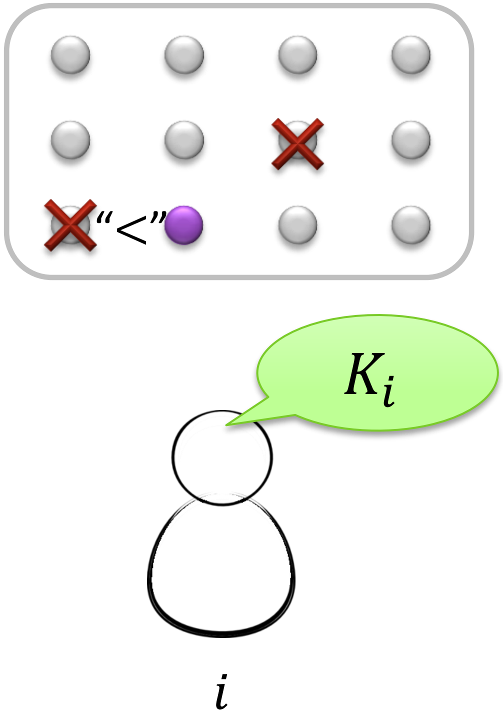

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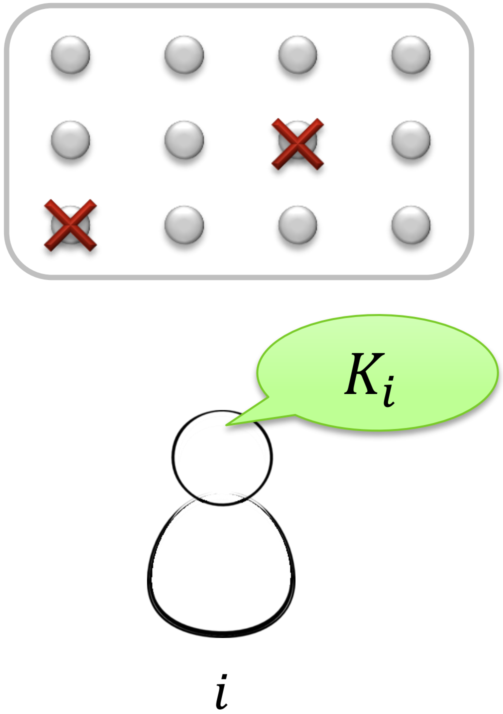

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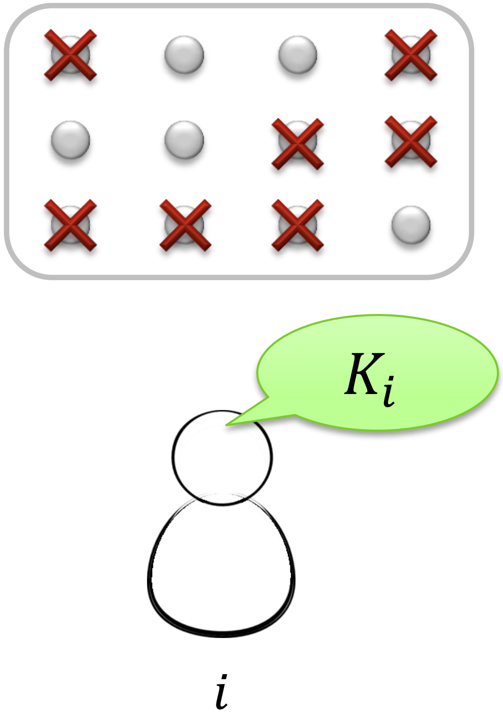

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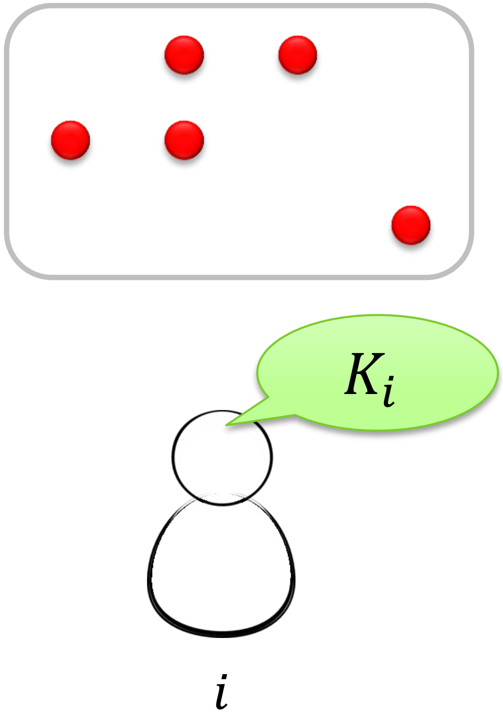

Thm 2: Second-price mechanism in undominated strats. guarantees $\left(\frac{1-\delta}{1+\delta}\right)^2 \cdot MSW$

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much better!!

(note that the second-price mechanism is not dominant-strategy anymore!)

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(note that the second-price mechanism is not dominant-strategy anymore!)

$\Longrightarrow$ a new role for undominated strategies!

Thm 2: Second-price mechanism in undominated strats. guarantees $\left(\frac{1-\delta}{1+\delta}\right)^2 \cdot MSW$

Thm 3: ∀ <u>deterministic</u> undom. strat. mechanism guarantees no more than $\left(\frac{1-\delta}{1+\delta}\right)^2 \cdot MSW$

Harder!

dominant strategies → ``single’’ mechanism (rev. principle)

undominated strategies → infinitely many mechanisms

Thm 2: Second-price mechanism in undominated strats. guarantees $\left(\frac{1-\delta}{1+\delta}\right)^2 \cdot MSW$

Thm 3: ∀ deterministic undom. strat. mechanism guarantees no more than $\left(\frac{1-\delta}{1+\delta}\right)^2 \cdot MSW$

And with randomness?

Implementation in Undomin. Strat’s

Thm 4: Our mechanism in undom. strategies guarantees $\frac{(1-\delta)^2+\frac{4\delta}{n}}{(1+\delta)^2}\cdot MSW$

$\delta = 0.5$	$n = 2$	5 times better
$\delta = 0.5$	$n = 4$	3 times better
$\delta = 0.25$	$n = 2$	2 times better

Implementation in Undomin. Strat's

Thm 4: Our mechanism in undom. strategies guarantees $\frac{(1-\delta)^2+\frac{4\delta}{n}}{(1+\delta)^2} \cdot MSW$

Thm 5: ∀ probabilistic undom. strat. mechanism guarantees no more than $\frac{(1-\delta)^2+\frac{4\delta}{n}}{(1+\delta)^2} \cdot MSW$

Summary

Dominant Strategies

Thm 1: Dominant Strategies don’t work

Undominated Strategies

Thm 2: Second-price mechanism guarantees $\left(\frac{1-\delta}{1+\delta}\right)^2 \cdot MSW$

Thm 3: & it is optimal among deterministic mechanisms

Thm 4: Our mechanism guarantees $\frac{(1-\delta)^2+\frac{4\delta}{n}}{(1+\delta)^2} \cdot MSW$

Thm 5: & it is optimal among probabilistic mechanisms

Summary

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Undominated Strategies

Thm 2: Second-price mechanism guarantees $\left(\frac{1-\delta}{1+\delta}\right)^2 \cdot MSW$

Thm 3: & it is optimal among deterministic mechanisms

Thm 4: Our mechanism guarantees $\frac{(1-\delta)^2+\frac{4\delta}{n}}{(1+\delta)^2} \cdot MSW$

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Structural Theorems

understanding undominated strategies with approximate knowledge

Summary

Dominant Strategies

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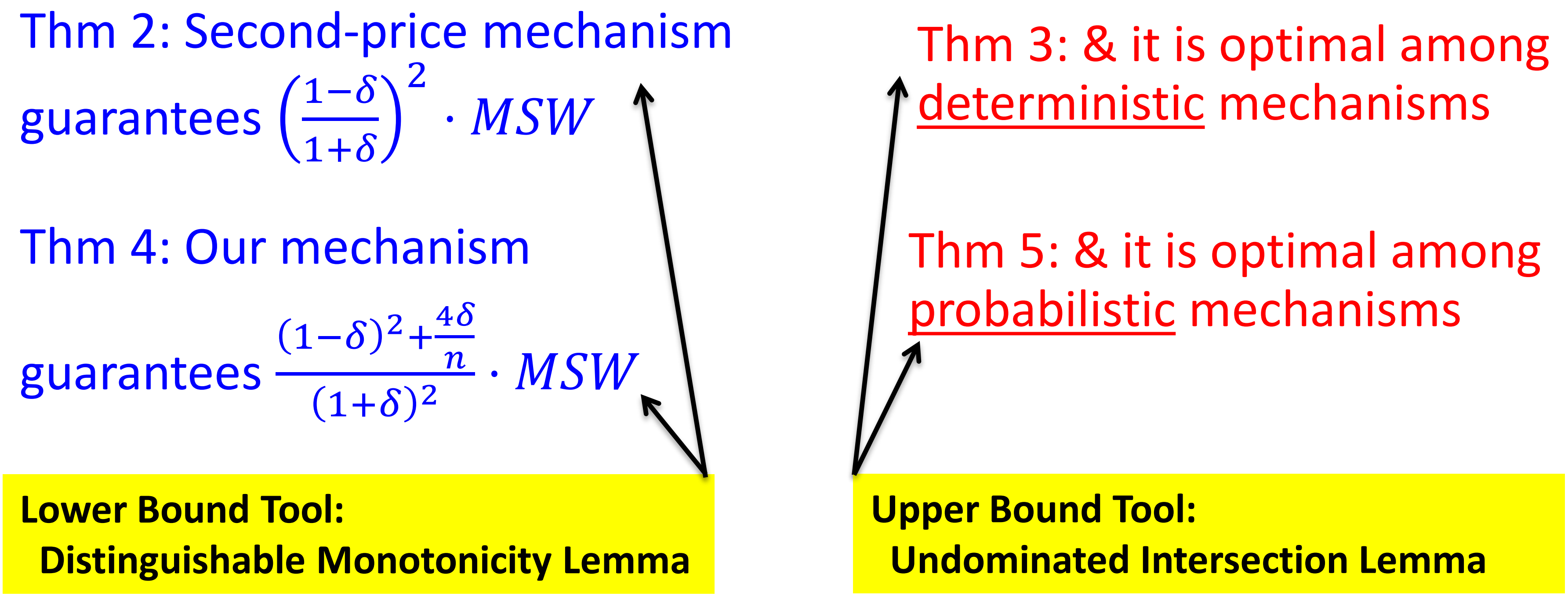

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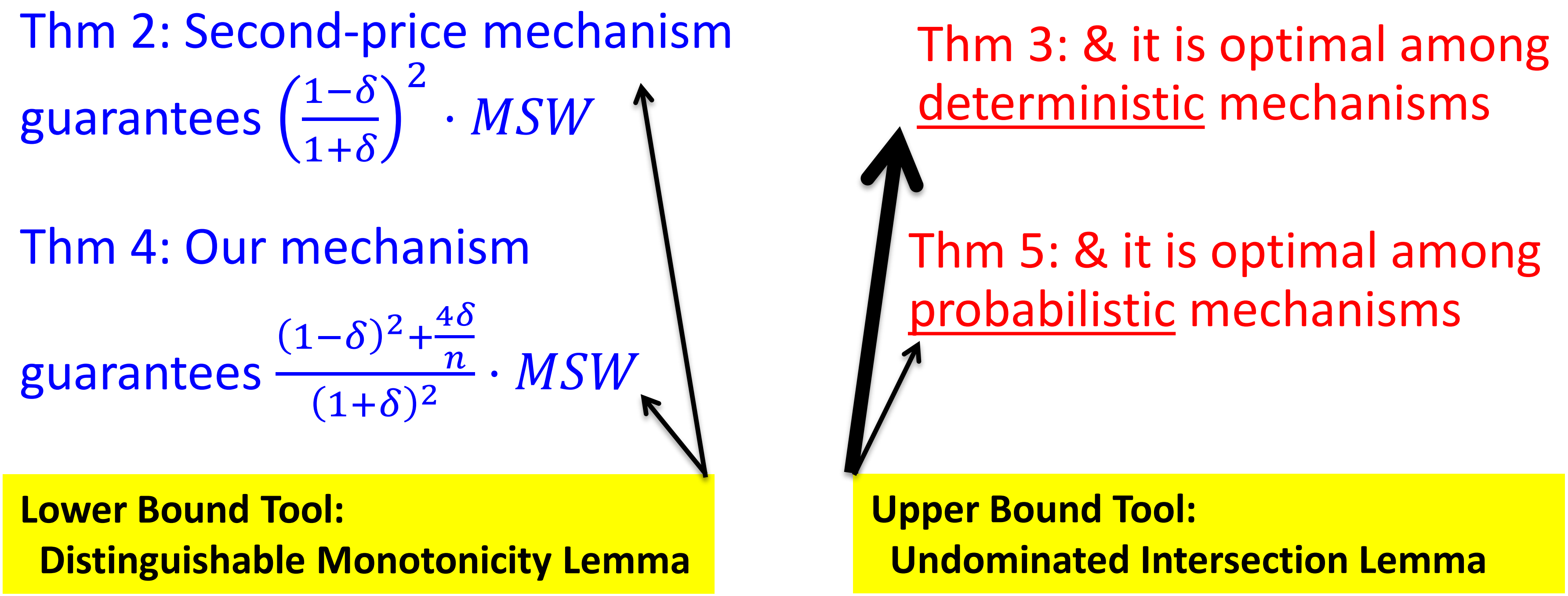

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Proving Theorem 3

Undominated Intersection Lemma:

$$|K_i \cap K_i'| \geq 2 \Longrightarrow \mathrm{UDed}_i(K_i) \cap \mathrm{UDed}_i(K_i') \neq \emptyset$$

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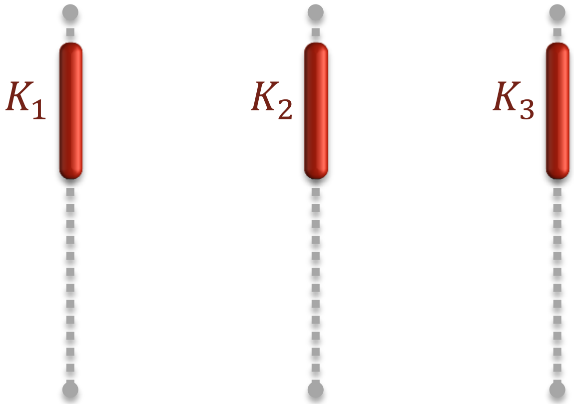

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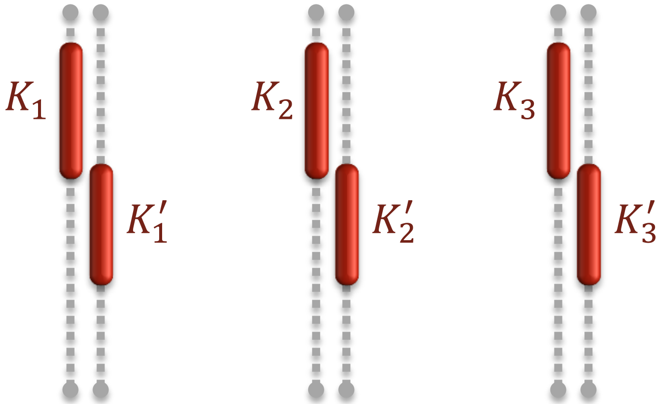

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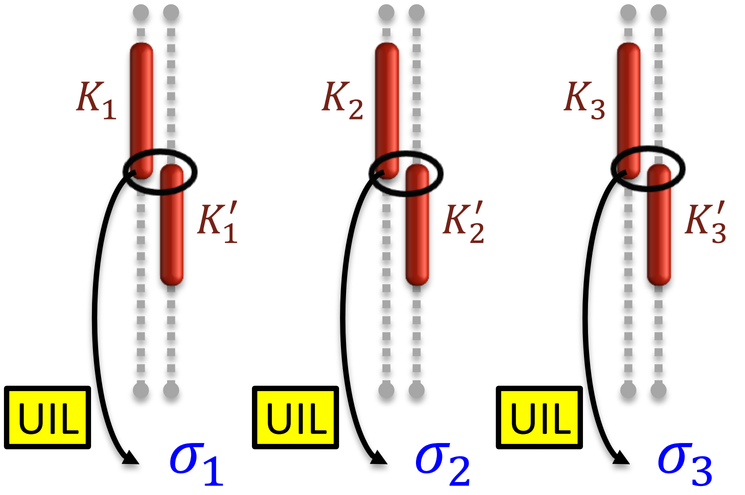

undominated in K_1 and K_1'

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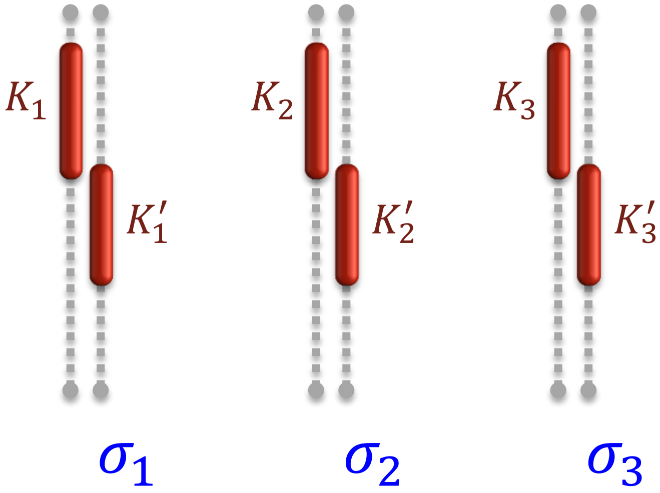

☹

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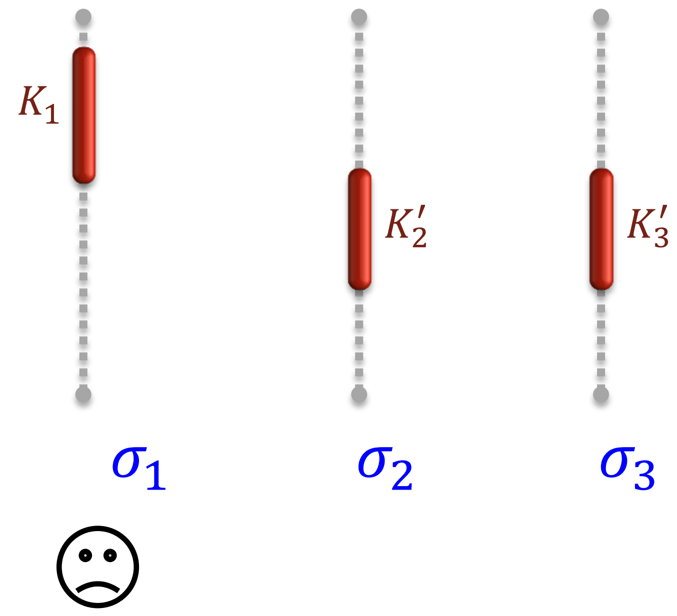

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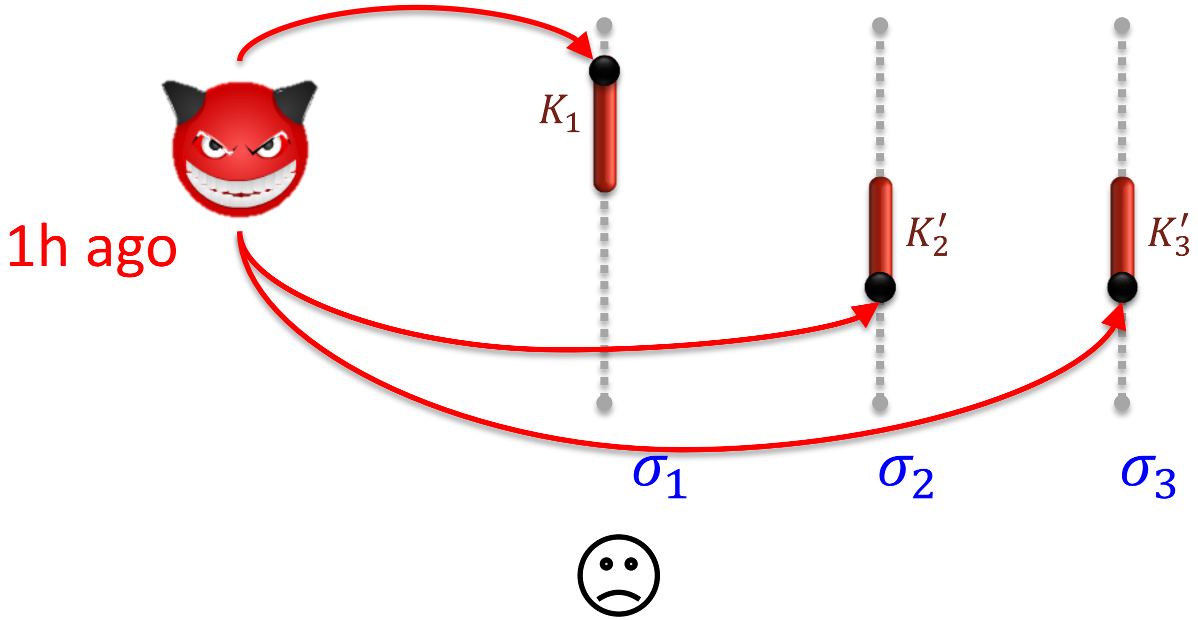

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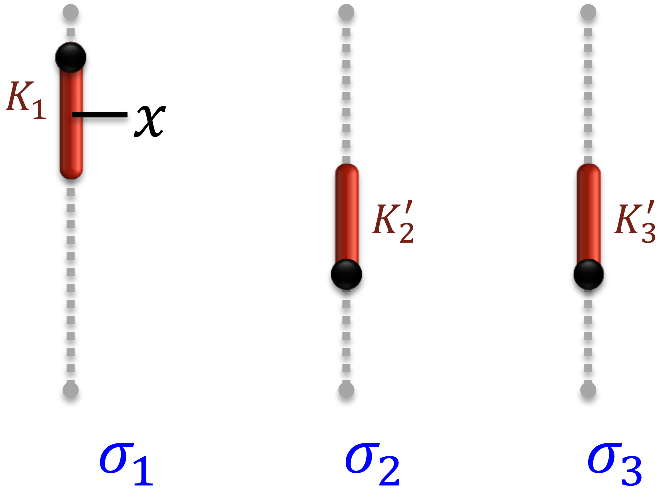

☹

$MSW = (1+\delta)x$

$$SW = \frac{(1-\delta)^2}{1+\delta}x$$

$$\Rightarrow \boxed{\varepsilon \leq \left(\frac{1-\delta}{1+\delta}\right)^2}$$

Deterministic: QED

Approximate Knowledge

more adversarial…

… more work (but doable)

… more fun!

Thank you!

Proving Theorem 3

Will use:

Undominated Intersection Lemma:

A tool for undominated strategy mechanisms

- No revelation principle to help
- Need to apply to **all** mechanisms

Proving Theorem 3

Undominated Intersection Lemma:

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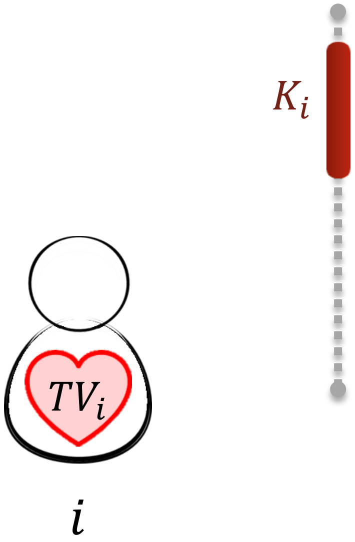
Example:



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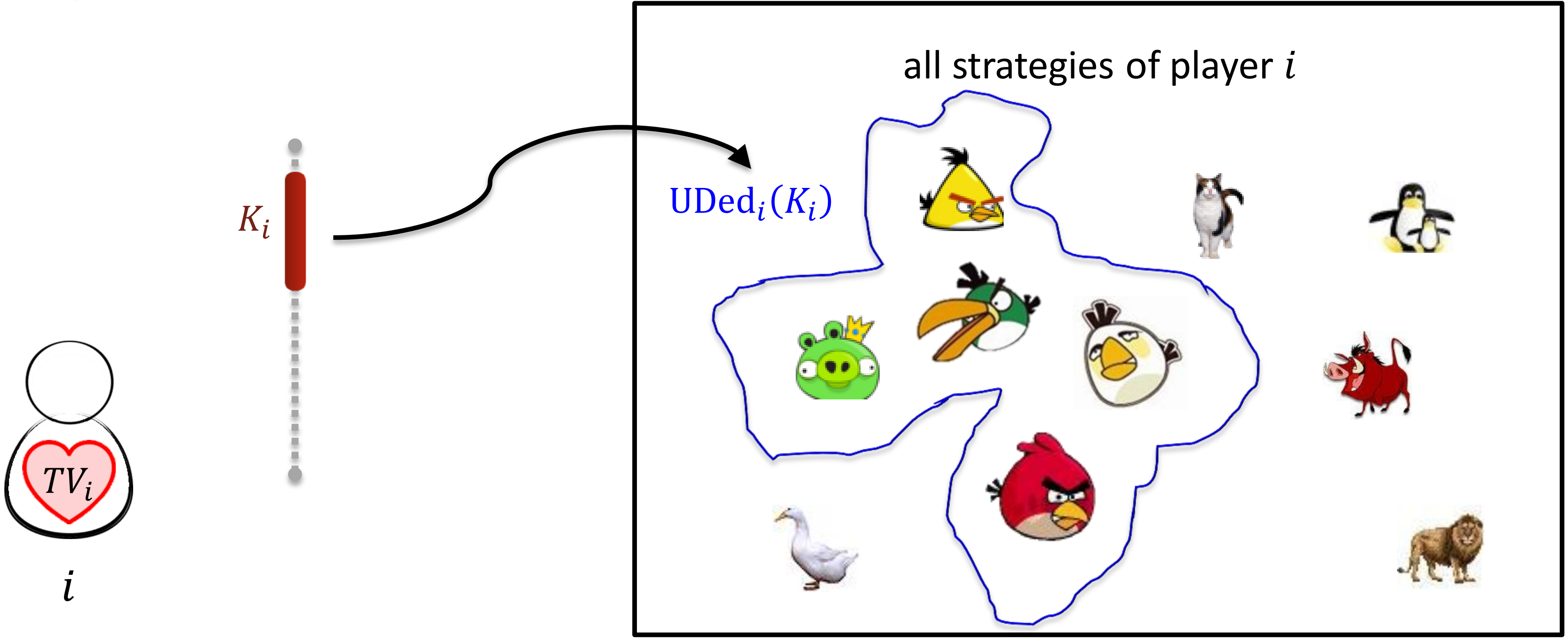

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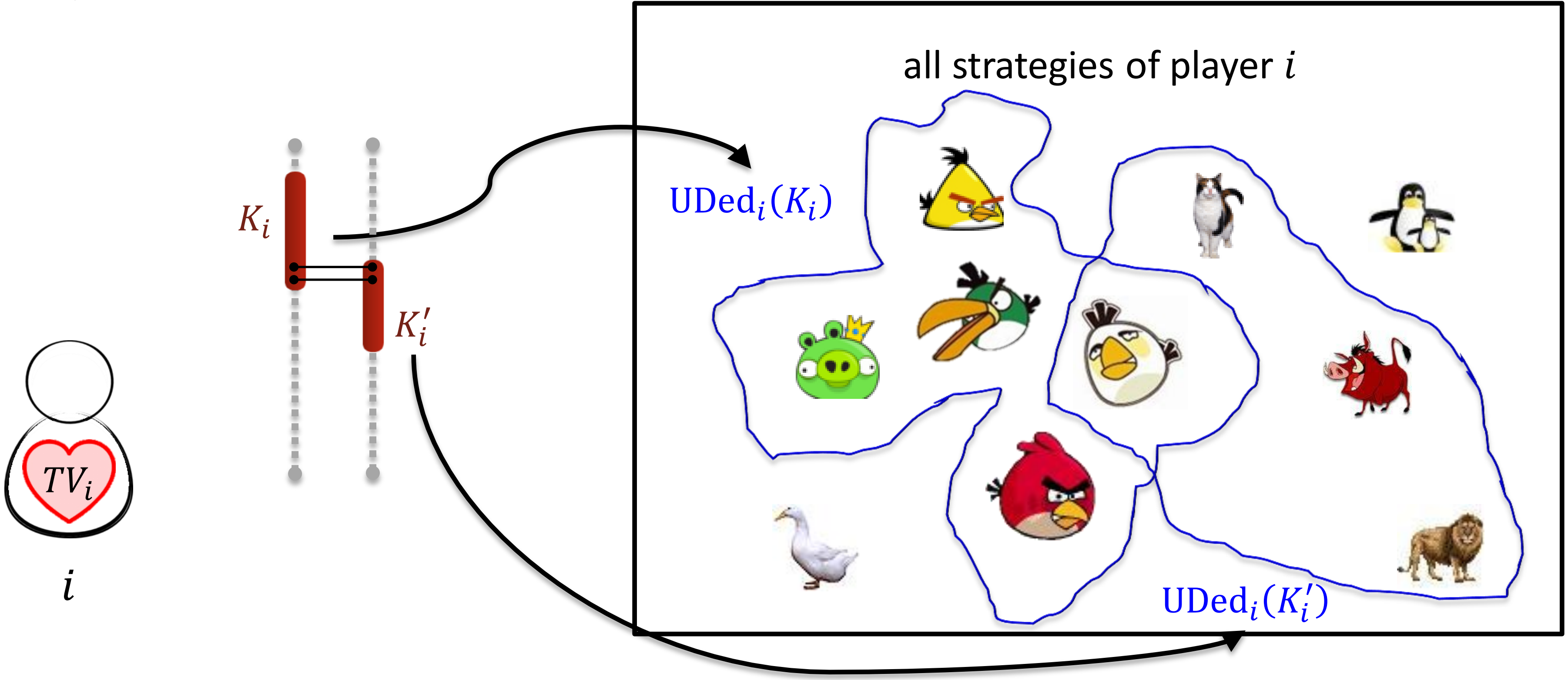

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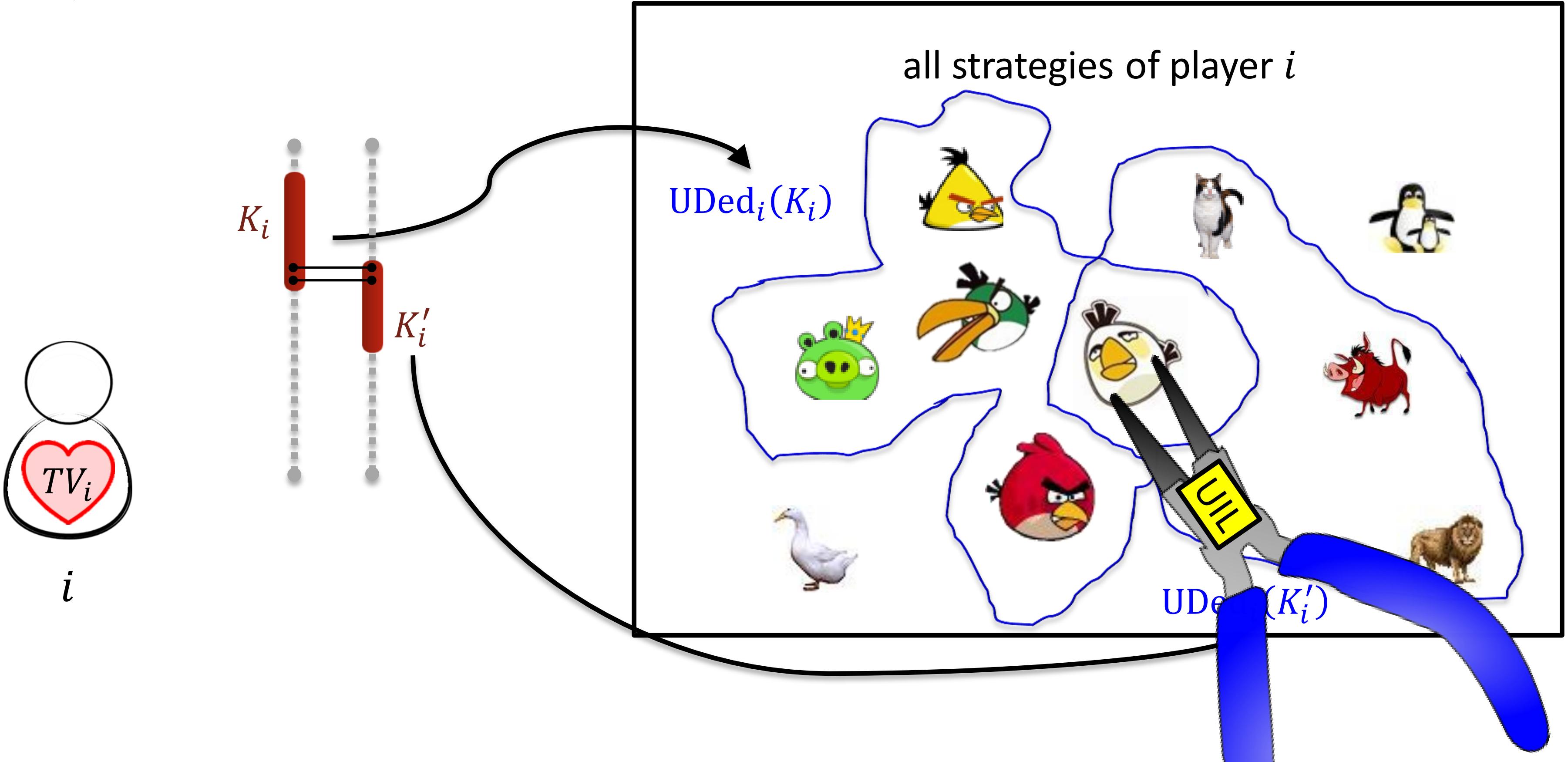

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↓

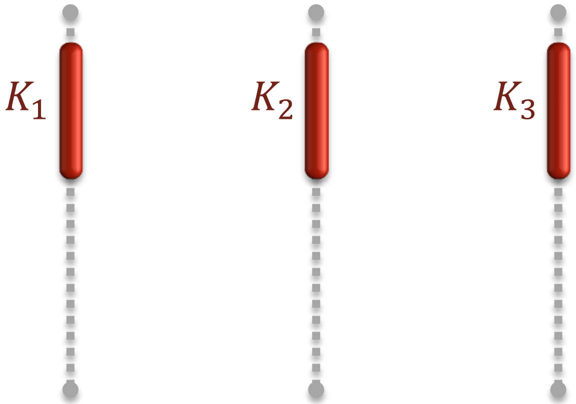
Recall Theorem 3:

in undominated strategies, no deterministic mechanism guarantees more than $\left(\frac{1-\delta}{1+\delta}\right)^2 \cdot MSW$

Proving Theorem 3

Proof:

1. Pick any x and set $K_1 = K_2 = K_3 = [(1-\delta)x, (1+\delta)x]$



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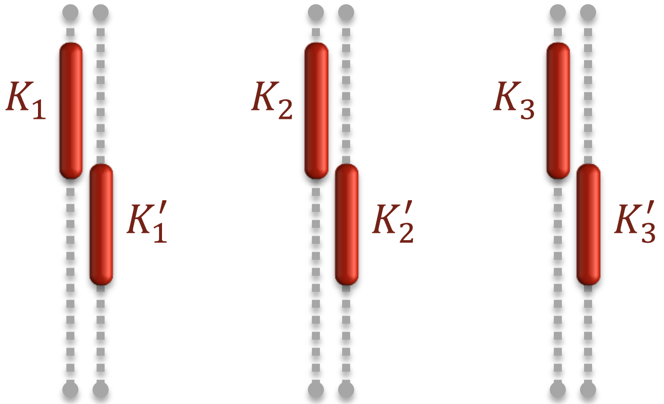
1. Pick any x and set $K_1 = K_2 = K_3 = [(1 - \delta)x, (1 + \delta)x]$
2. Set $K_1' = K_2' = K_3'$ to “just touch K_i from below”



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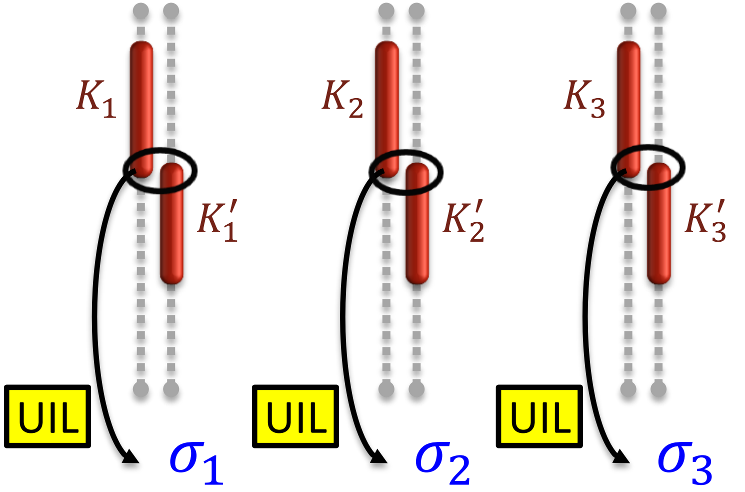
3. Apply UIL to obtain $\sigma_i \in \mathrm{UDed}_i(K_i) \cap \mathrm{UDed}_i(K_i')$ for each i



undominated in K_1 and K_1'

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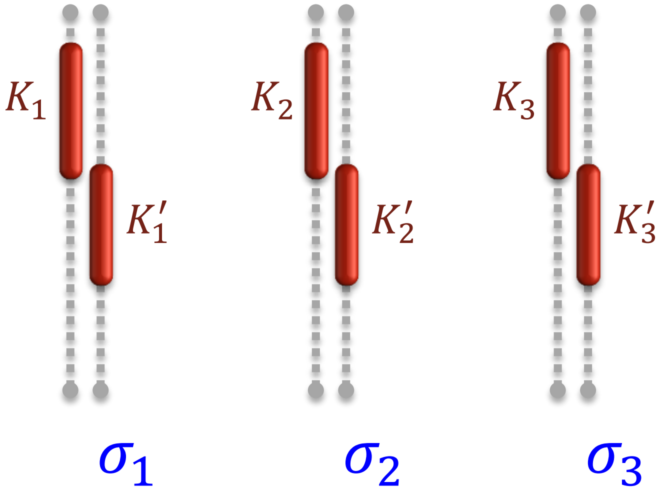

$\Pr[1 \text{ wins}] = 0$ ☹

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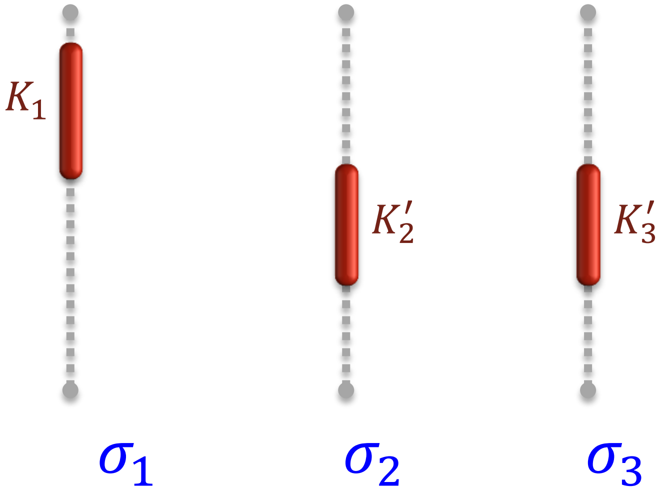
5. Choose the “world” of (K_1, K_2', K_3')… This is the hard instance!



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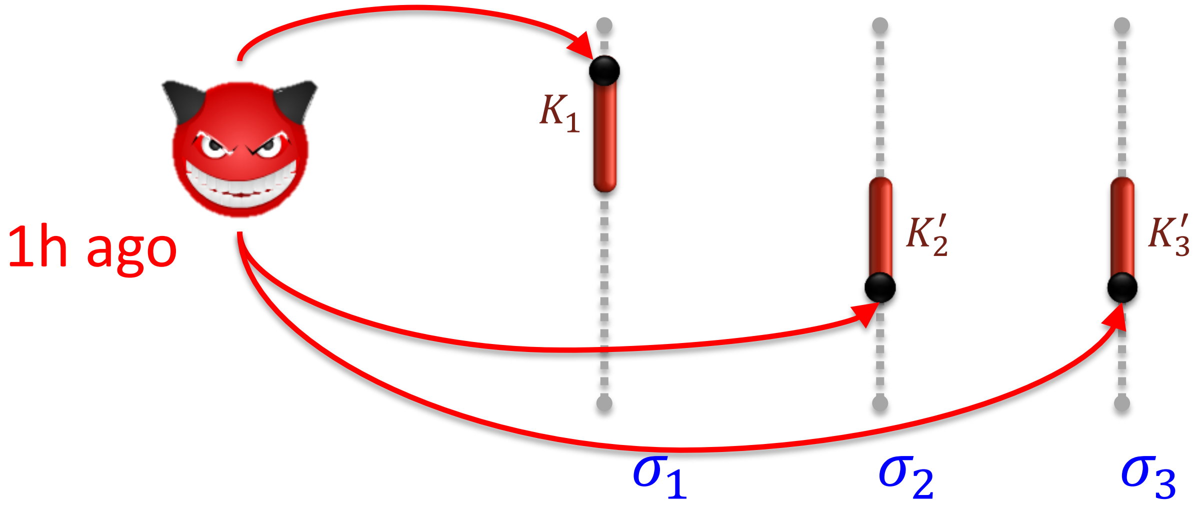

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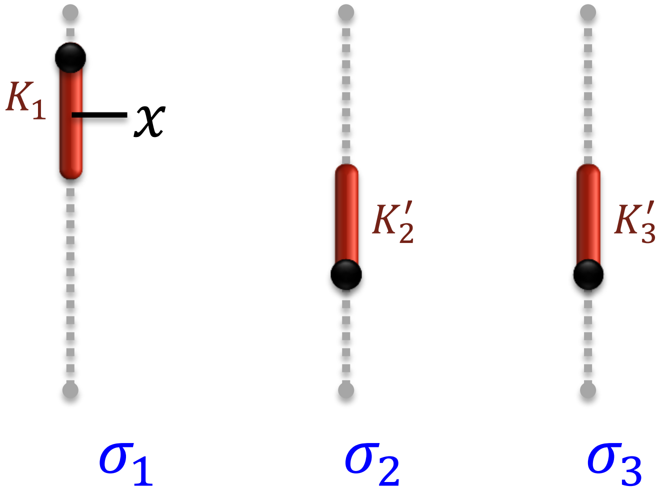

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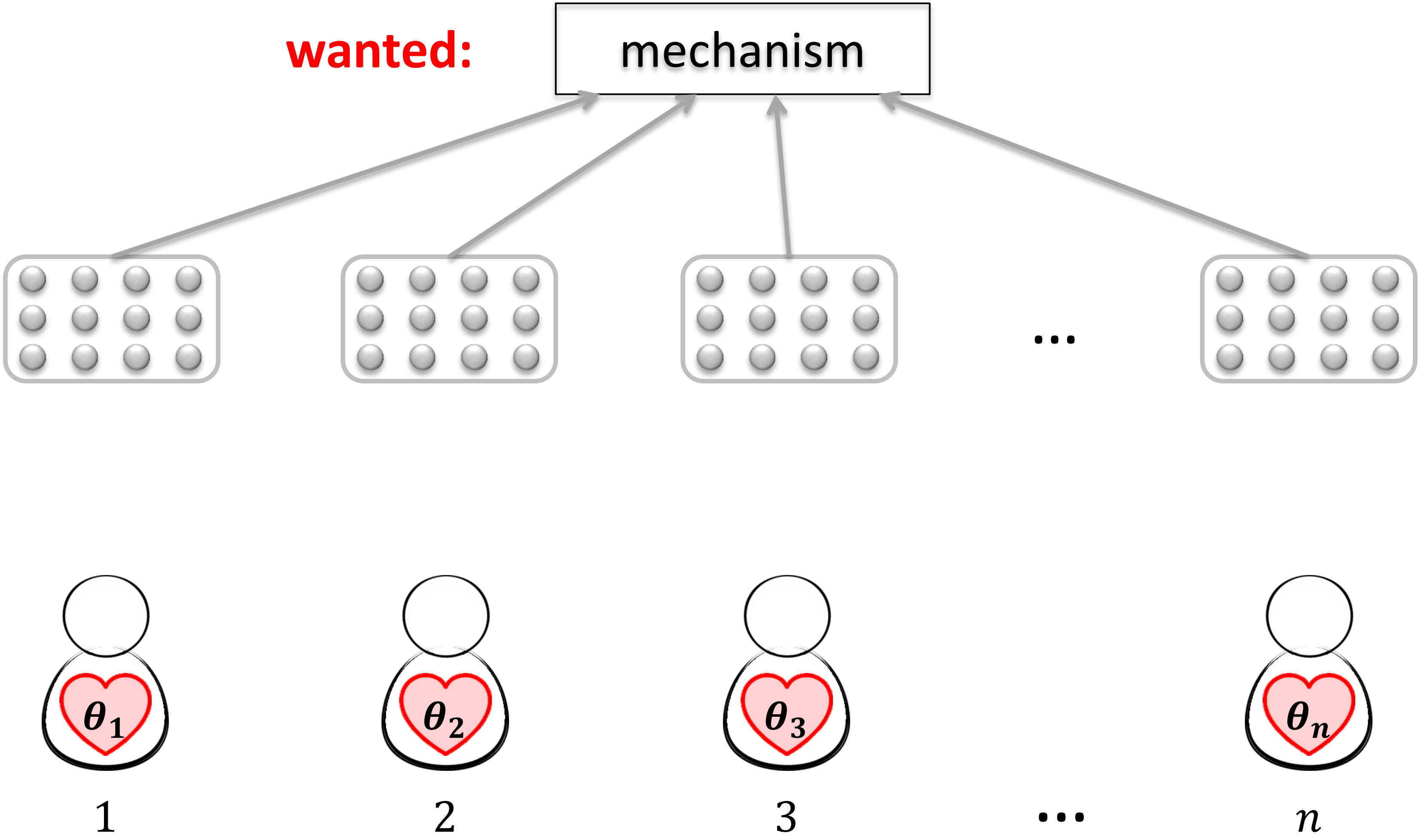
Deterministic: QED

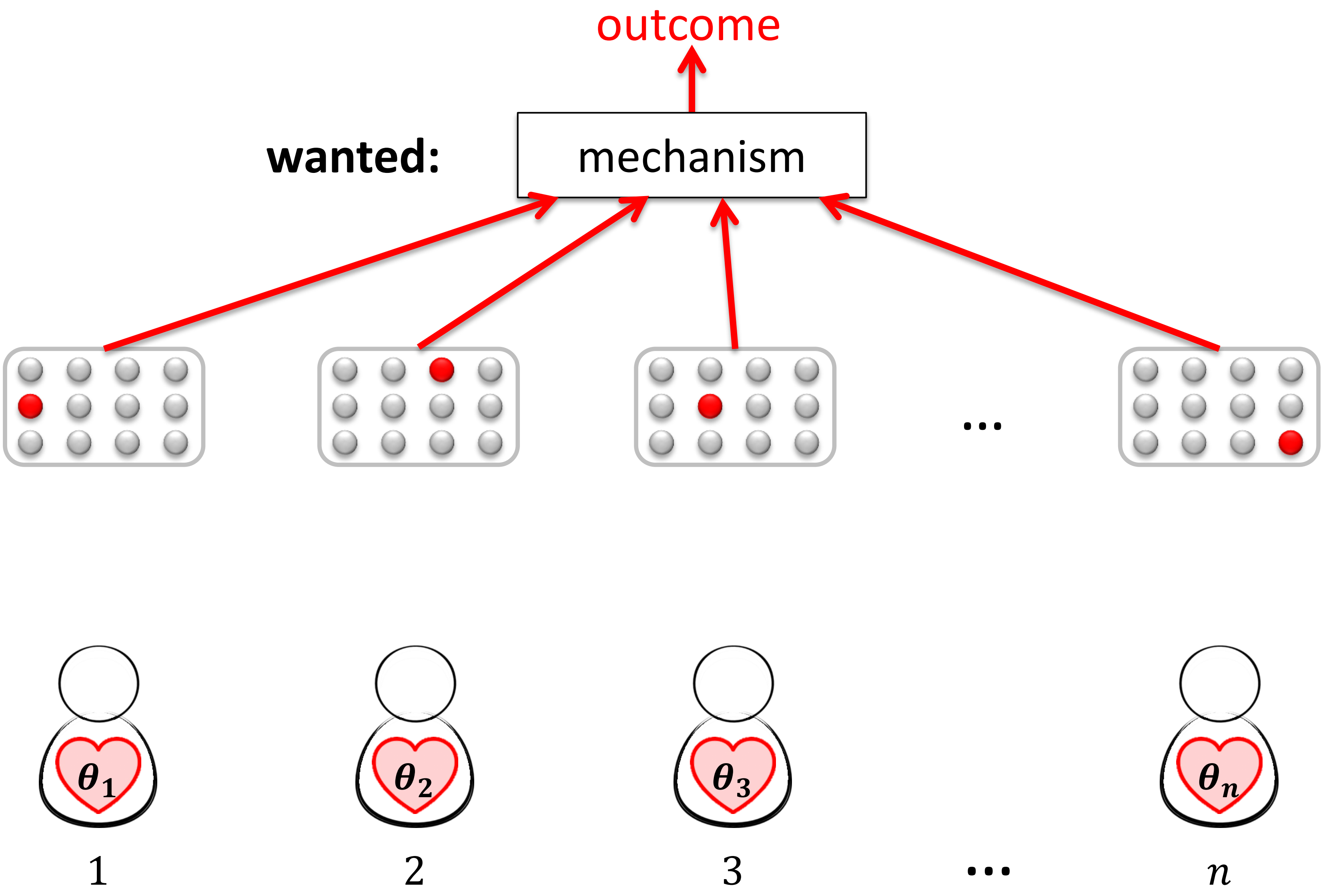
$\Pr[1 \text{ wins}] = 0$ ☹

Recall…



wanted: mechanism

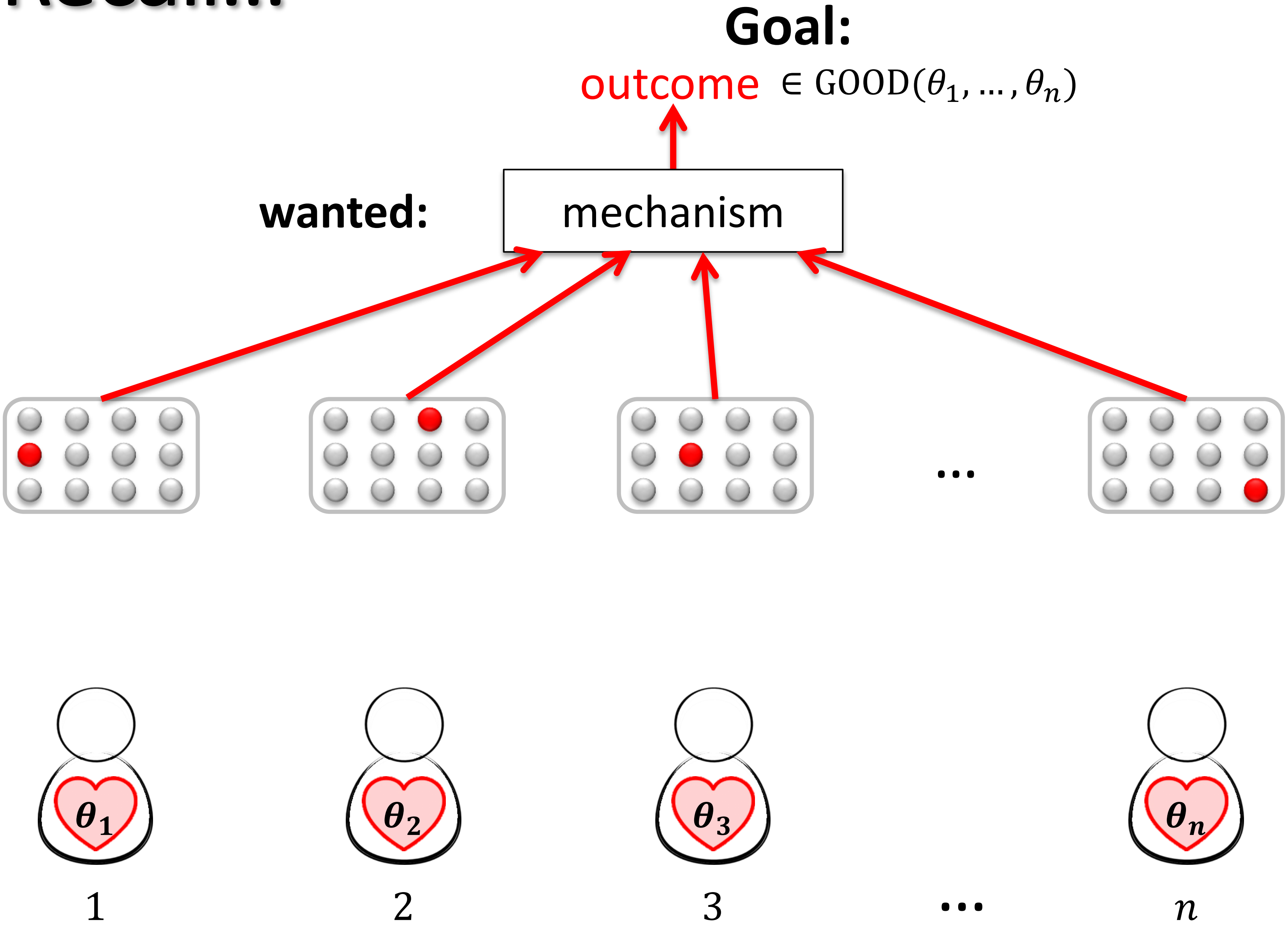
Recall…



outcome

wanted: mechanism

Recall…

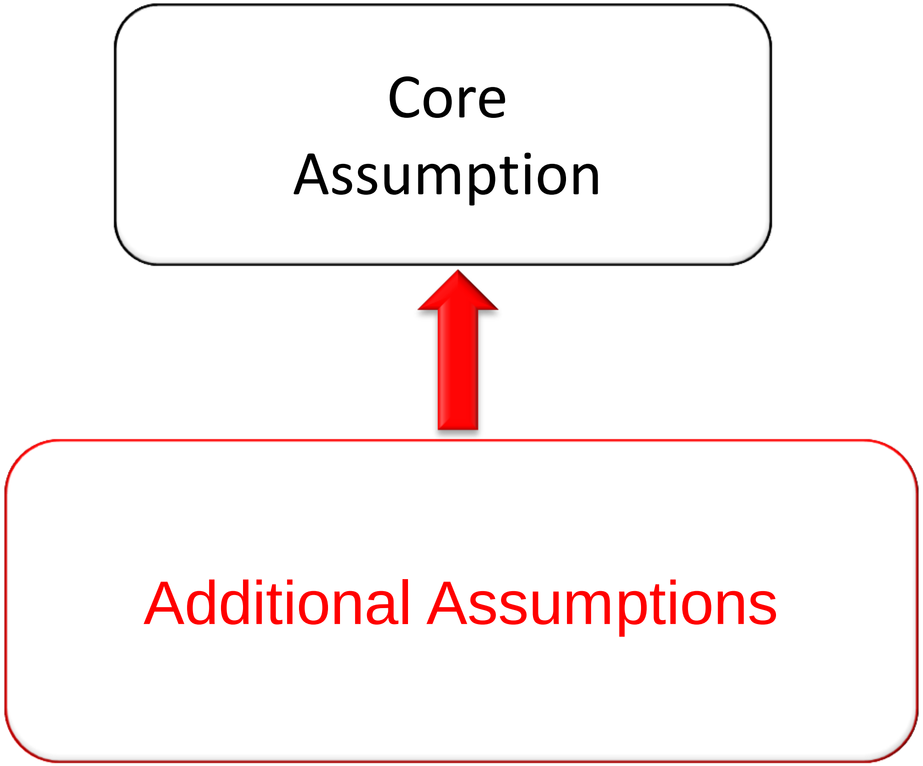


Goal:

outcome $\in \mathrm{GOOD}(\theta_1, \ldots, \theta_n)$

wanted: mechanism

1. Motivation

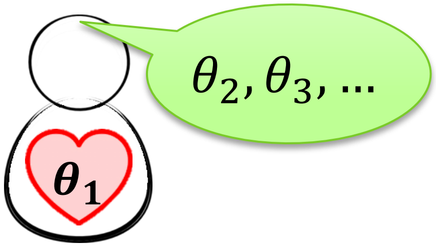


Core
Assumption

Additional Assumptions

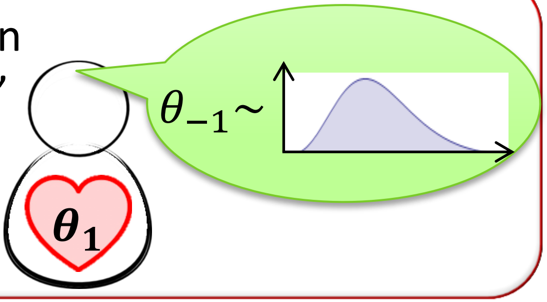
Core Assumption

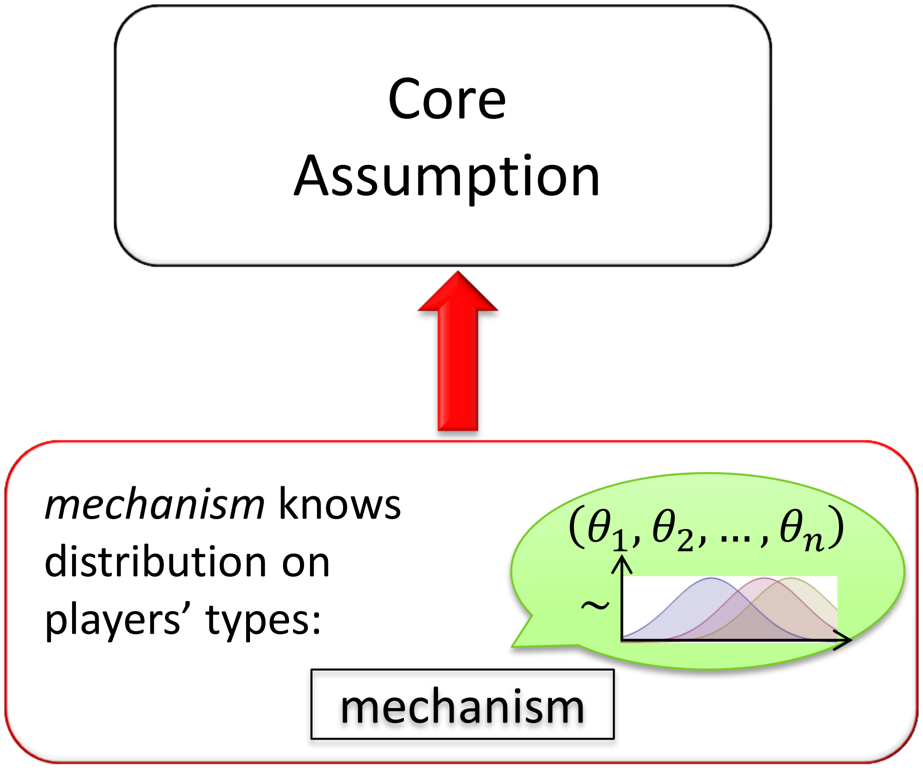
know other players' types:



Core Assumption

know distribution on other players' types:



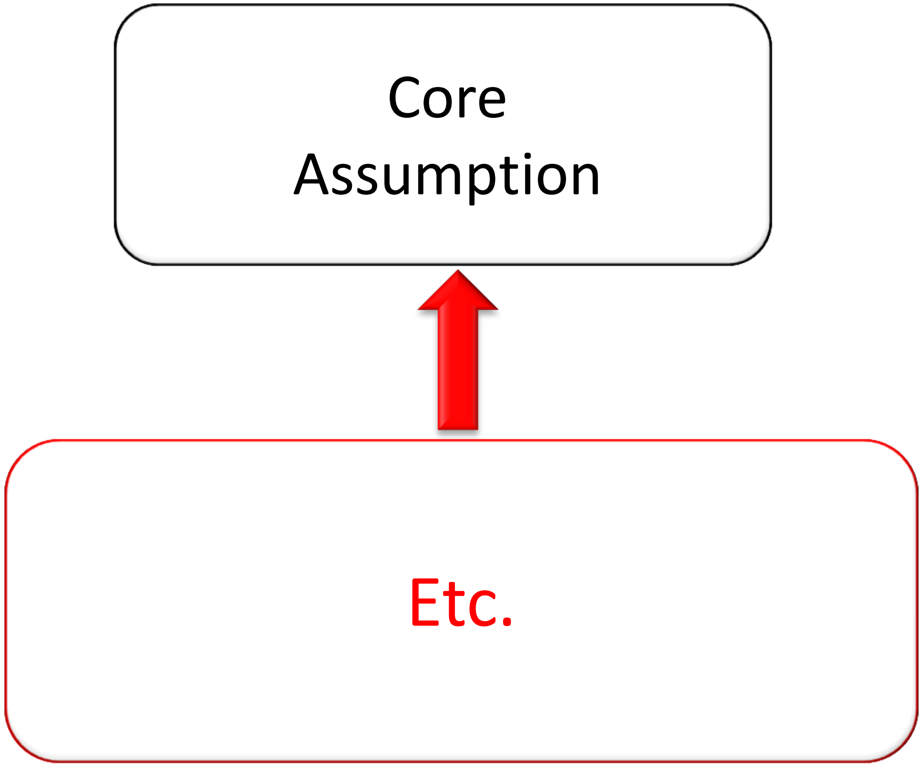


Core Assumption

mechanism knows distribution on players' types:

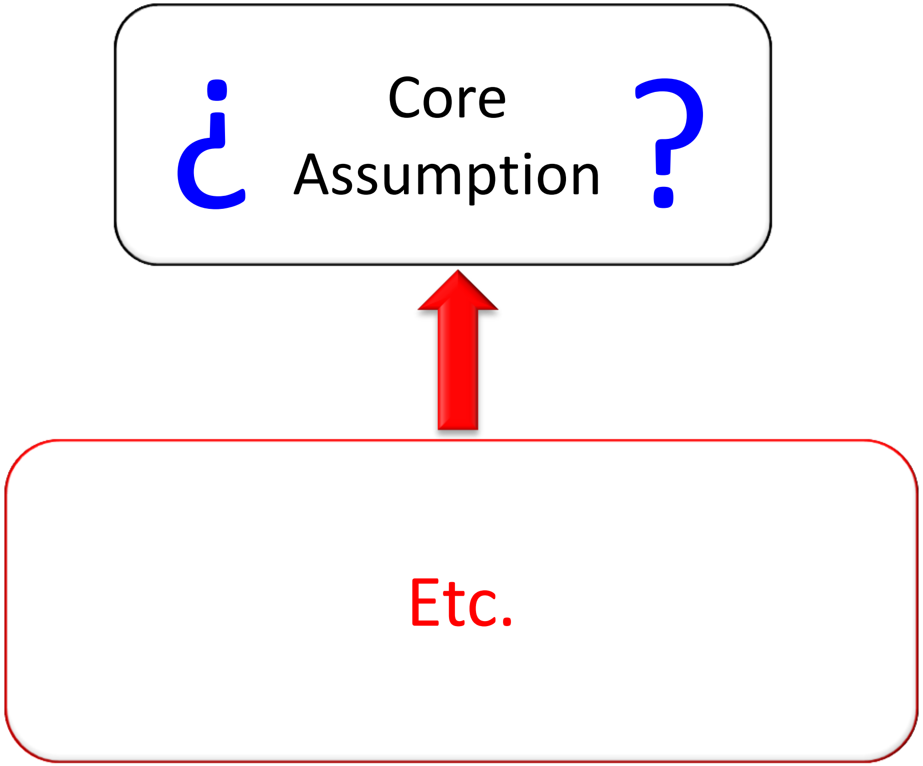
$(\theta_1, \theta_2, \dots, \theta_n) \sim$

mechanism



Core Assumption

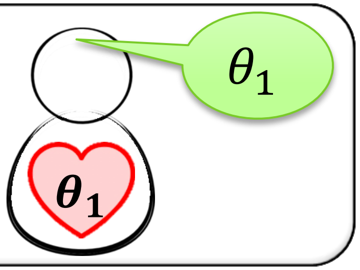
Etc.



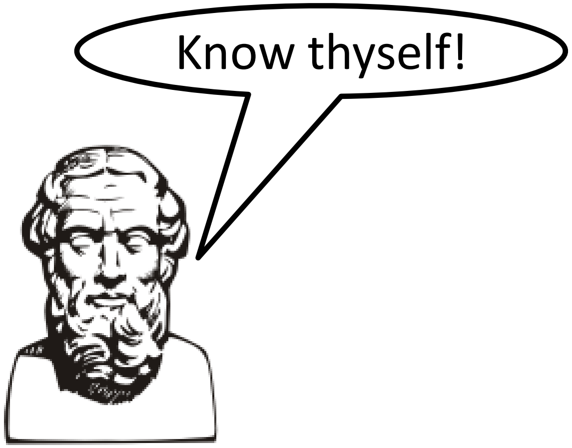
¿ Core Assumption ?

Etc.

know your own (payoff) type exactly:



Etc.



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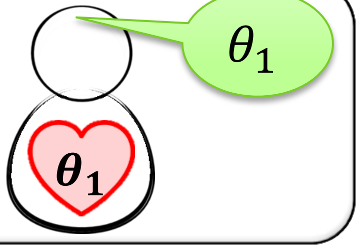
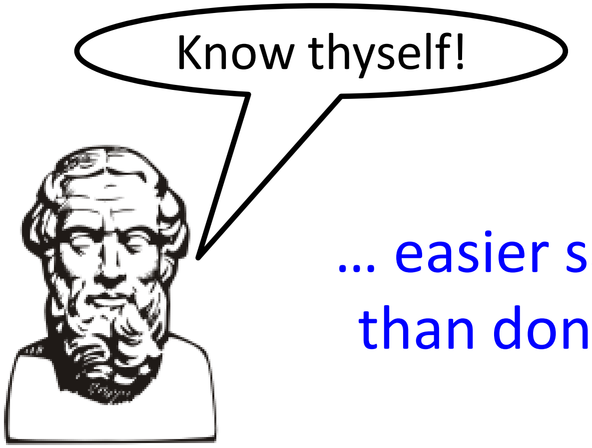
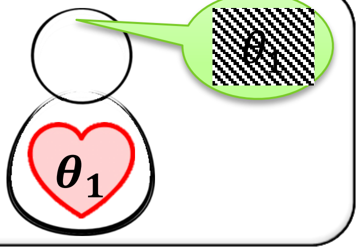


Etc.



... easier said than done!

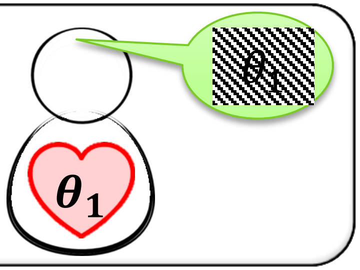
know your own (payoff) type exactly:



Etc.

Today's focus

know your
own (payoff)
type exactly:



Today’s focus

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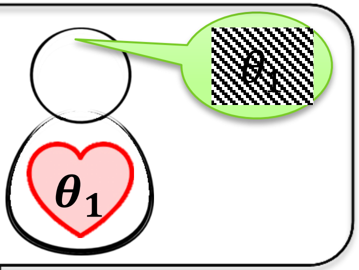

approximately

Today's focus

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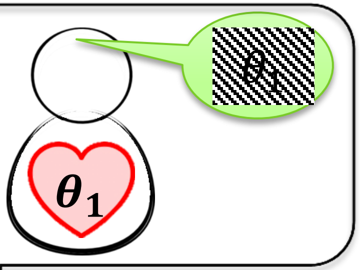
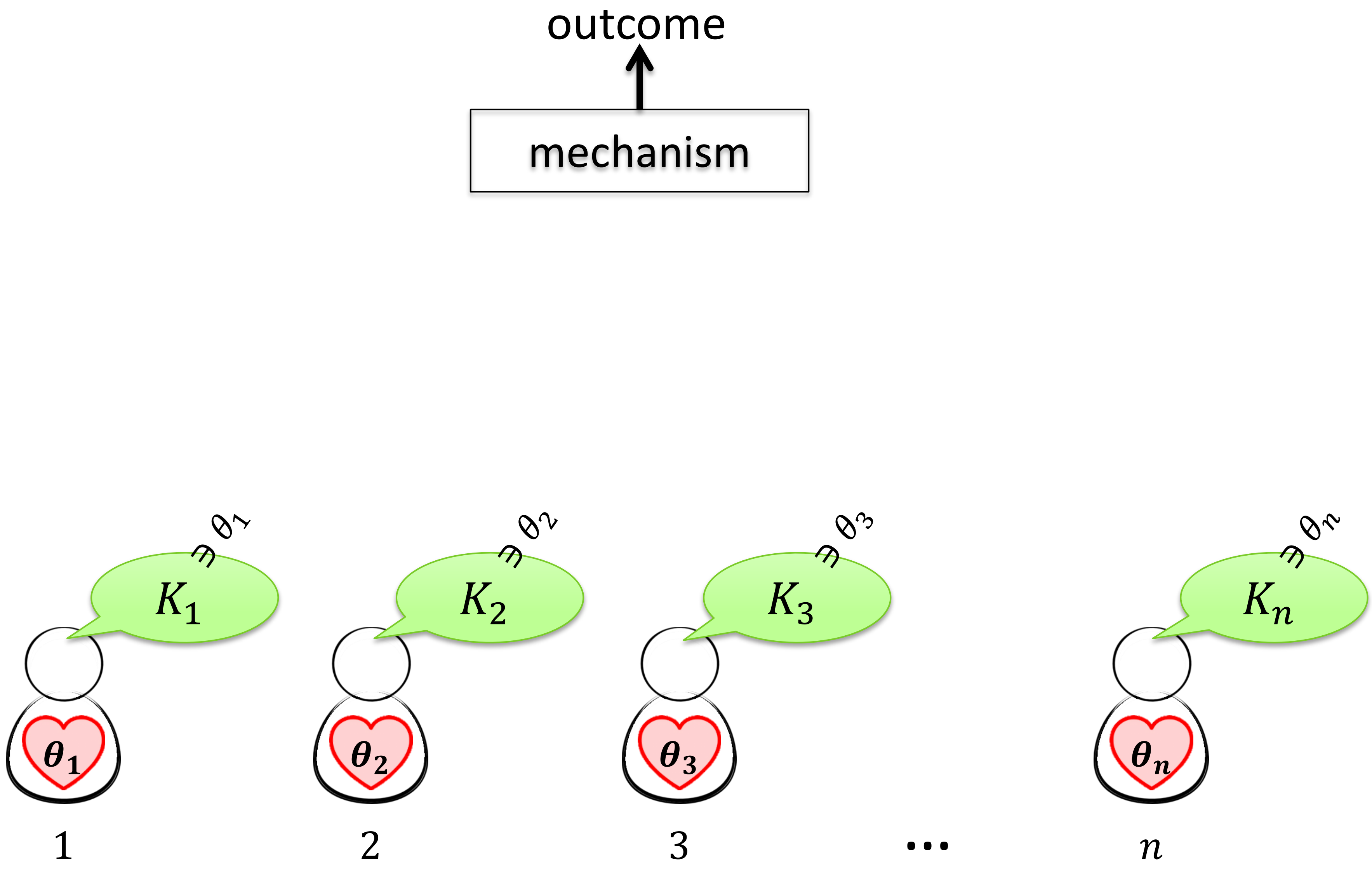


approximately

Within 1% or 10% or 25% …

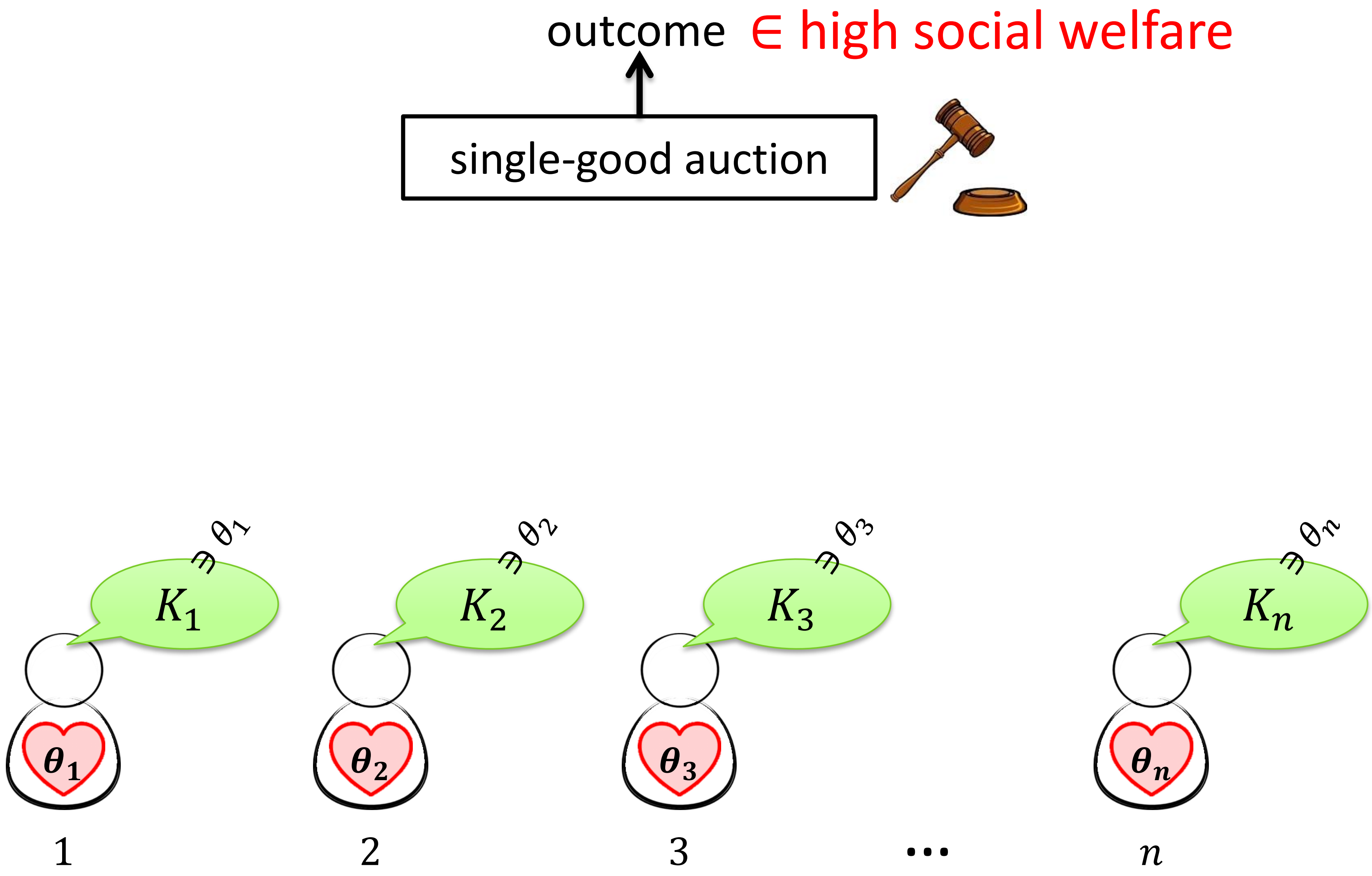
Approximate Types



outcome

mechanism

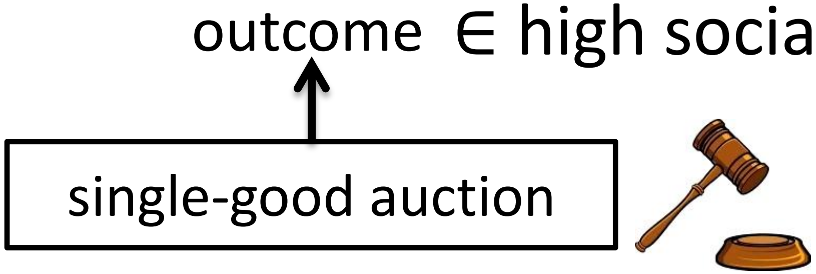
Approximate Types



outcome $\in$ high social welfare

single-good auction

Approximate Valuations

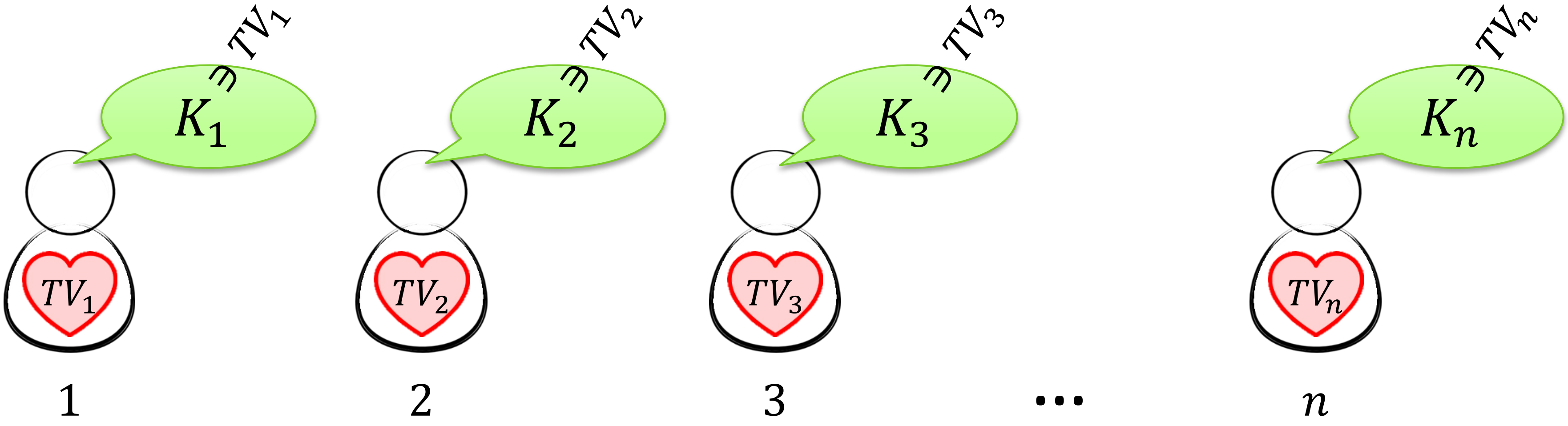


outcome ∈ high social welfare

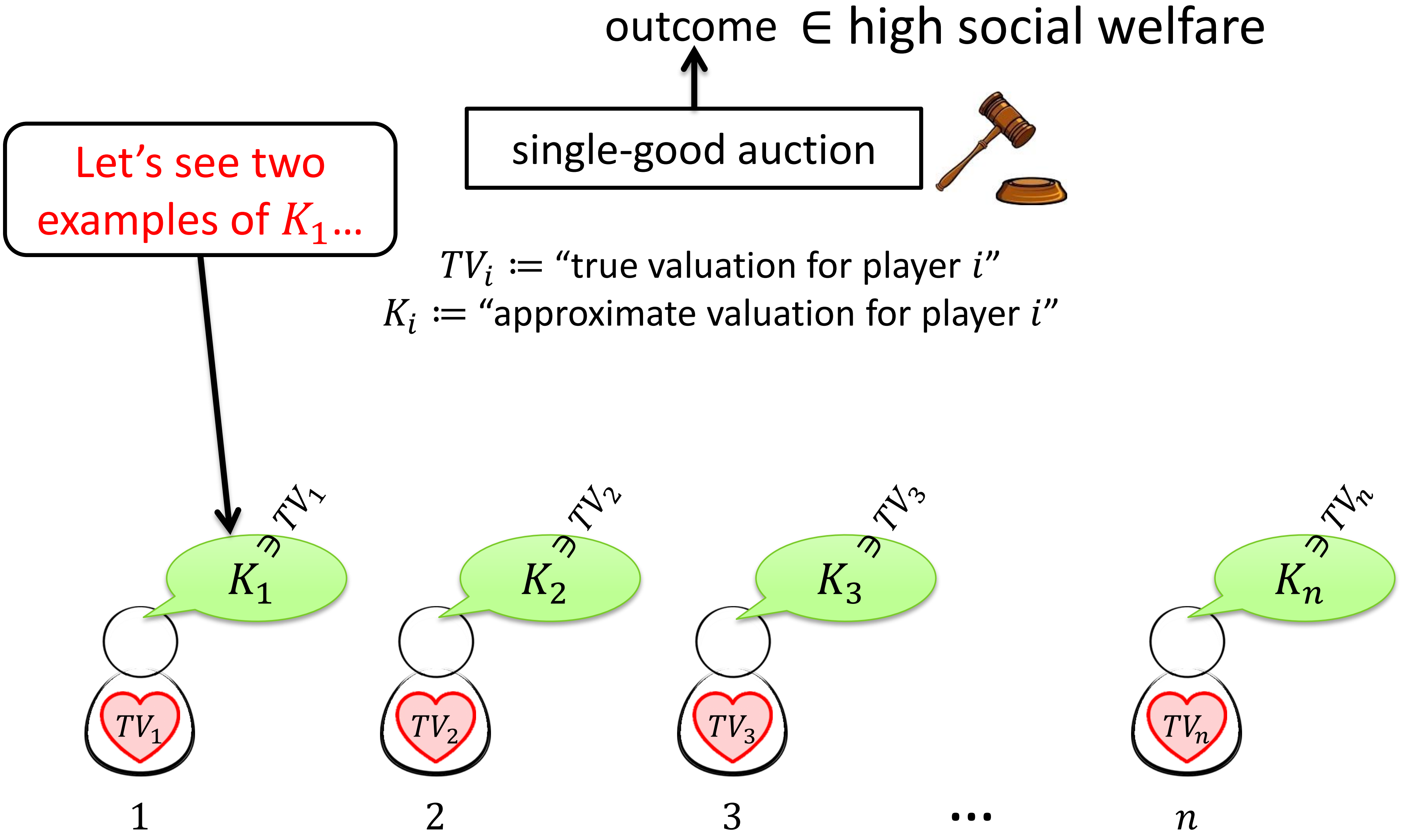
single-good auction

TV_i := “true valuation for player i”

K_i := “approximate valuation for player i”



Approximate Valuations



outcome $\in$ high social welfare

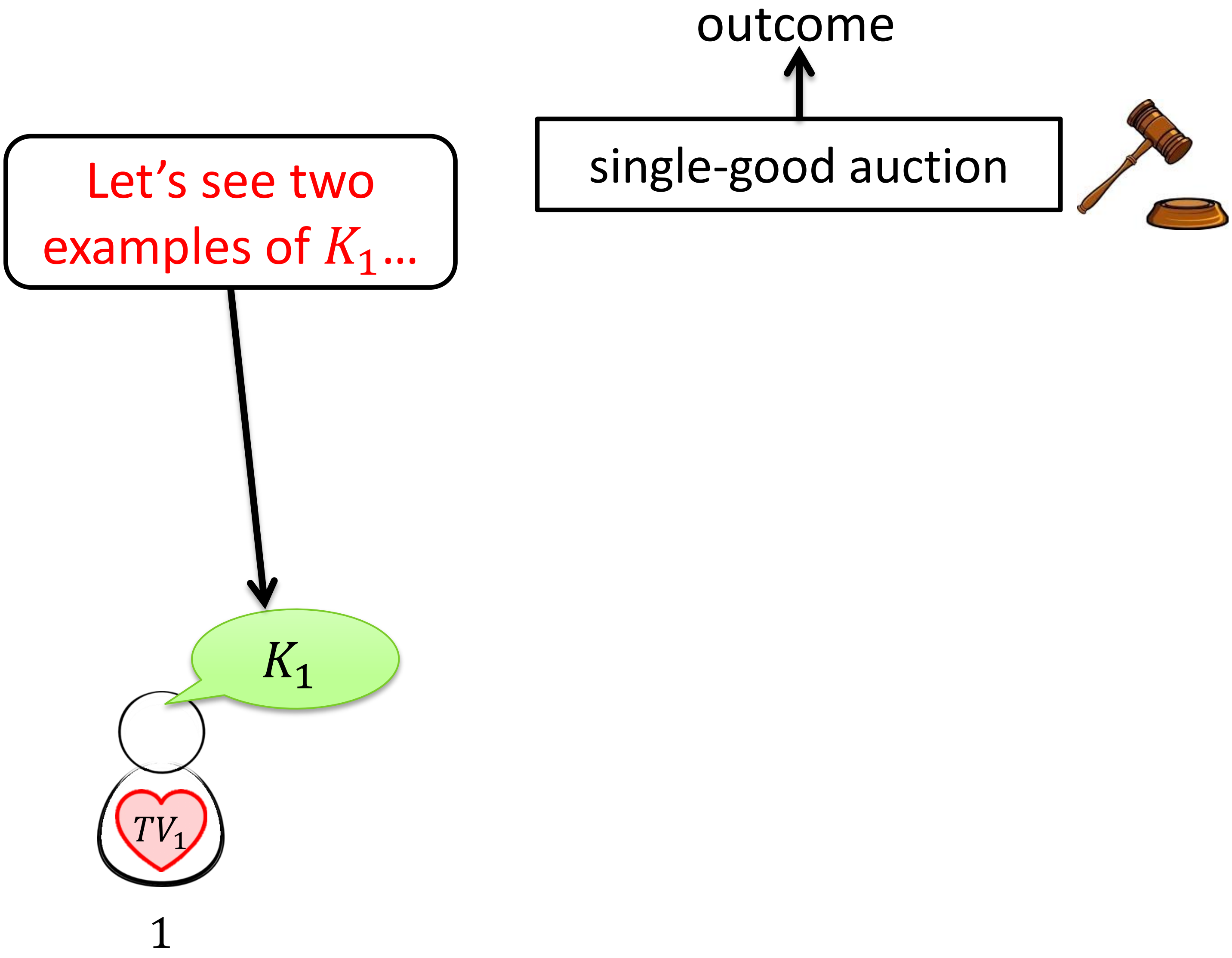
single-good auction

Let’s see two examples of K_1…

TV_i ≔ “true valuation for player i”

K_i ≔ “approximate valuation for player i”

Approximate Valuations



outcome

single-good auction

Let’s see two examples of K_1…

K_1

TV_1

1

Approximate Valuations

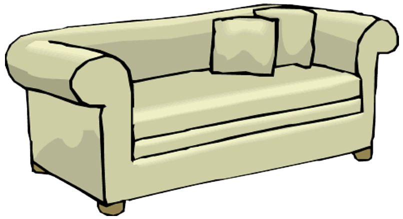
outcome

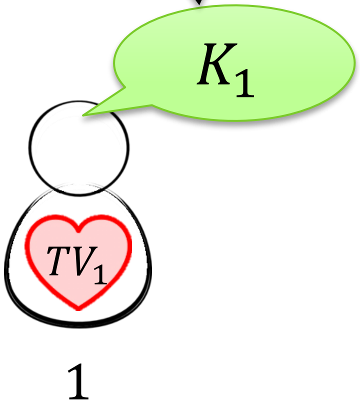
single-good auction

Let's see two examples of K_1…

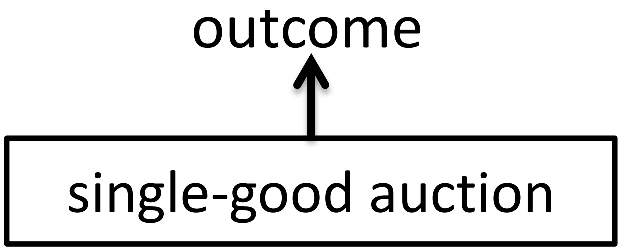
Example 1:

Example 2:





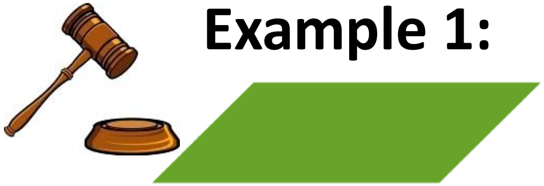
Approximate Valuations



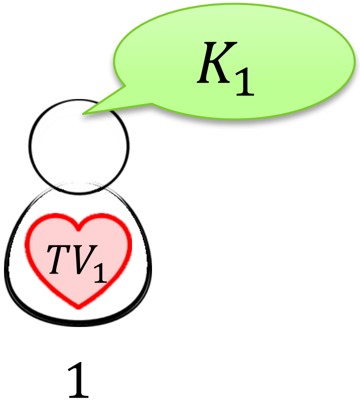
outcome

single-good auction

Example 1:



constructible plot of land

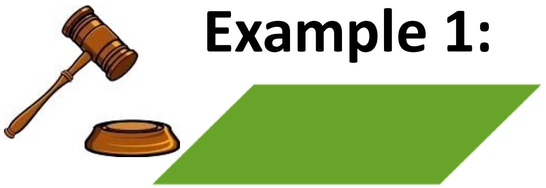


Approximate Valuations

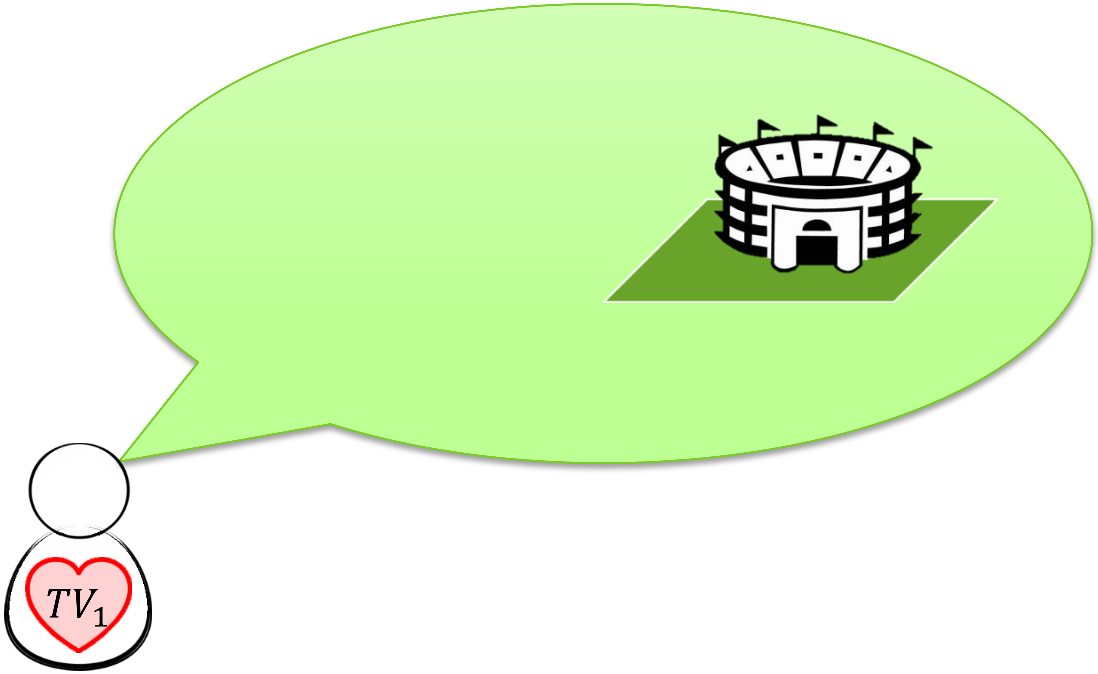
outcome

single-good auction

Example 1:



constructible plot of land

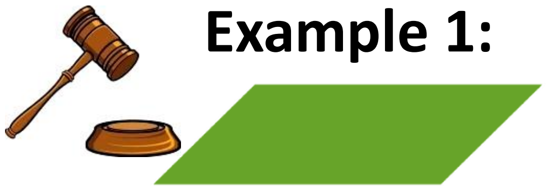


1

Approximate Valuations

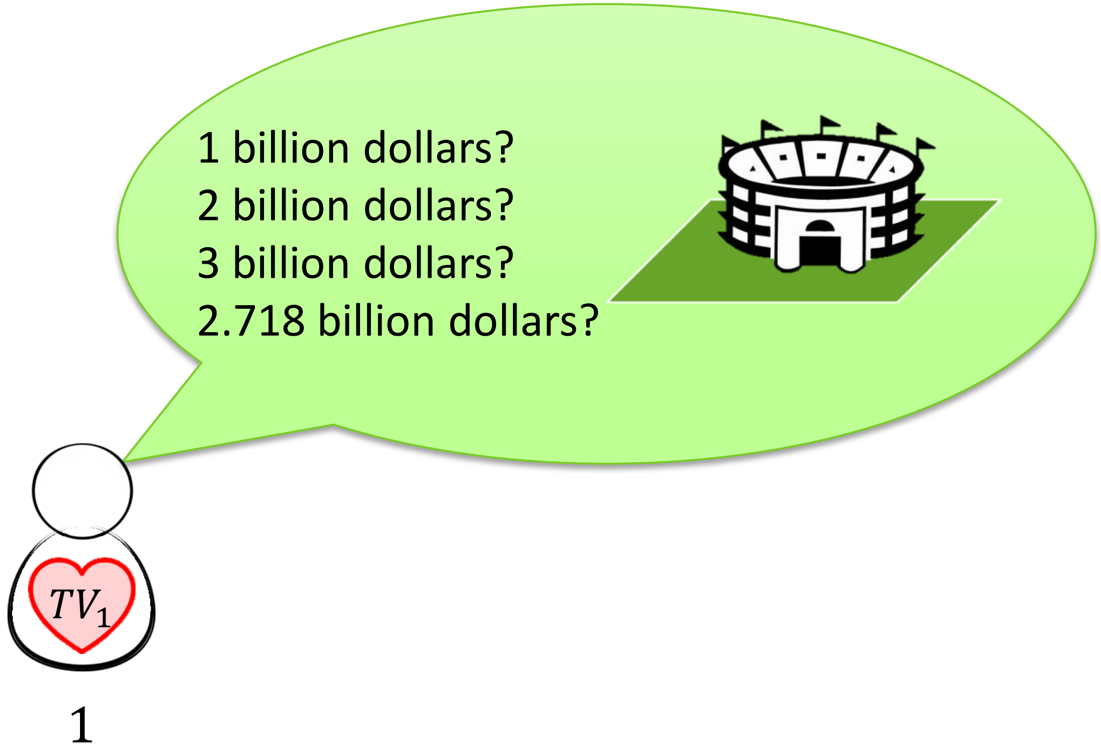
outcome

single-good auction



Example 1:

constructible plot of land



1 billion dollars?
2 billion dollars?
3 billion dollars?
2.718 billion dollars?

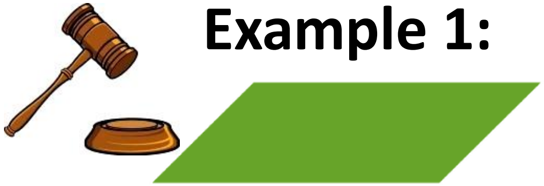
Approximate Valuations

outcome

↑

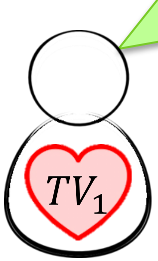
single-good auction

Example 1:



constructible plot of land

$$K_1 = [1 \times 10^9, 3 \times 10^9]$$



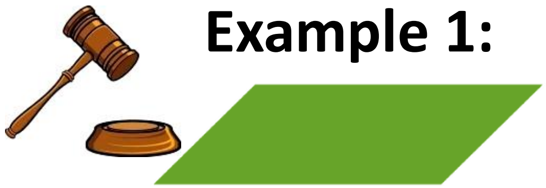
1

Approximate Valuations

outcome

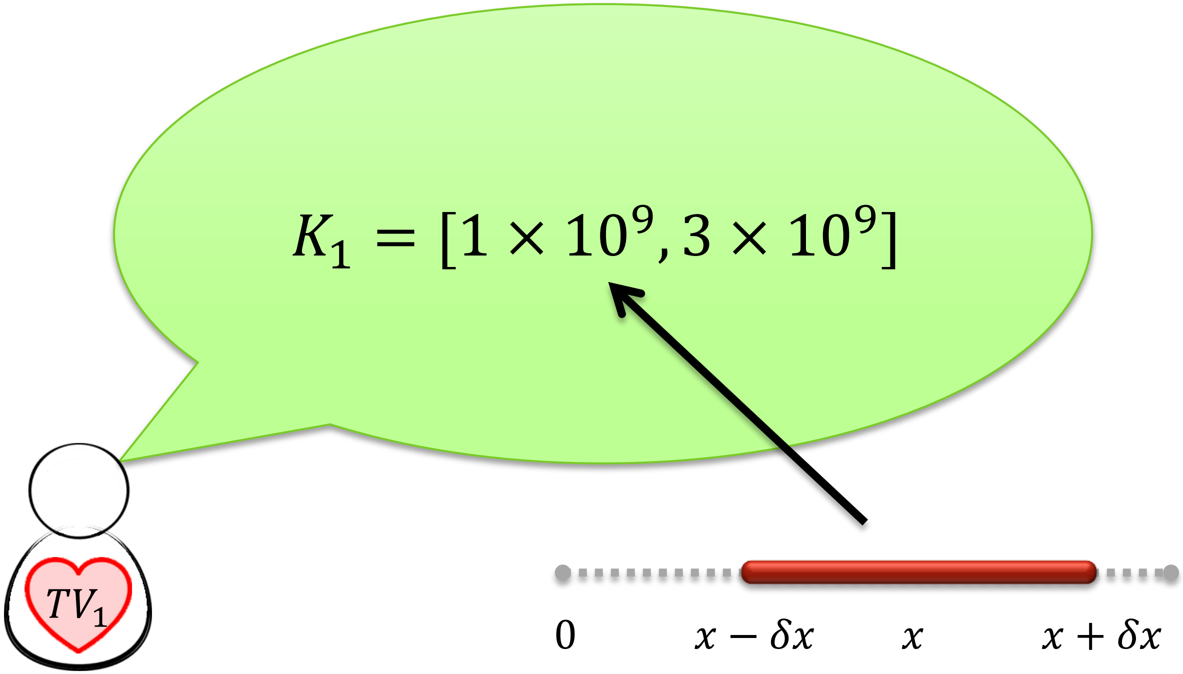
↑

single-good auction



Example 1:

constructible plot of land



$x = 2 \times 10^9$
$\delta = 0.5$

1

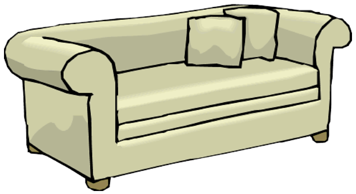
Approximate Valuations

outcome

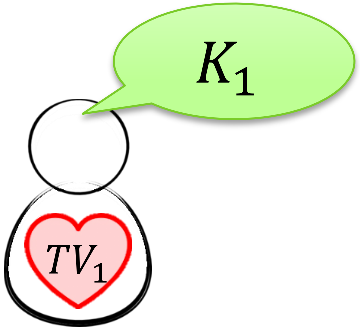
↑

single-good auction

Example 2:



yard sale



1

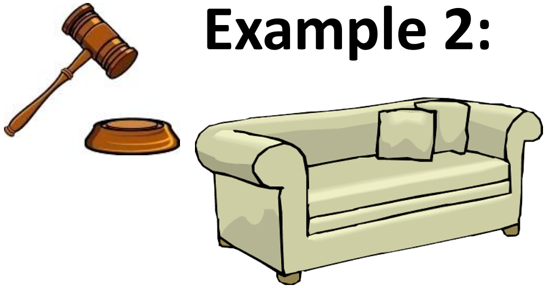
Approximate Valuations

outcome

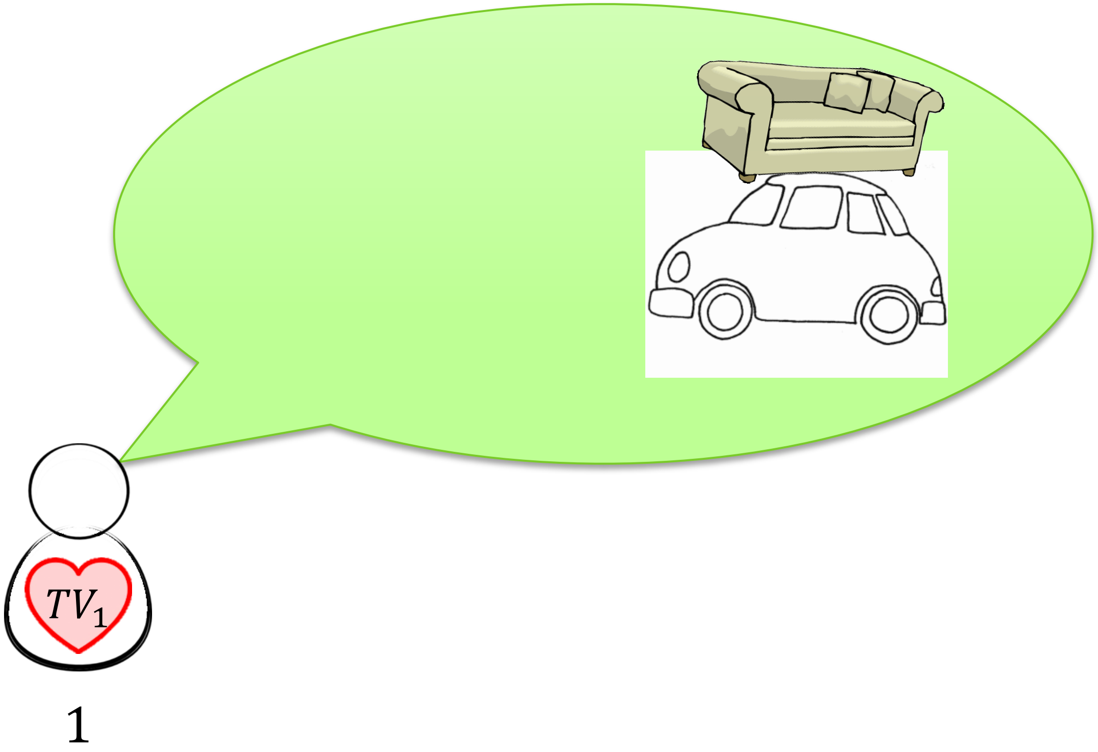
↑

single-good auction

Example 2:



yard sale

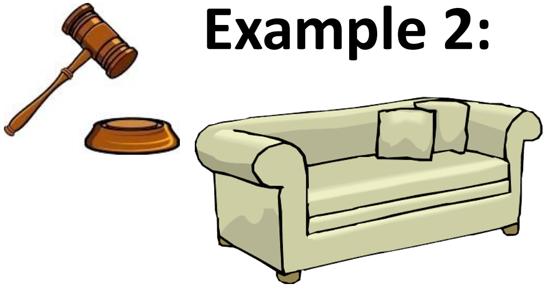


Approximate Valuations

outcome

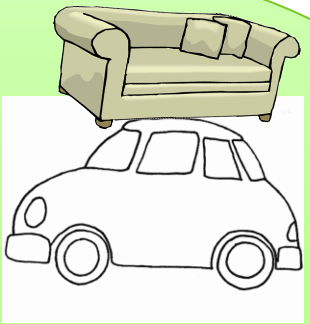
single-good auction

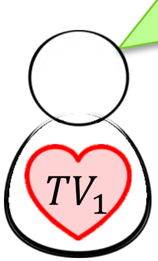
Example 2:



yard sale

- $600 if caught by the police;
- $1000 if not.





1

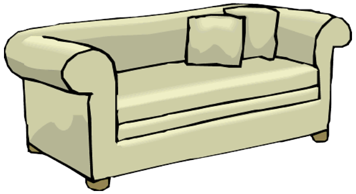
Approximate Valuations

outcome

↑

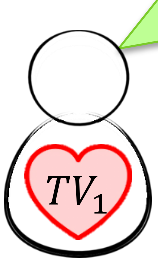
single-good auction

Example 2:



yard sale

$$K_1 = \{600,1000\}$$



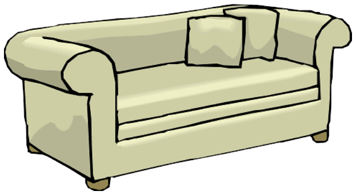
1

Approximate Valuations

outcome

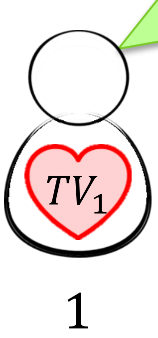
single-good auction

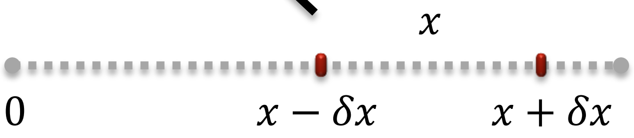
Example 2:



yard sale

$$K_1 = \{600,1000\}$$





$$x = 800$$
$$\delta = 0.25$$

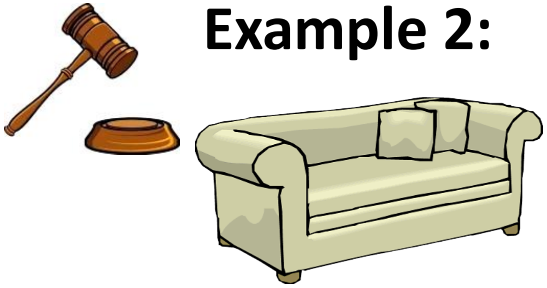
$$K_1 \subseteq [(1-\delta)800, (1+\delta)800]$$

Approximate Valuations

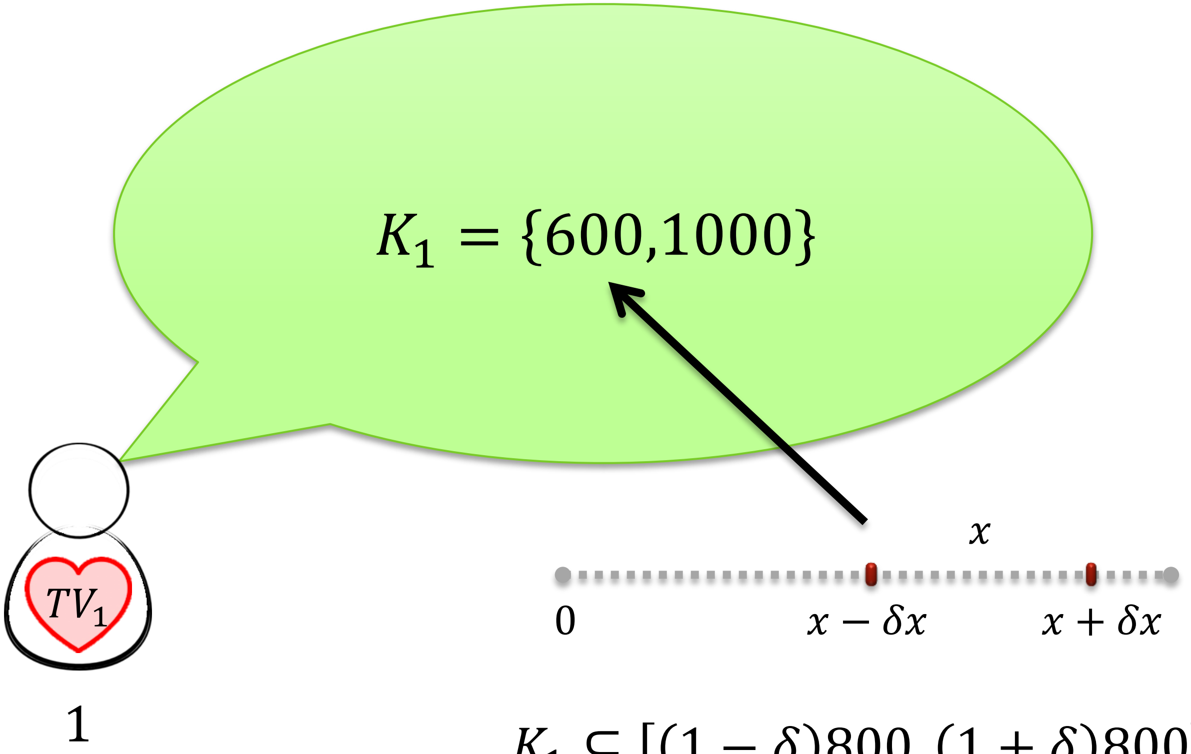
outcome

single-good auction

Example 2:



yard sale



$$K_1 = \{600,1000\}$$

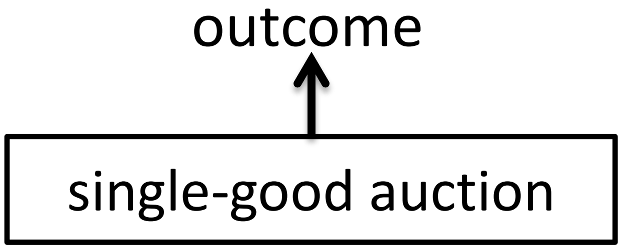
δ = "approximation inaccuracy"

$x = 800$

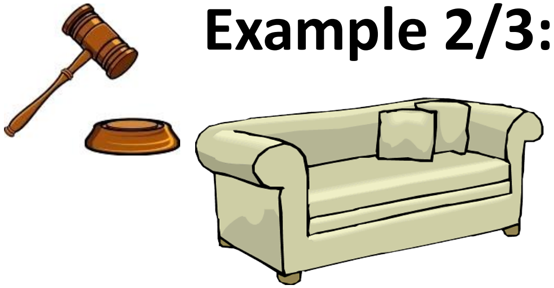
$\delta = 0.25$

$$K_1 \subseteq [(1-\delta)800, (1+\delta)800]$$

Approximate Valuations

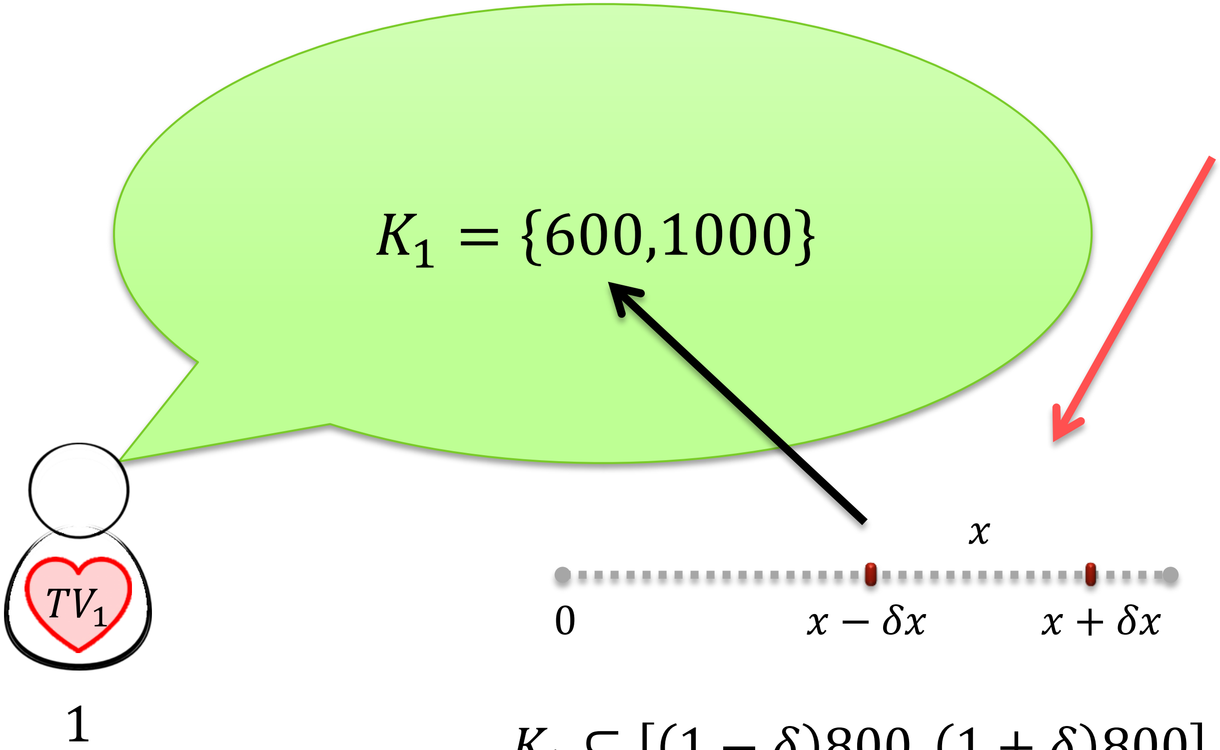


Example 2/3:



yard sale

different from Bayesian!



$$K_1 = \{600{,}1000\}$$

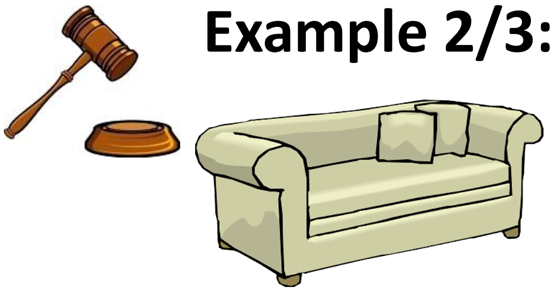
$$K_1 \subseteq [(1-\delta)800, (1+\delta)800]$$

Approximate Valuations

outcome

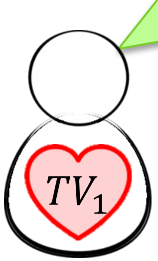
single-good auction

Example 2/3:

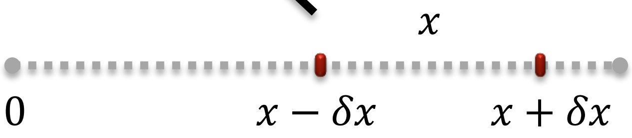


yard sale

$K_1 = \{600{,}1000\}$



1



$$K_1 \subseteq [(1-\delta)800, (1+\delta)800]$$

different from Bayesian!

$$\Pr\left[\begin{matrix}\text{police in another}\\ \text{neighborhood}\end{matrix}\right]?$$

$$\Pr\left[\begin{matrix}\text{police is on}\\ \text{lunch break}\end{matrix}\right]?$$

$$\Pr\left[\begin{matrix}\text{police chasing}\\ \text{a thief}\end{matrix}\right]?$$

$\vdots$

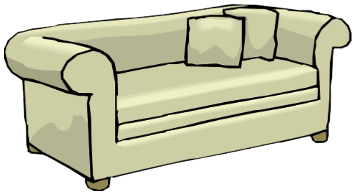
Approximate Valuations

outcome

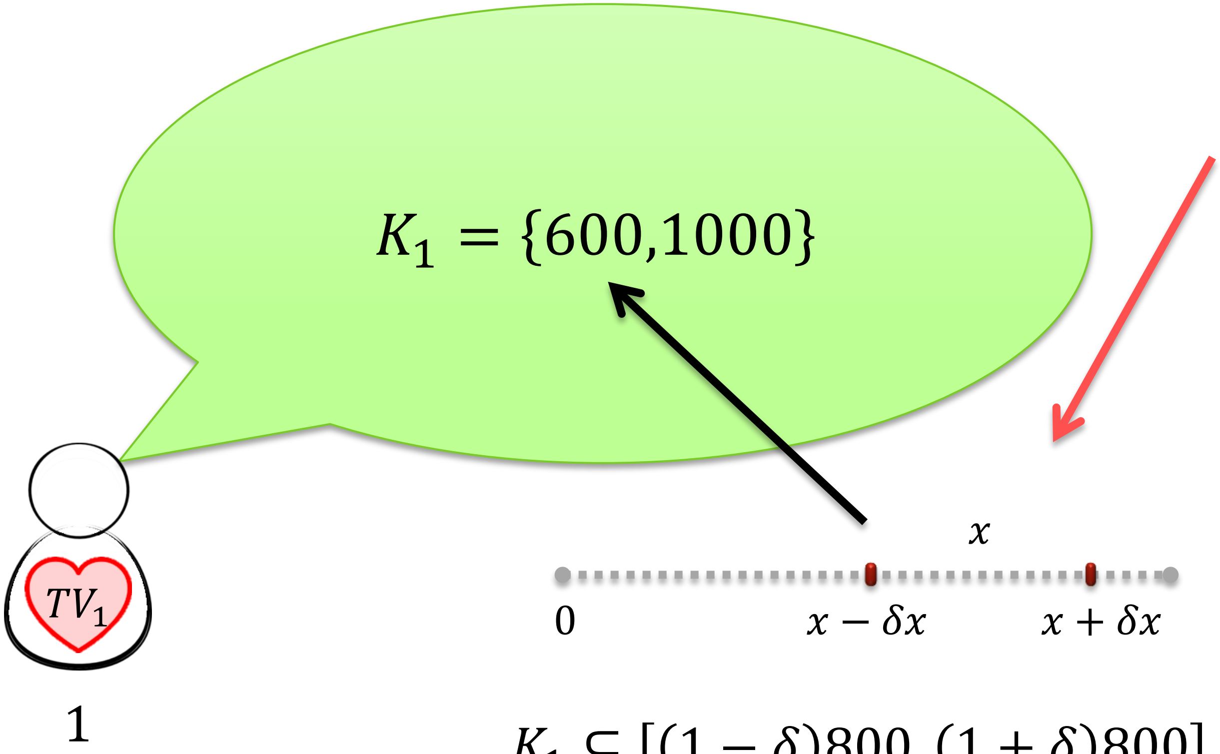
↑

single-good auction

Example 2:



yard sale



$K_1 = \{600{,}1000\}$

different from Bayesian!

$\Pr\begin{bmatrix}\text{police in another}\\ \text{neighborhood}\end{bmatrix}?$

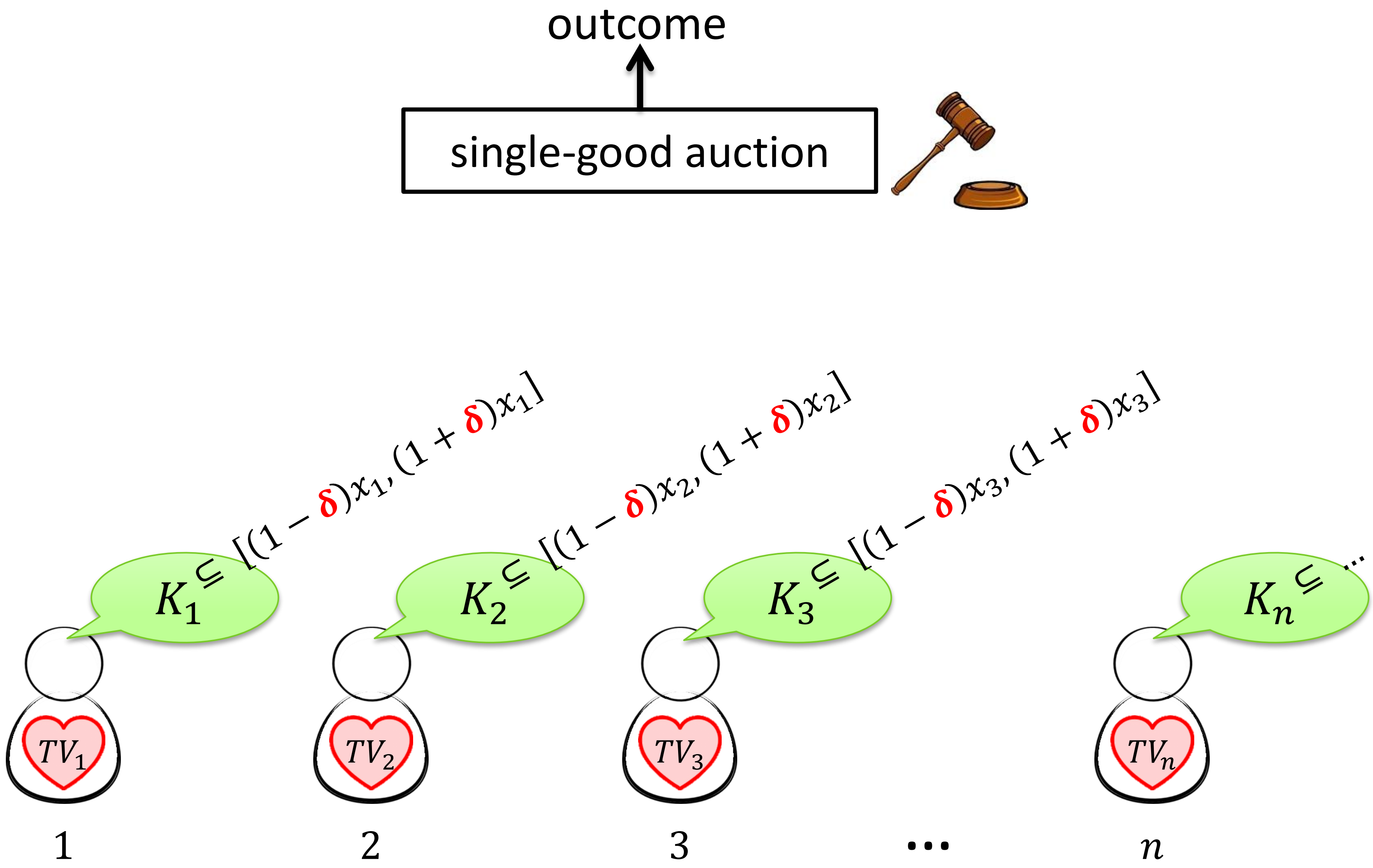
$\Pr\begin{bmatrix}\text{police is on}\\ \text{lunch break}\end{bmatrix}?$

$\Pr\begin{bmatrix}\text{police chasing}\\ \text{a thief}\end{bmatrix}?$

⋮

$$K_1 \subseteq [(1-\delta)800, (1+\delta)800]$$

Approximate Valuations (summary)



outcome

single-good auction

$K_1 \subseteq [(1-\boldsymbol{\delta})x_1, (1+\boldsymbol{\delta})x_1]$

$K_2 \subseteq [(1-\boldsymbol{\delta})x_2, (1+\boldsymbol{\delta})x_2]$

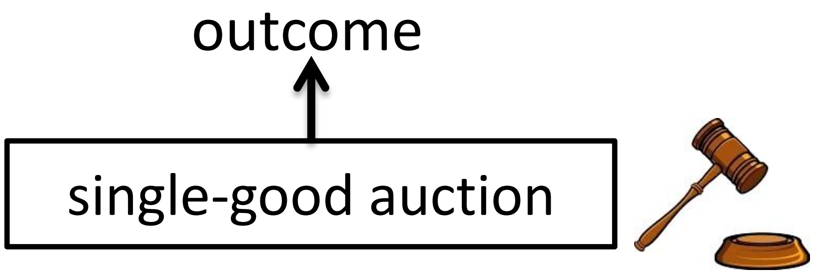
$K_3 \subseteq [(1-\boldsymbol{\delta})x_3, (1+\boldsymbol{\delta})x_3]$

$K_n \subseteq \ldots$

TV_1 TV_2 TV_3 TV_n

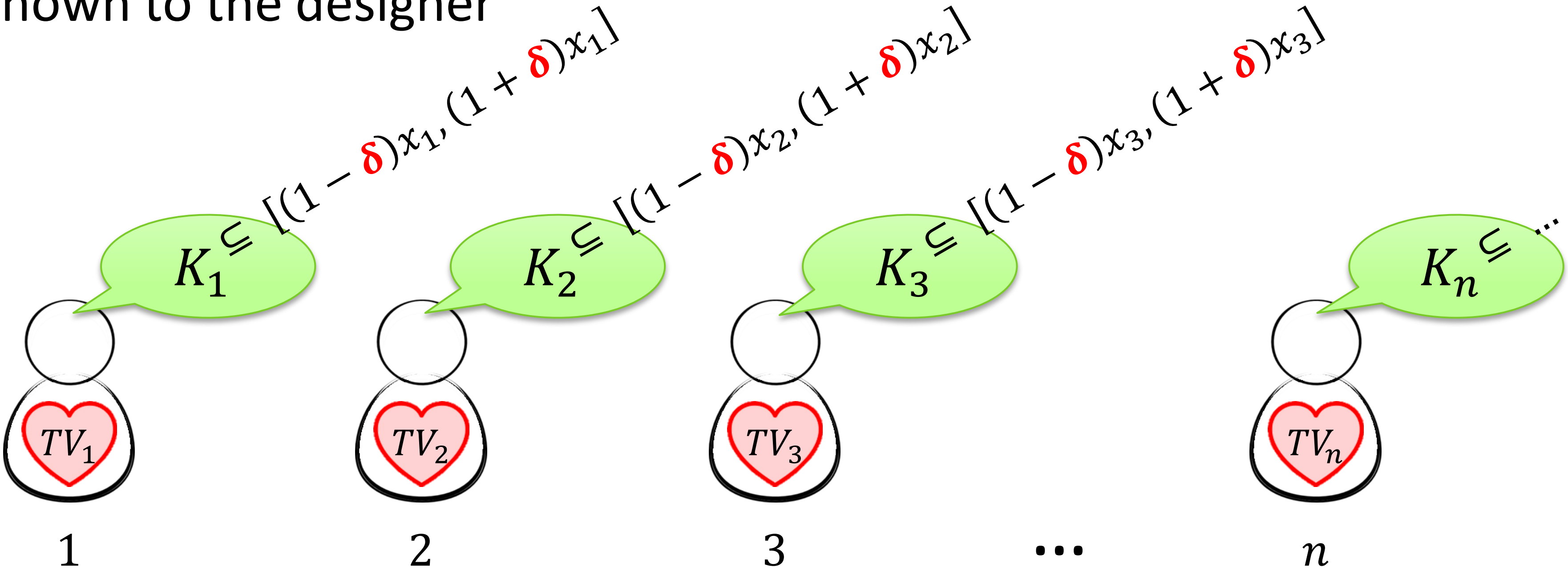
1 2 3 … n

Approximate Valuations (summary)



approximation inaccuracy δ is:

- guaranteed
- known to the designer



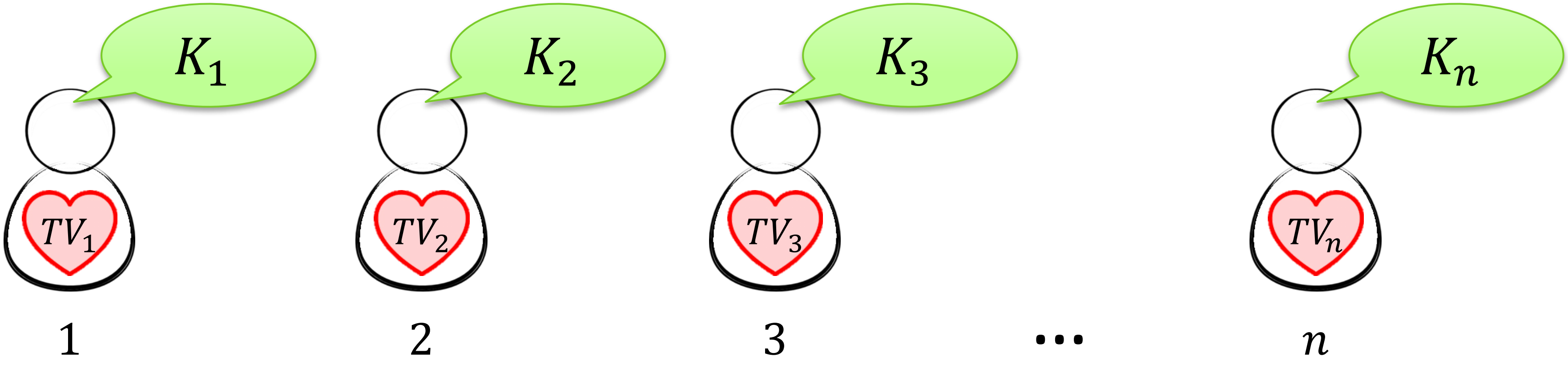
3. Our Question

Which Social Welfare?

outcome ∈ high social welfare

single-good auction

w.r.t. what?

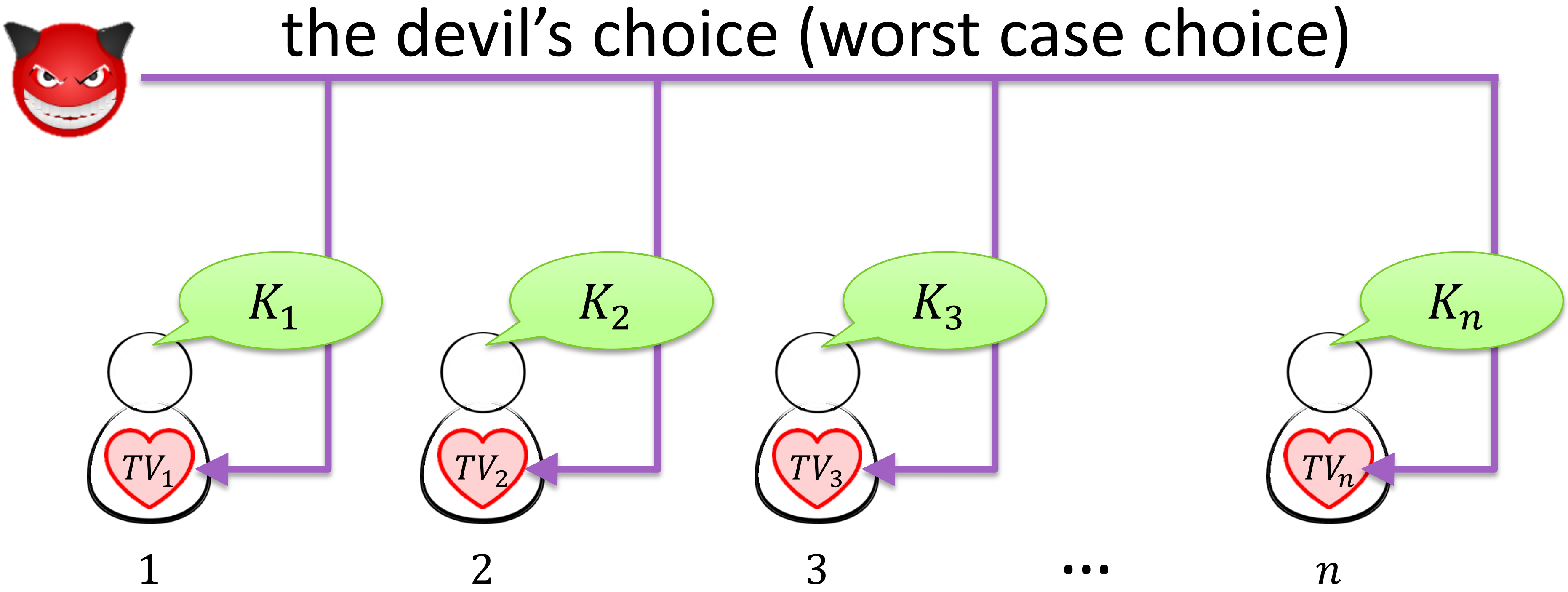


Which Social Welfare?

outcome ∈ high social welfare

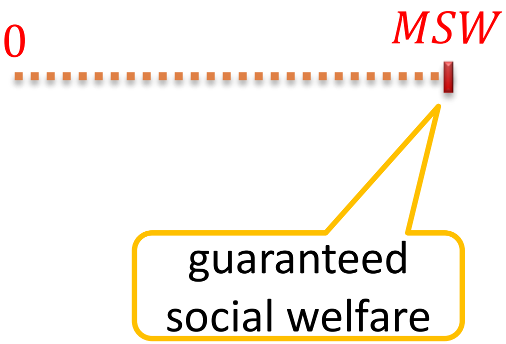
single-good auction

w.r.t. what?



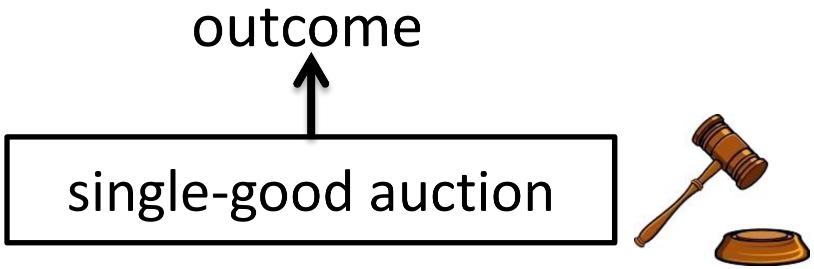
the devil's choice (worst case choice)

How Much SW Can We Get?



0 MSW

guaranteed social welfare



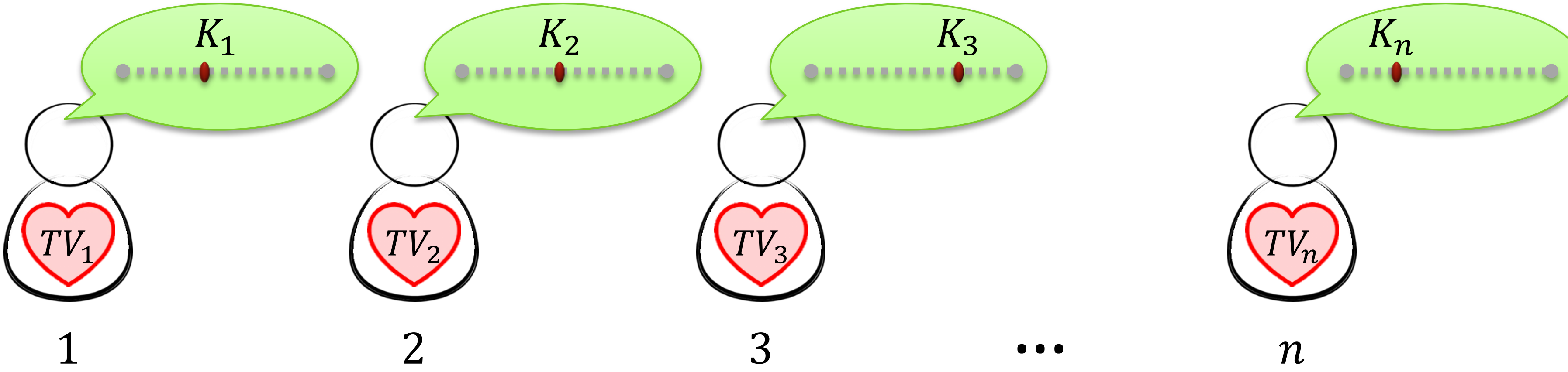
outcome

single-good auction

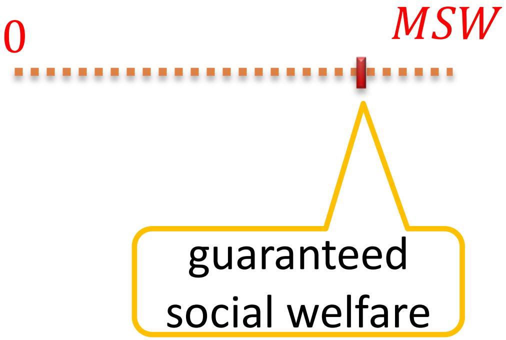
$\delta = 0$

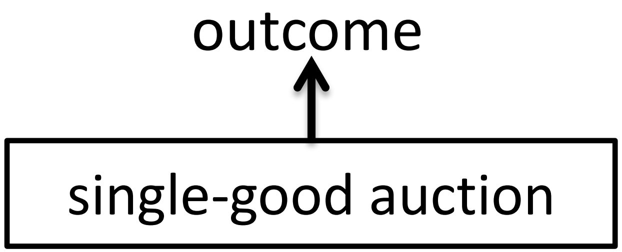
$\Longrightarrow$ guaranteed SW is maximum

(by the second-price mechanism)



How Much SW Can We Get?

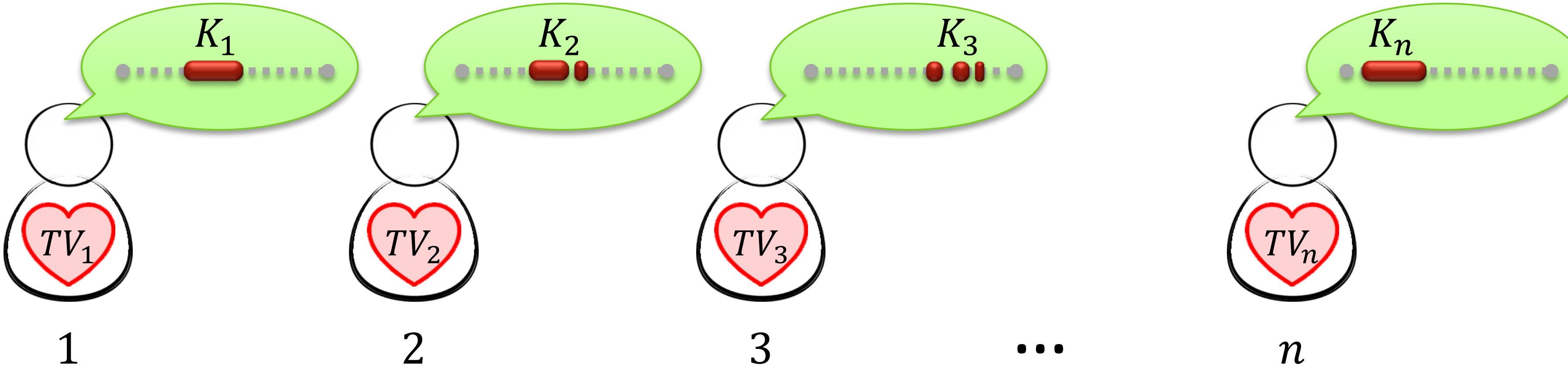



outcome

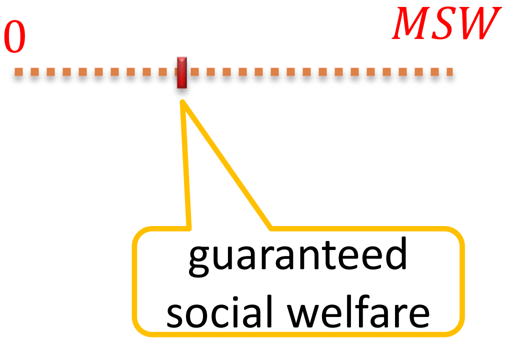
single-good auction

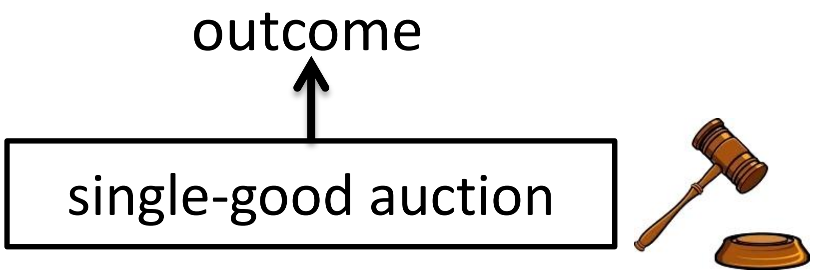
δ increases

$\Longrightarrow$ guaranteed SW decreases



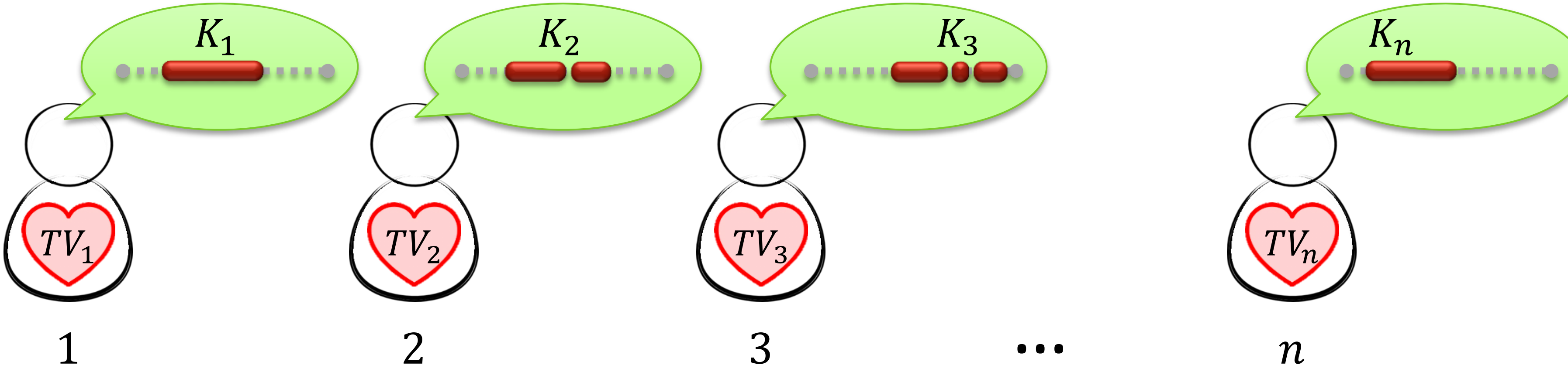
How Much SW Can We Get?



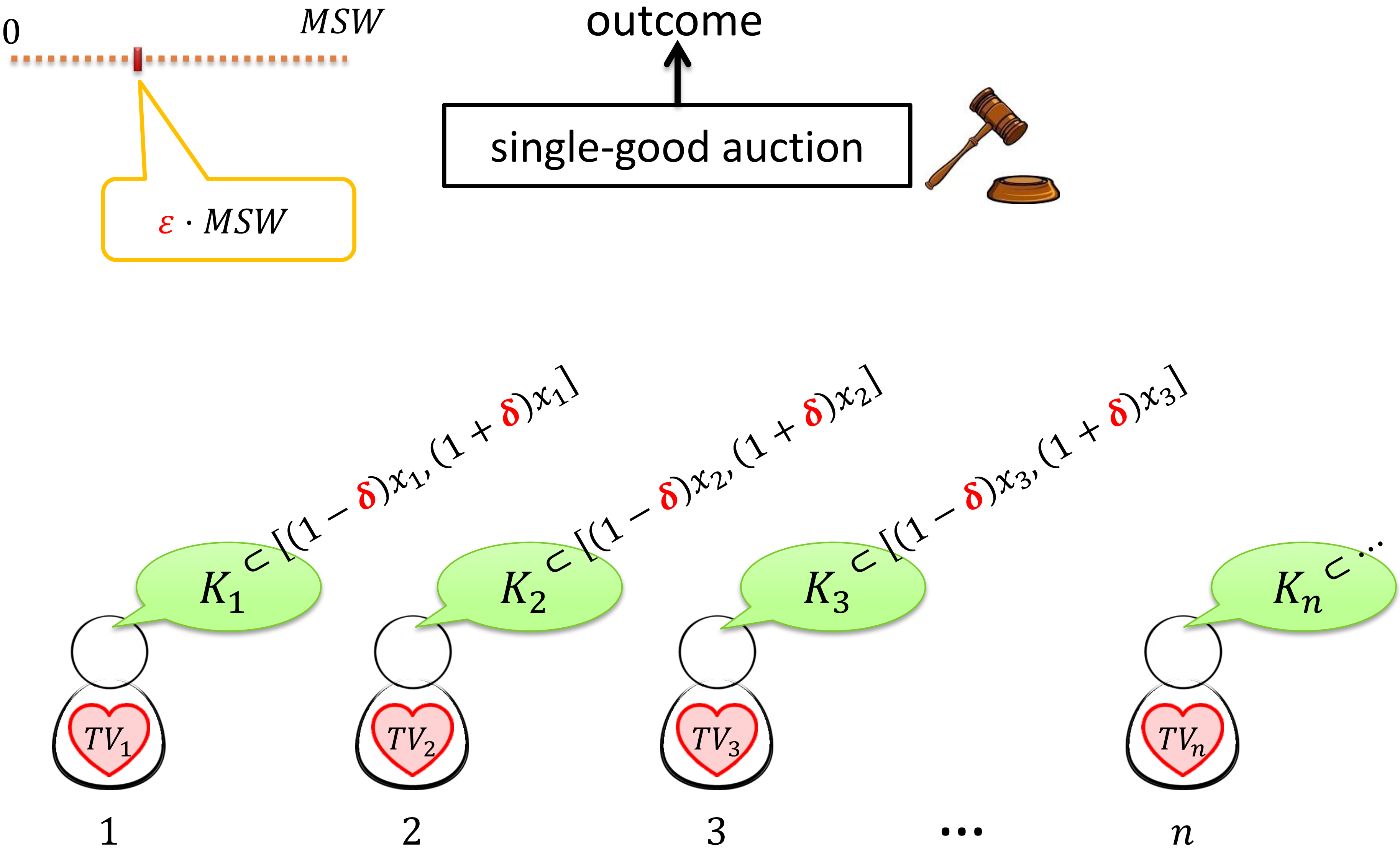


δ increases further

$\Longrightarrow$ guaranteed SW decreases further



How Much SW Can We Get?



0 MSW

$\varepsilon \cdot MSW$

outcome

single-good auction

$K_1 \subset [(1-\delta)x_1, (1+\delta)x_1]$

$K_2 \subset [(1-\delta)x_2, (1+\delta)x_2]$

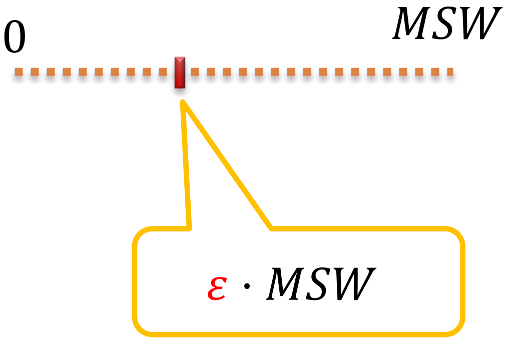
$K_3 \subset [(1-\delta)x_3, (1+\delta)x_3]$

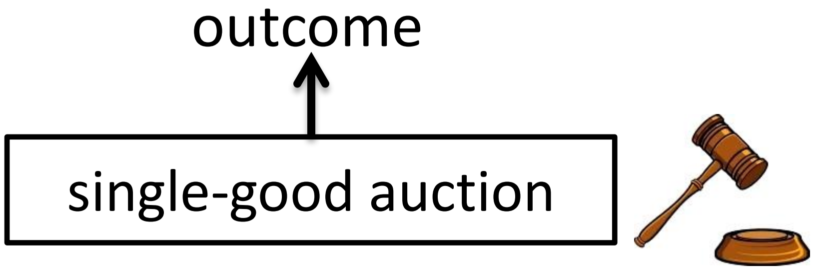
$K_n \subset \ldots$

TV_1 TV_2 TV_3 TV_n

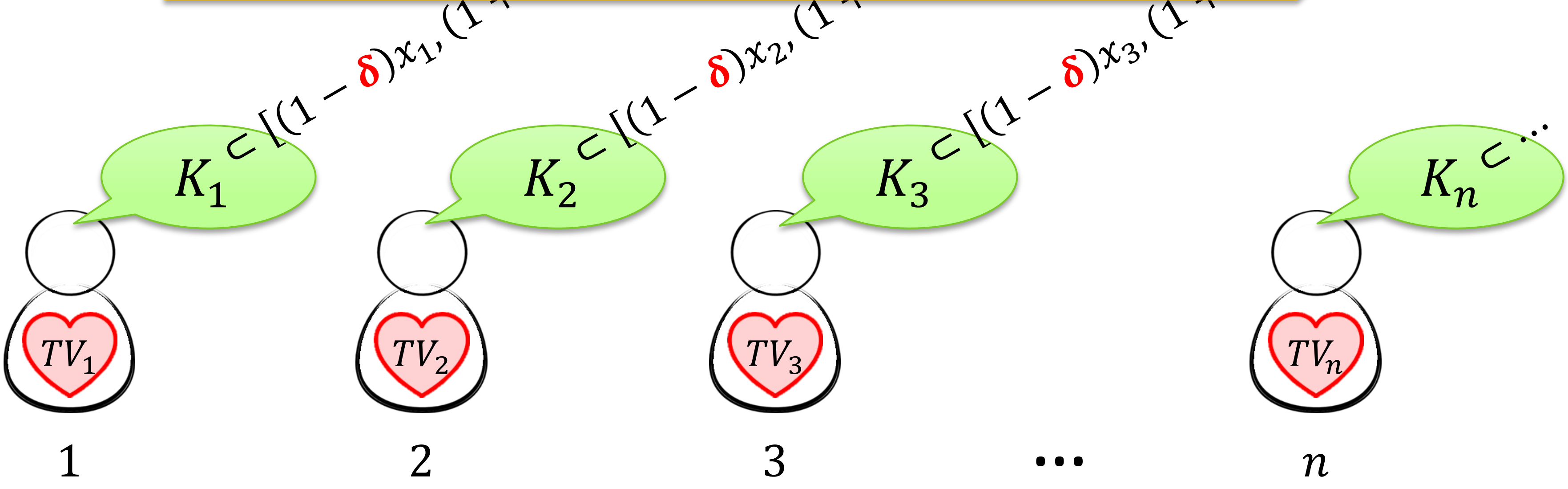
1 2 3 … n

Our Question

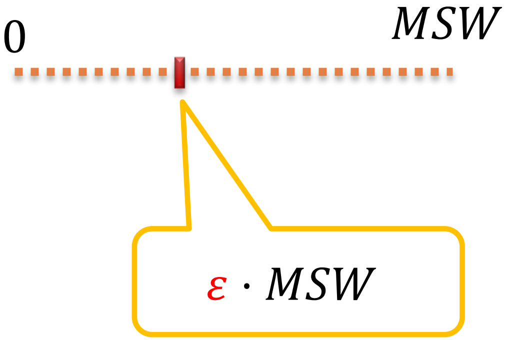


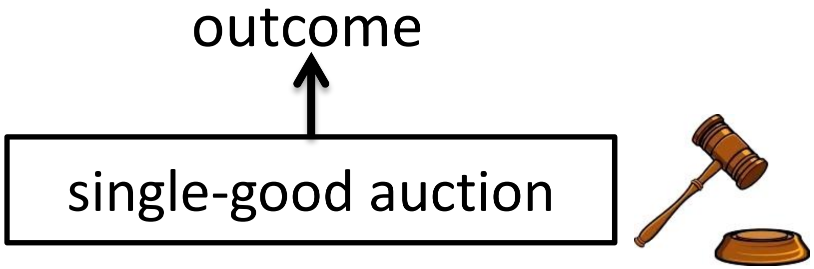


What is the best $\varepsilon(\delta, n)$?



Our Question

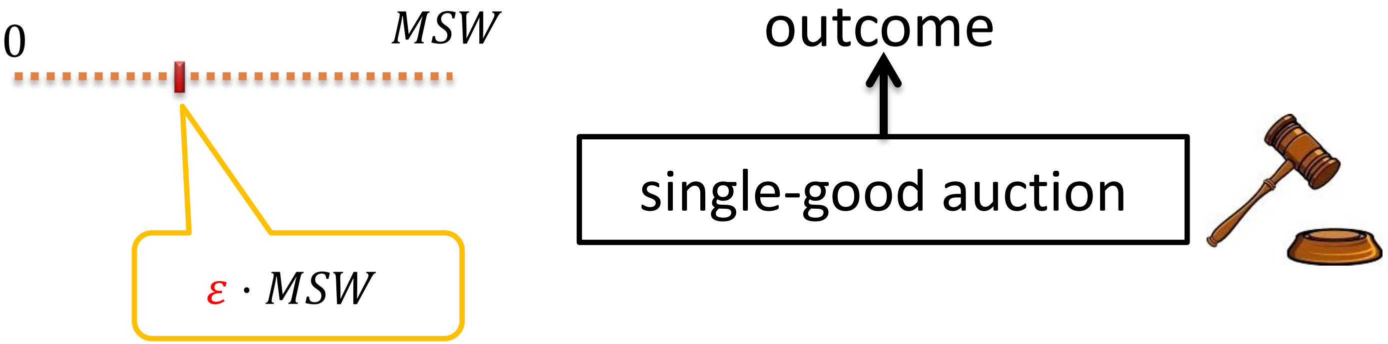




What is the best $\boldsymbol{\varepsilon(\delta, n)}$?

Under which solution concepts should we ask the question?

Our Question



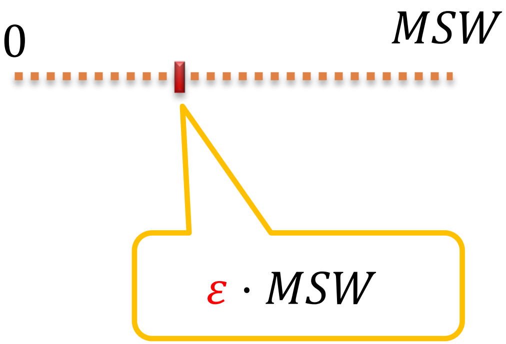
What is the best $\boldsymbol{\varepsilon(\delta, n)}$?

Under which solution concepts should we ask the question?

(non-Bayesian) incomplete information,
so two natural notions to consider:

1. implementation in **dominant** strategies
2. implementation in **undominated** strategies

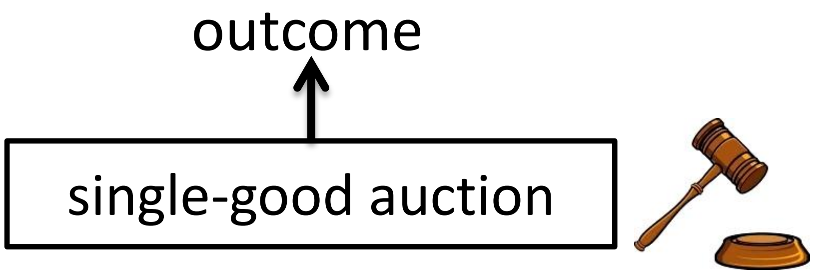
Our Question



0

MSW

$\varepsilon \cdot MSW$



outcome

single-good auction

What is the best $\varepsilon(\delta, n)$?

Under which solution concepts should we ask the question?

(non-Bayesian) incomplete information,
so two natural notions to consider:

1. implementation in **dominant** strategies
2. implementation in **undominated** strategies

(3. ex-post NE reduces to dominant strategies)

4. Our Results

Our Results

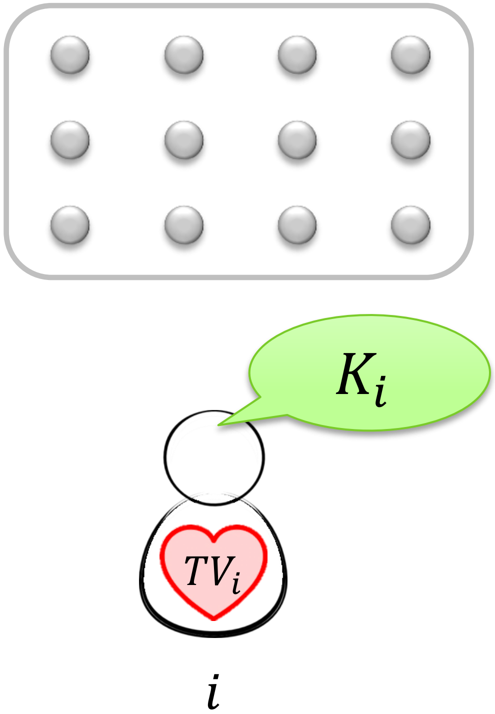
Implementation in …

… Dominant Strategies

Our Results

Implementation in …

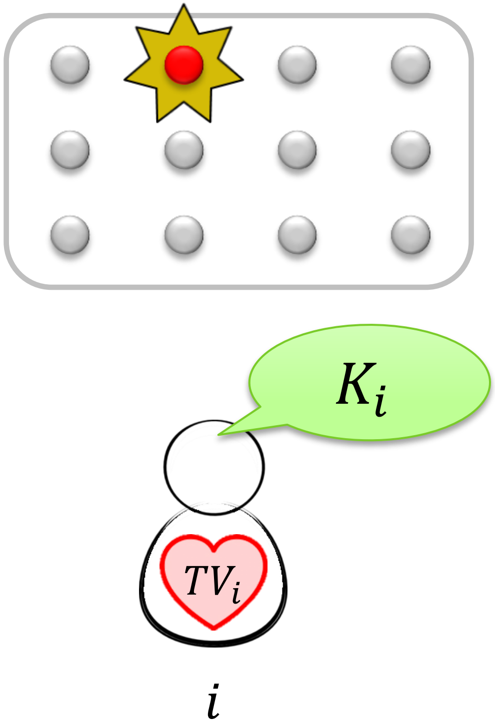
… Dominant Strategies



Our Results

Implementation in …

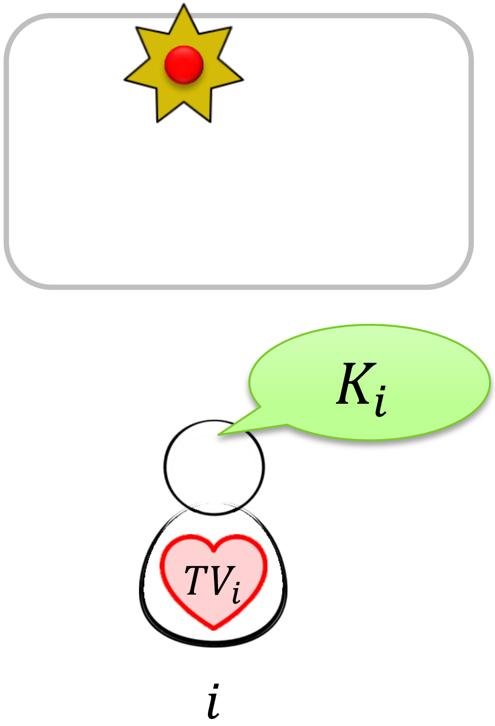
… Dominant Strategies



Our Results

Implementation in …

… Dominant Strategies



Our Results

Implementation in …

… Dominant Strategies

irrelevant!?

Our Results

Implementation in …

… Dominant Strategies

irrelevant!?

maybe not …
as there is more to reveal

Our Results

Implementation in …

… Dominant Strategies

- Thm 1:
  Can guarantee $\varepsilon(\delta, n) \cdot MSW$

Our Results

Implementation in …

… Dominant Strategies

- Thm 1:
  Can guarantee (?) $\cdot MSW$?

Our Results

Implementation in …

… Dominant Strategies

- Thm 1:
  Can guarantee $(1-\delta)\cdot MSW$?

Our Results

Implementation in …

… Dominant Strategies

- Thm 1:
  Can guarantee $\left(\frac{1-\delta}{1+\delta}\right) \cdot MSW$?

Our Results

Implementation in …

… Dominant Strategies

- Thm 1:
  Can guarantee $\frac{(1-\delta)^5}{(1+\delta)^3} \cdot MSW$?

Our Results

Implementation in …

… Dominant Strategies

- Thm 1:
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Our Results

Implementation in …

… Dominant Strategies

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Terrible!

can trivially achieve by assigning good at random
(after all, some player has the highest valuation!)

Our Results

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Interpretation:
dominant strategy
if and only if
exact knowledge of valuation
or individual Bayesian

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$70(1 \pm 0.1)$

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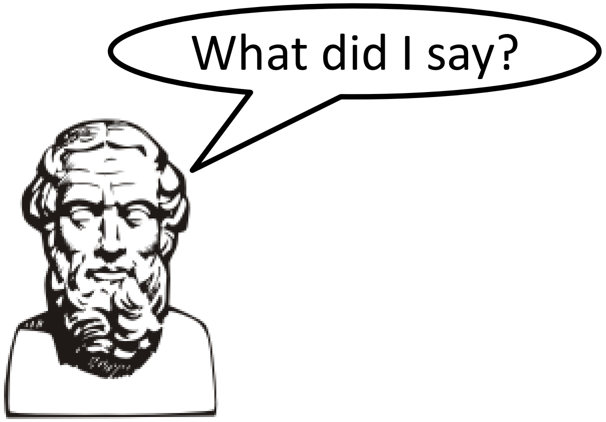

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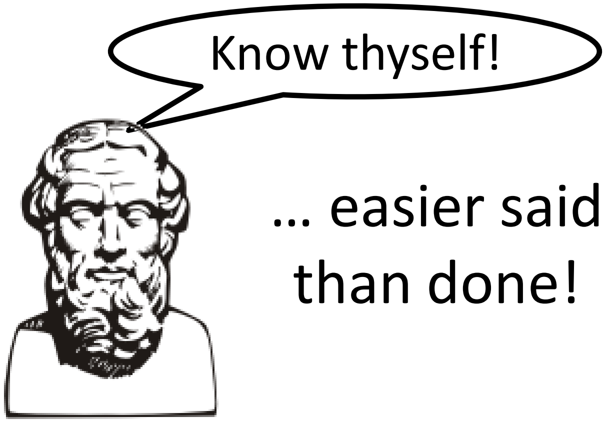

… easier said than done!

Our Results

Implementation in …

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… Undominated Strategies

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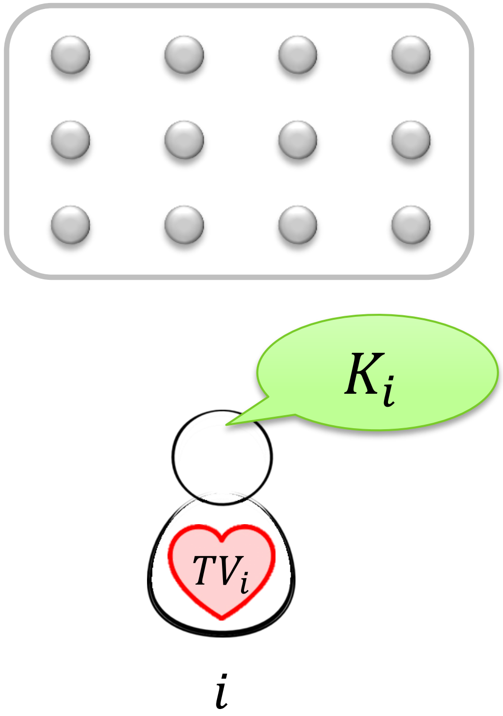

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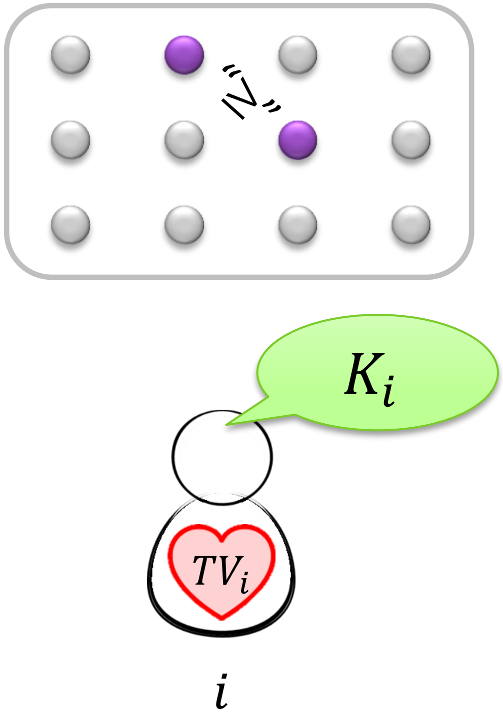

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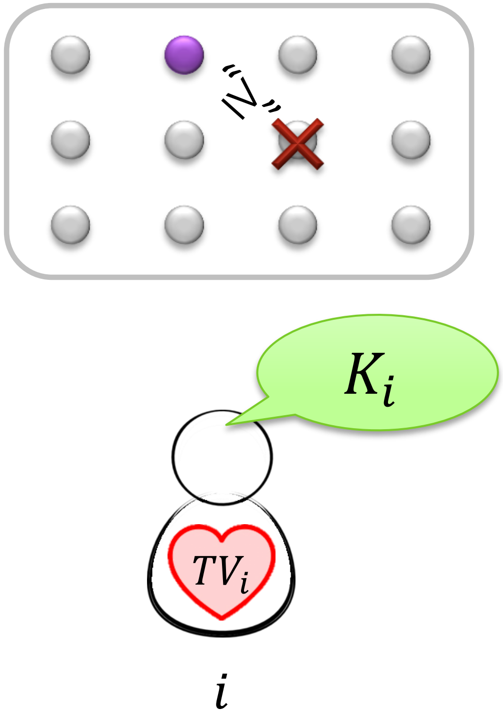

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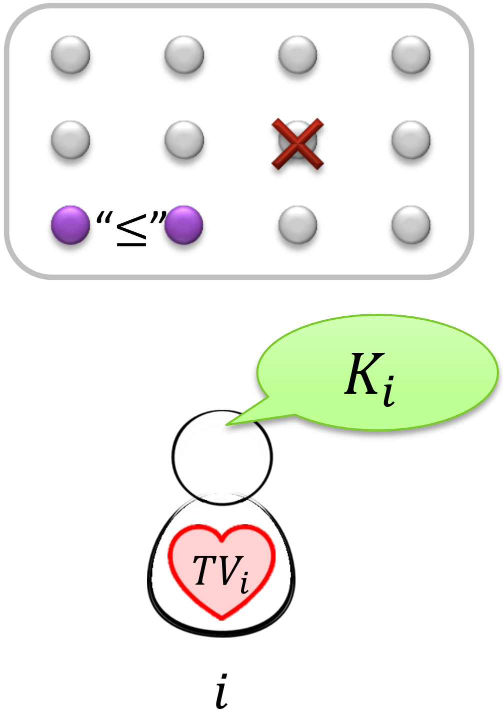

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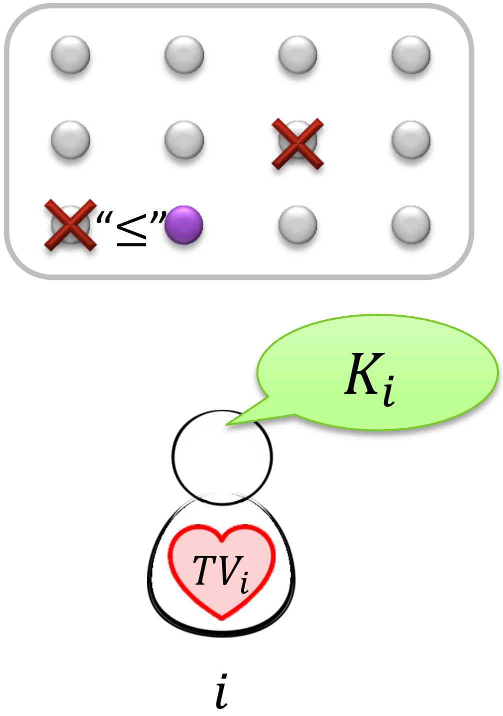

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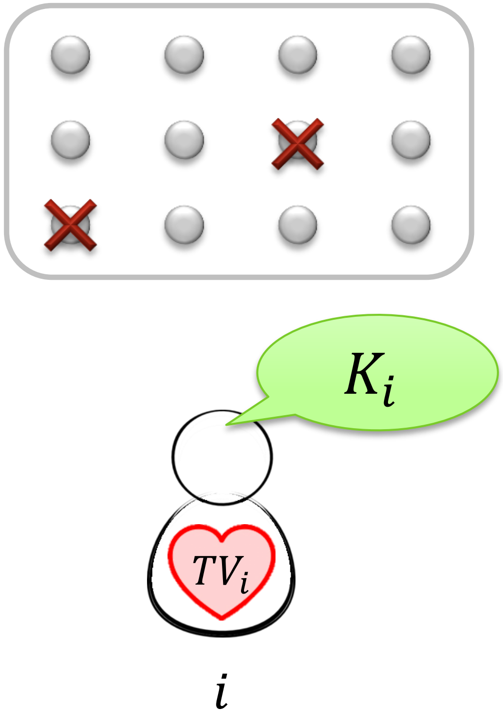

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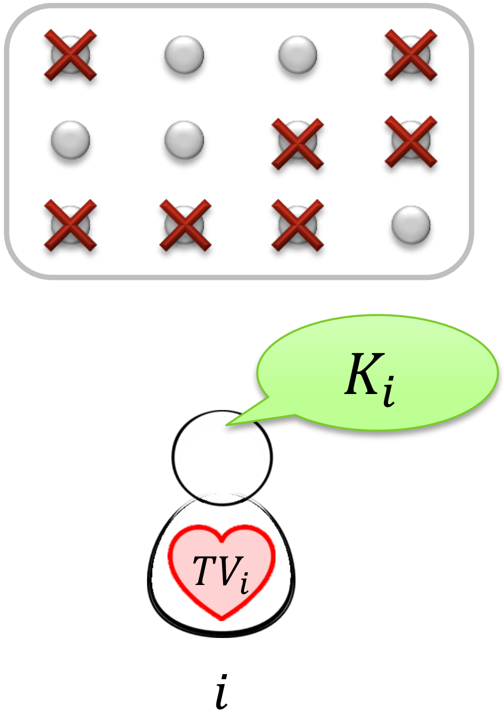

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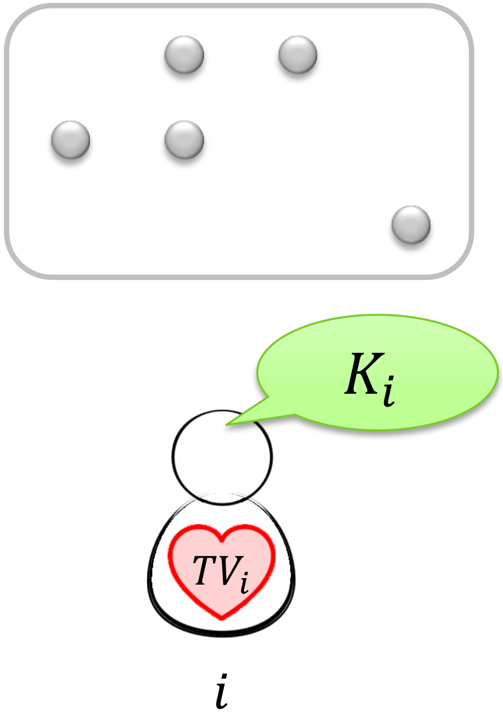

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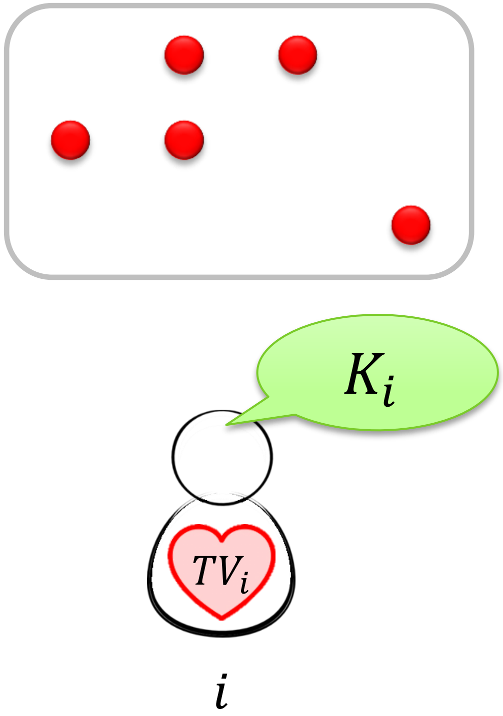

Our Results

Implementation in …

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much better!!

(note that the second-price mechanism is not dominant-strategy anymore!)

Our Results

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$\Longrightarrow$ a new role for undominated strategies!

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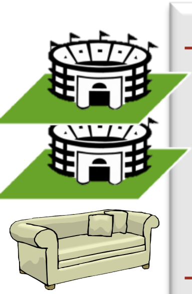

$\delta = 0.5$	$n = 2$	5 times better
$\delta = 0.5$	$n = 4$	3 times better
$\delta = 0.25$	$n = 2$	2 times better

… Undominated Strategies

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Our Results

Implementation in …

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Upper Bound Tool:
Undominated Intersection Lemma

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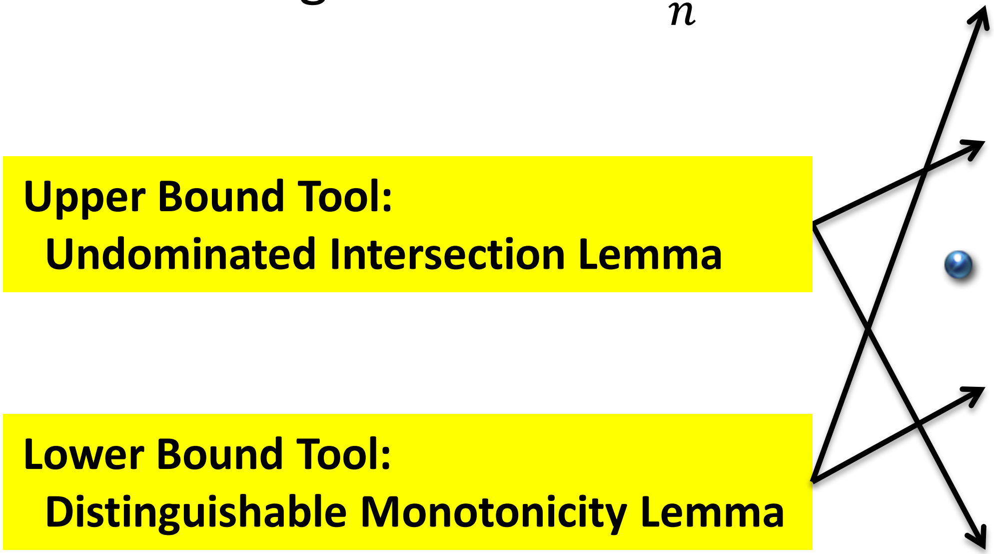

Upper Bound Tool:
Undominated Intersection Lemma

Lower Bound Tool:
Distinguishable Monotonicity Lemma

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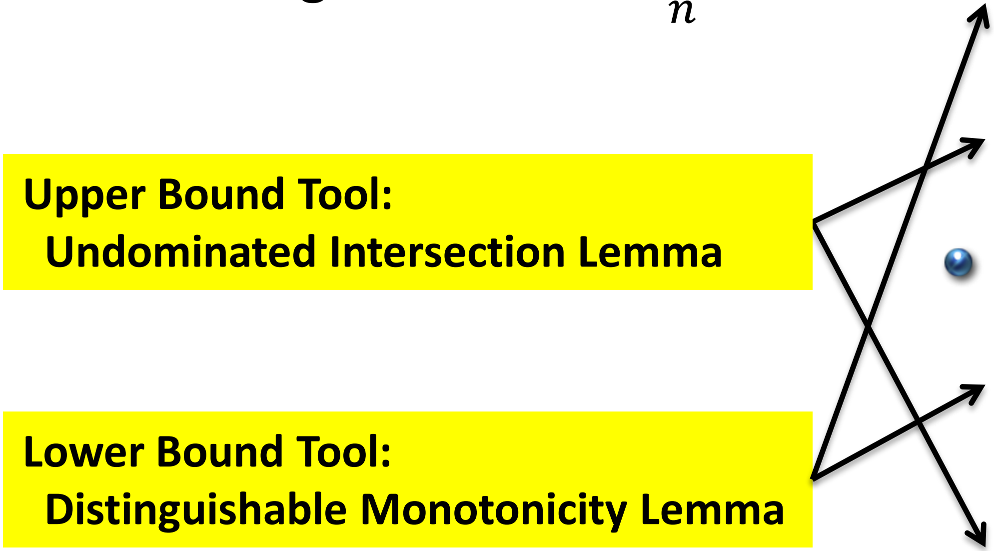

Upper Bound Tool:
Undominated Intersection Lemma

Lower Bound Tool:
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Full version at http://arxiv.org/abs/1112.1147

5. Our Techniques

Our Results

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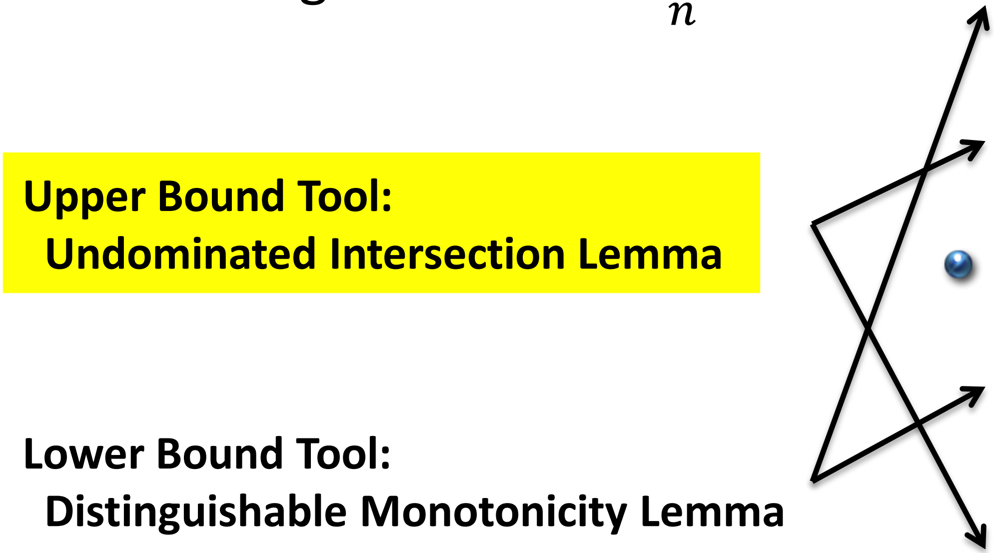

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- A tool for undominated strategy mechanisms
  - No revelation principle to help
  - Need to apply to **all** mechanisms

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Upper Bound Tool

Tool #1

Undominated Intersection Lemma:

$$|K_i \cap K_i'| \geq 2 \Longrightarrow \mathrm{UDed}_i(K_i) \cap \mathrm{UDed}_i(K_i') \neq \emptyset$$

Example:

Upper Bound Tool

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Undominated Intersection Lemma:

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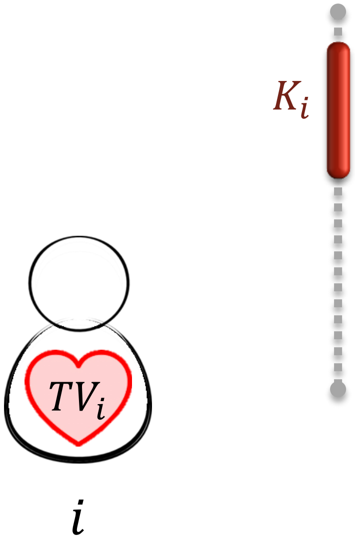

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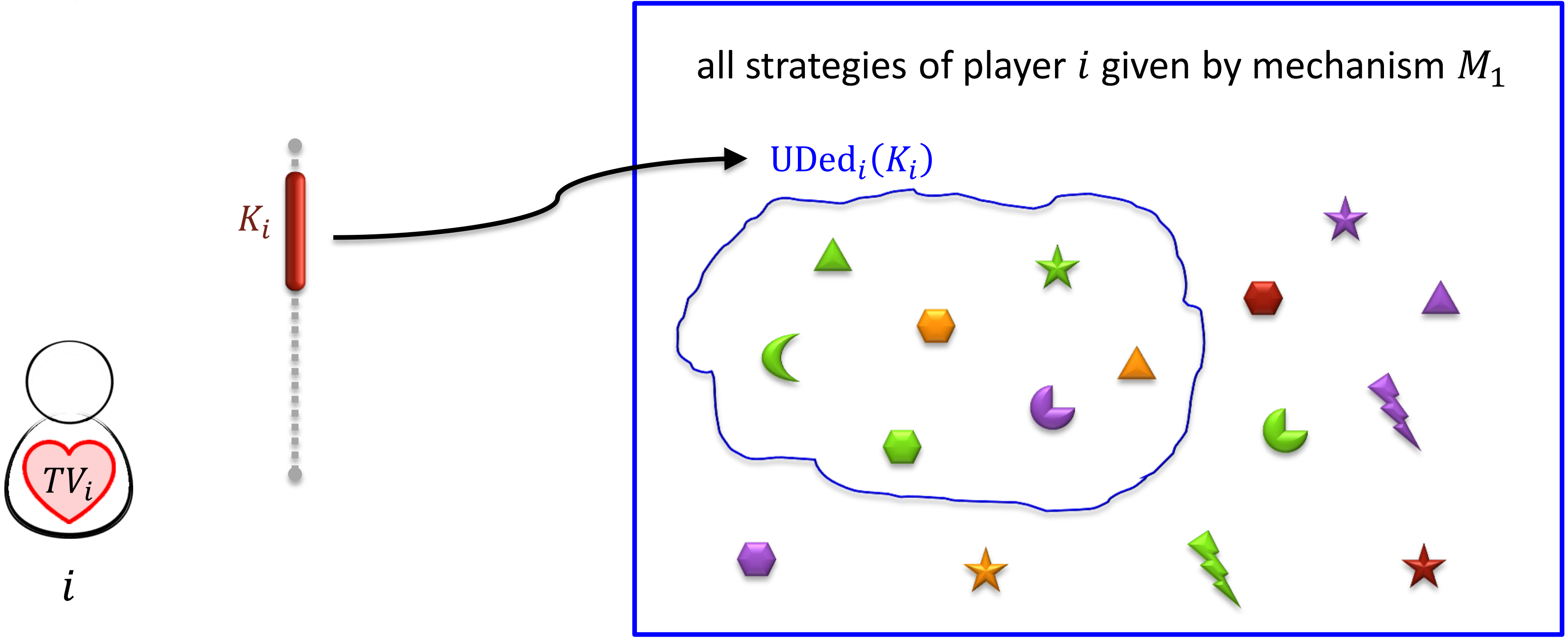

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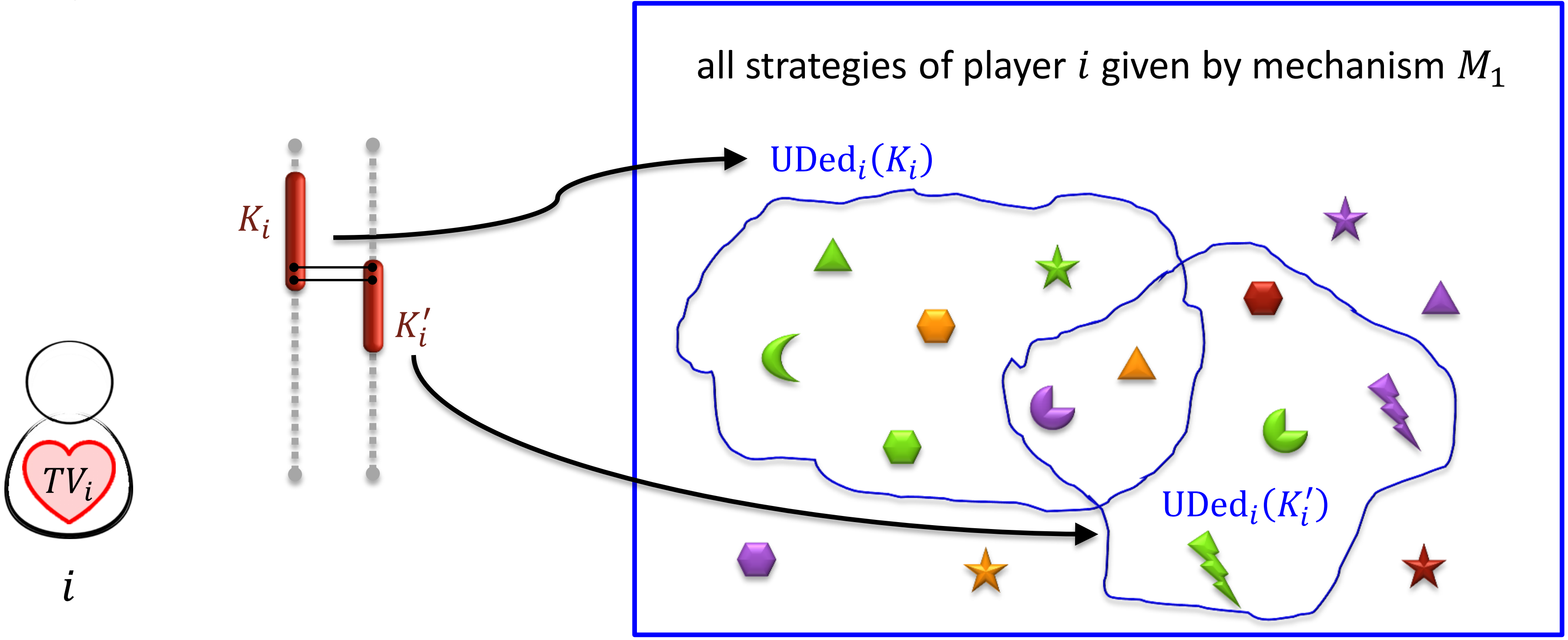

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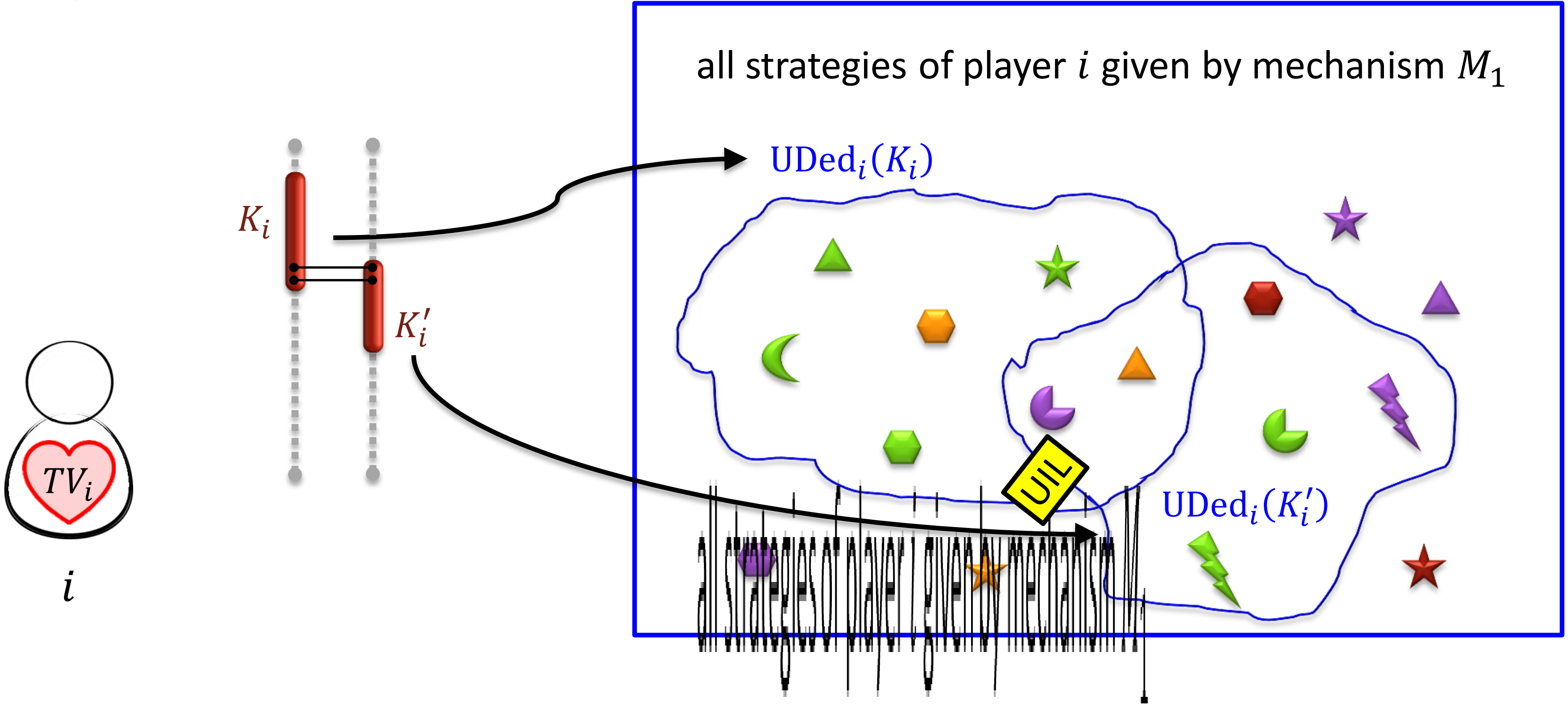

all strategies of player i given by mechanism M

Upper Bound Tool

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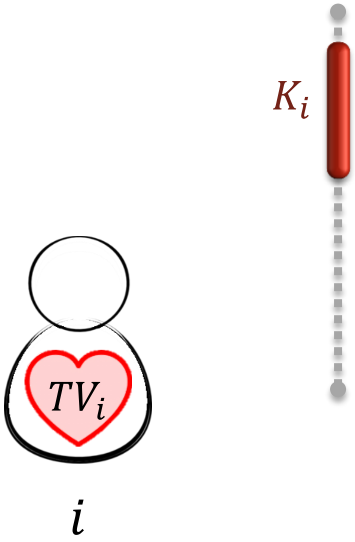


all strategies of player i given by mechanism M_2

Upper Bound Tool

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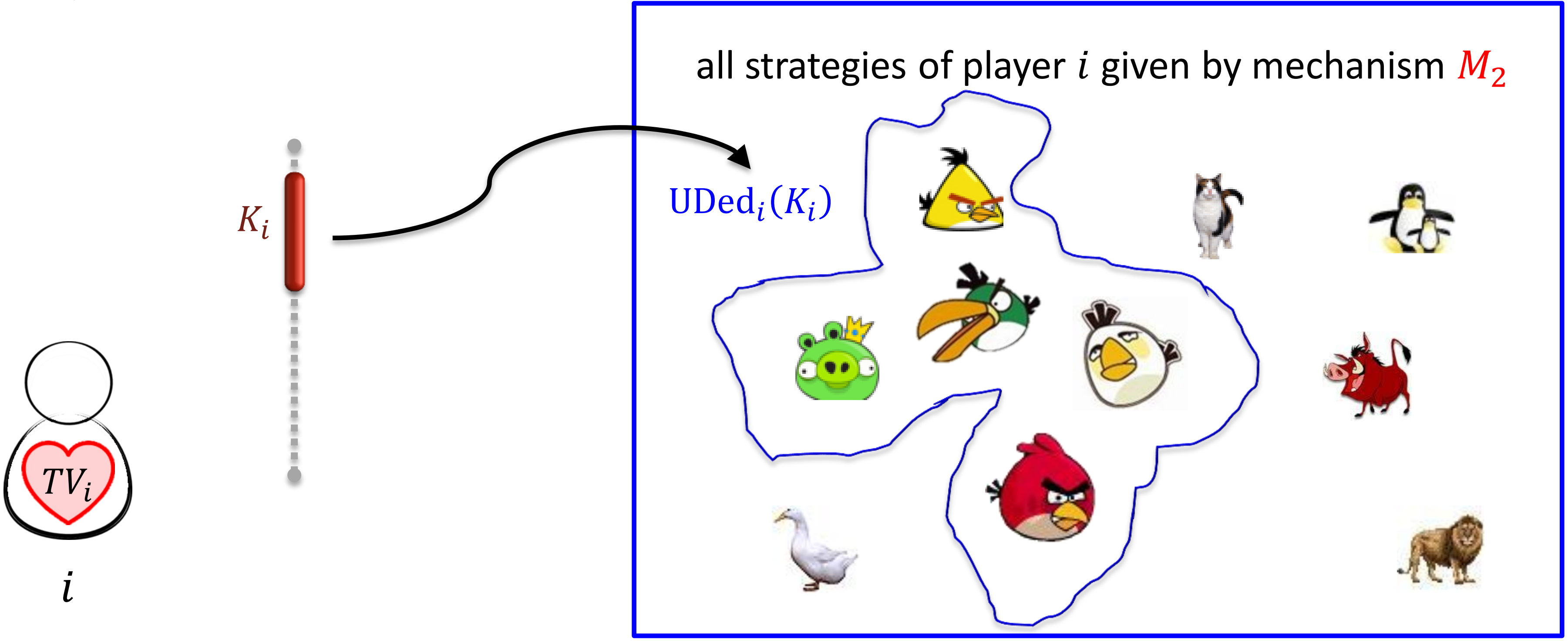

all strategies of player i given by mechanism M_2

$\mathrm{UDed}_i(K_i)$

K_i

TV_i

i

Upper Bound Tool

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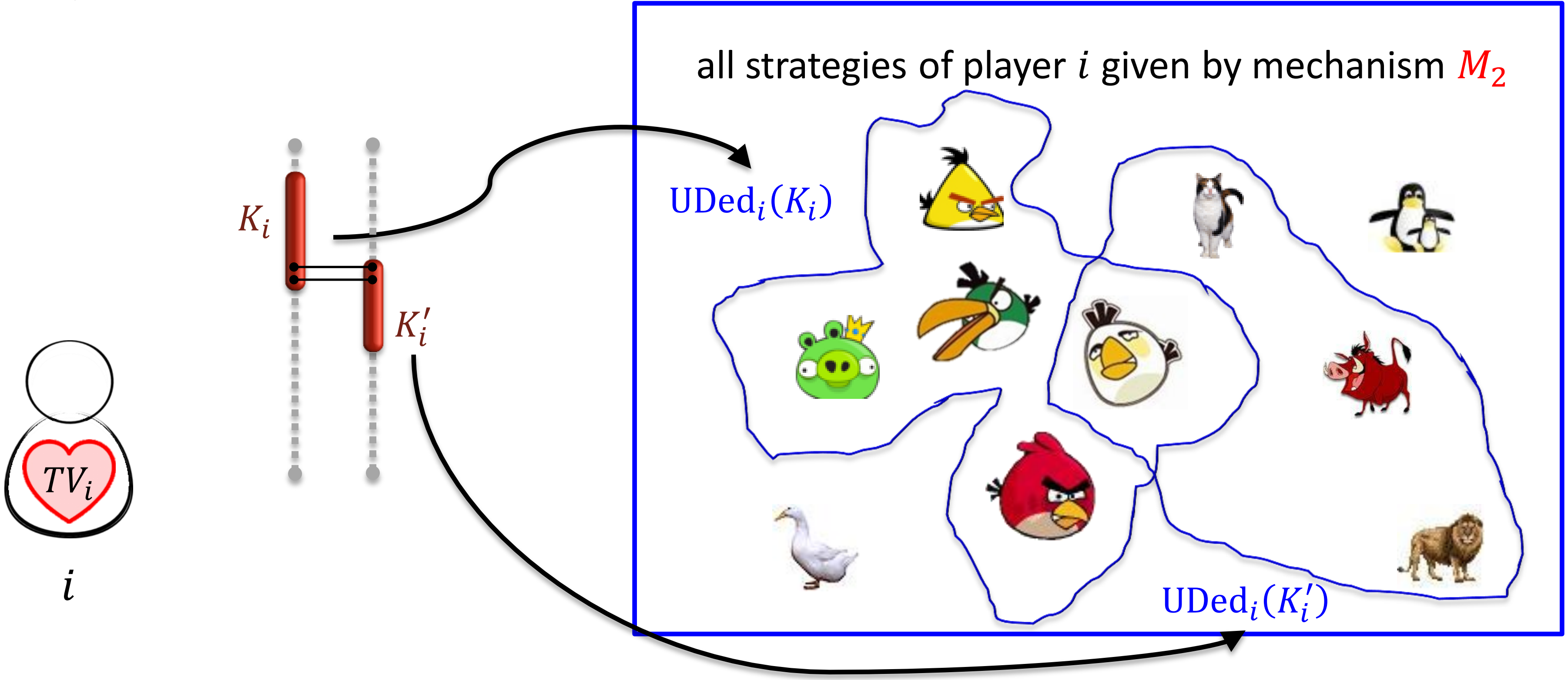

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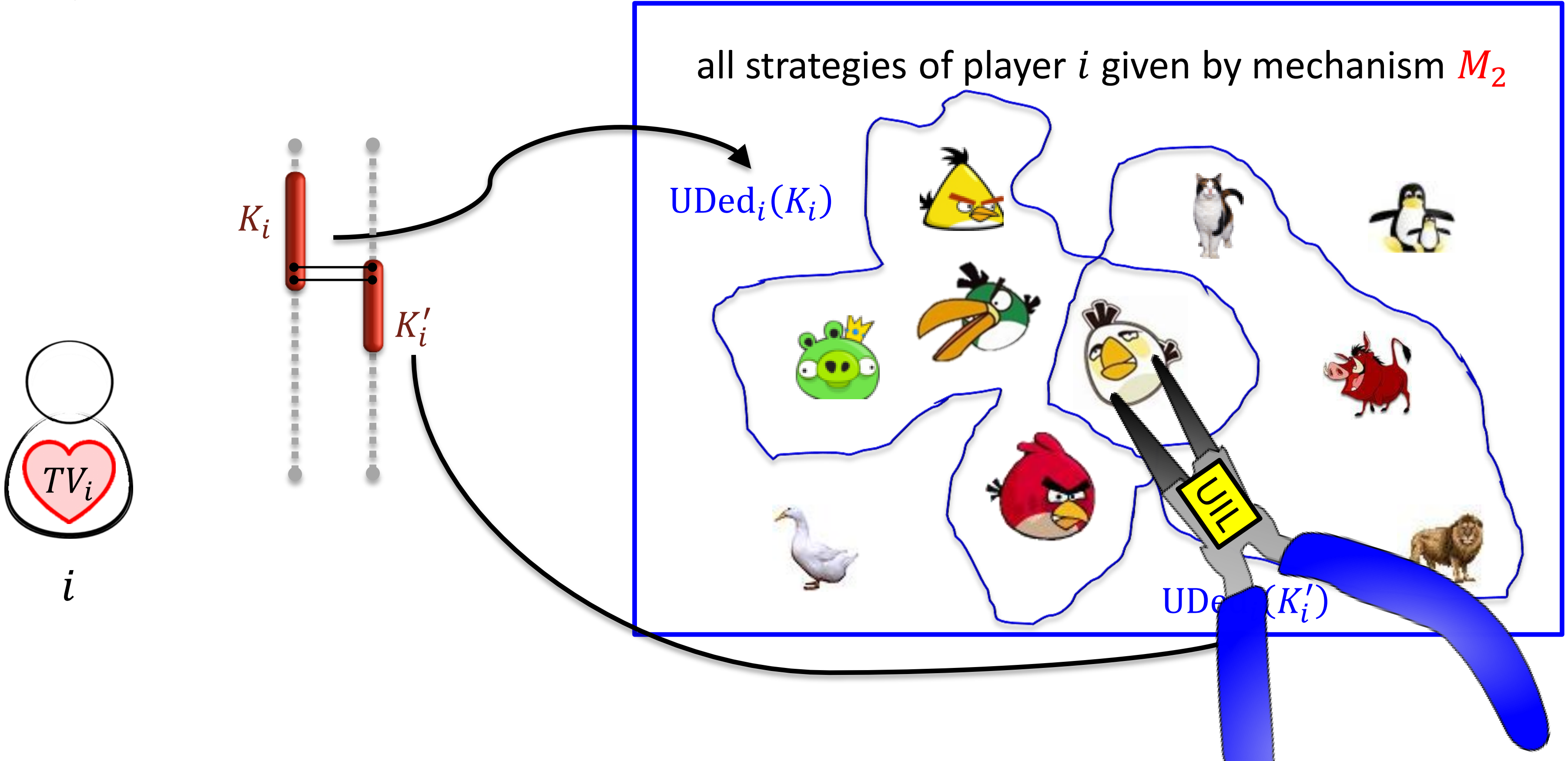

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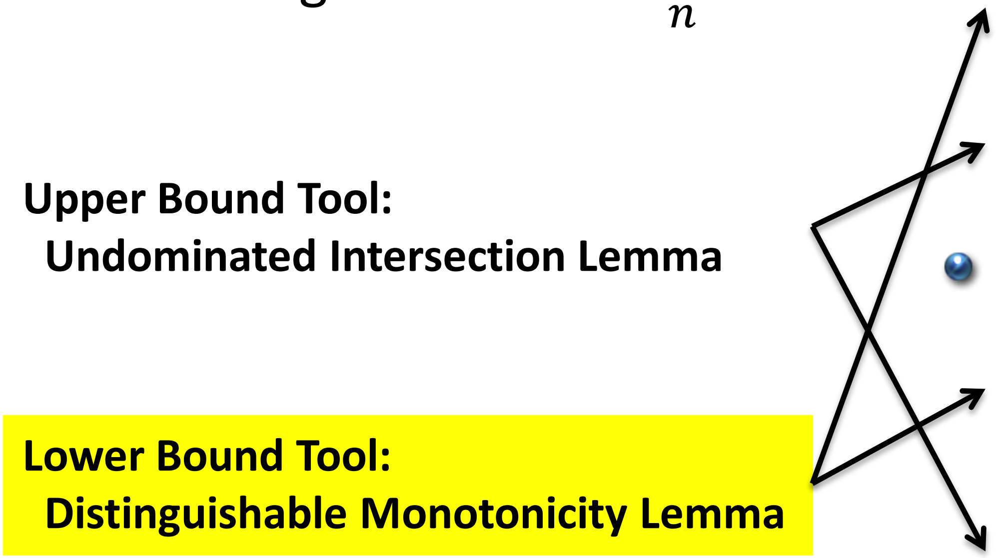

Upper Bound Tool:
Undominated Intersection Lemma

Lower Bound Tool:
Distinguishable Monotonicity Lemma

Lower Bound Tool

- Establishing lower bounds on ε involves (possibly finding and then) analyzing mechanisms
- DIFFICULTY: understanding the set of undominated strategies is not an easy task

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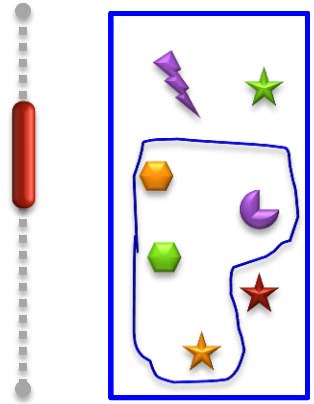

$K_i \Longrightarrow \mathrm{UDed}_i(K_i)$

Lower Bound Tool

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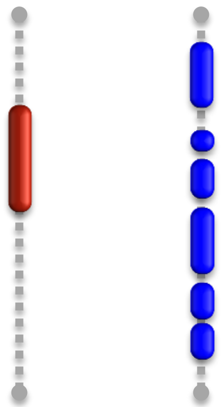
What if…



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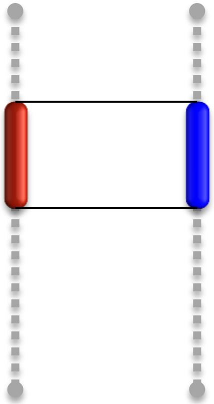


$K_i \Longrightarrow \mathrm{UDed}_i(K_i)$

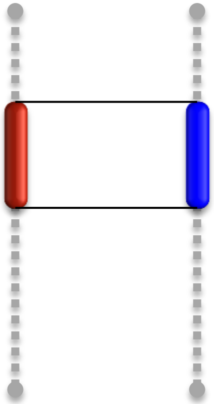
Lower Bound Tool

Tool #2

Approximate truthfulness:

$$\mathrm{UDed}_i(K_i) \subset [\min K_i\,, \max K_i]$$

What if…



$K_i \implies \mathrm{UDed}_i(K_i)$

Lower Bound Tool

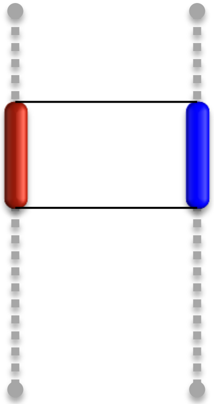
Tool #2

Distinguishable Monotonicity Lemma:

For any mechanism satisfying good property:

$$\mathrm{UDed}_i(K_i) \subset [\min K_i \,, \max K_i]$$

What if…



$K_i \Longrightarrow \mathrm{UDed}_i(K_i)$

Lower Bound Tool

Tool #2

Distinguishable Monotonicity Lemma:

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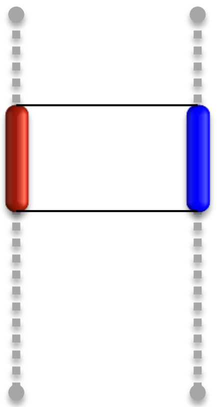
Examples:



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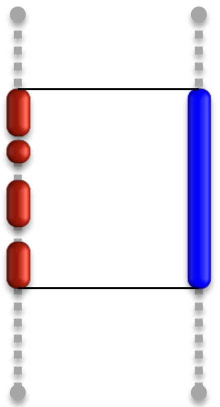

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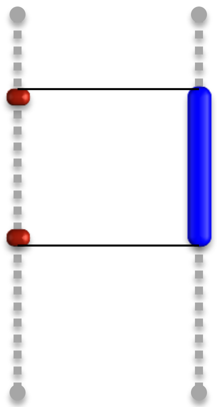

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Lower Bound Tool

Tool #2

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For any mechanism satisfying good property:

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What property?

Lower Bound Tool

Tool #2

Distinguishable Monotonicity Lemma:

For any mechanism satisfying good property:

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- Recall a classical result:

  $\forall$ monotonic $f: \mathbb{R}^n \to [0,1]^n$, $\quad M_f$ is DST

Lower Bound Tool

Tool #2

Distinguishable Monotonicity Lemma:

For any mechanism satisfying good property:

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- Recall a classical result:

  $$\forall \text{ monotonic } f: \mathbb{R}^n \to [0,1]^n, \qquad M_f \text{ is DST}$$

- How does M_f look like? On input bid-profile v
  - Player i wins w.p. $f_i(v)$;
  - Player i (if wins), pays $v_i - \frac{1}{f_i(v)} \int_{z=0}^{v_i} f_i(z \sqcup v_{-i}) dz$

Lower Bound Tool

Tool #2

Distinguishable Monotonicity Lemma:

$\forall$ monotonic $f: \mathbb{R}^n \to [0,1]^n$, M_f satisfies

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- Our result:

Lower Bound Tool

Tool #2

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$\forall$ monotonic* $f: \mathbb{R}^n \to [0,1]^n$, M_f satisfies

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- Our result:

* = distinguishably monotonic

$\Longrightarrow$ only need to focus on finding good allocation function f

Our Results

Implementation in …

… Dominant Strategies

- Thm 1:

  Cannot get more than $\frac{1}{n} \cdot MSW$

… Undominated Strategies

- Thm 2:
  - Second-price mechanism guarantees $\left(\frac{1-\delta}{1+\delta}\right)^2 \cdot MSW$
  - And it is optimal among deterministic mechanisms
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  - Our mechanism guarantees $\frac{(1-\delta)^2+\frac{4\delta}{n}}{(1+\delta)^2} \cdot MSW$
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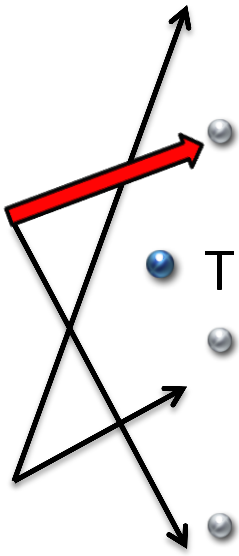


Upper Bound Tool:
Undominated Intersection Lemma

Lower Bound Tool:
Distinguishable Monotonicity Lemma

Upper Bounds on ε

Upper Bound Tool

Undominated Intersection Lemma:

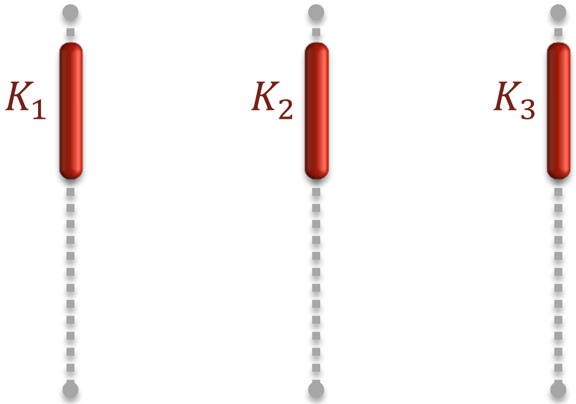
$$|K_i \cap K_i'| \geq 2 \Longrightarrow \mathrm{UDed}_i(K_i) \cap \mathrm{UDed}_i(K_i') \neq \emptyset$$

- Thm: in undominated strategies, no deterministic mechanism guarantees more than $\left(\frac{1-\delta}{1+\delta}\right)^2 \cdot MSW$.

Upper Bounds on ε

Proof:

1. Pick any x and set $K_1 = K_2 = K_3 = [(1-\delta)x, (1+\delta)x]$



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1. Pick any x and set $K_1 = K_2 = K_3 = [(1-\delta)x, (1+\delta)x]$
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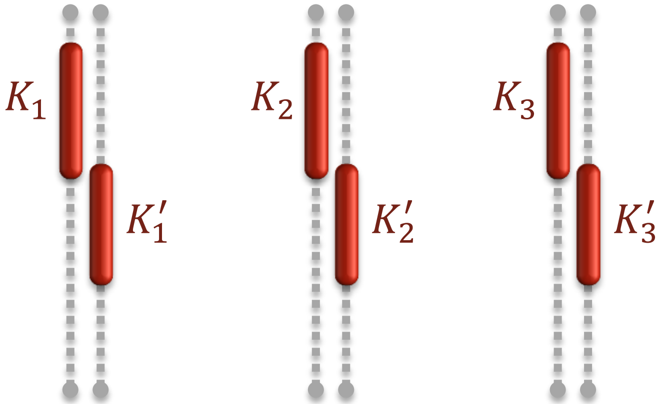

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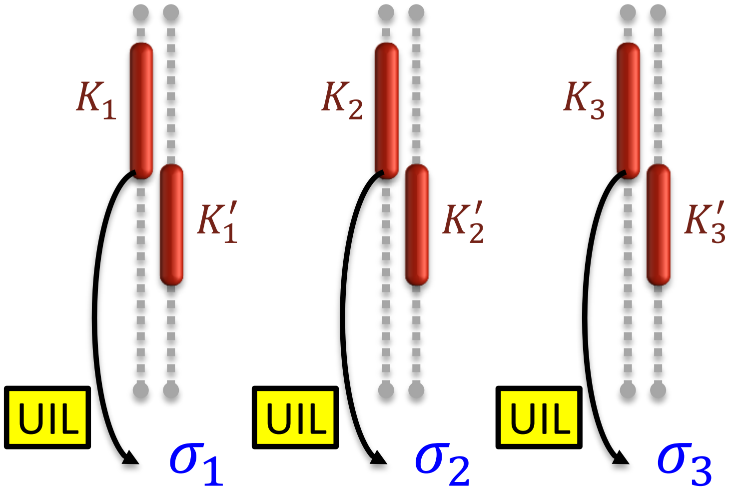
3. Apply UIL to obtain $\sigma_i \in \mathrm{UDed}_i(K_i) \cap \mathrm{UDed}_i(K_i')$ for each i



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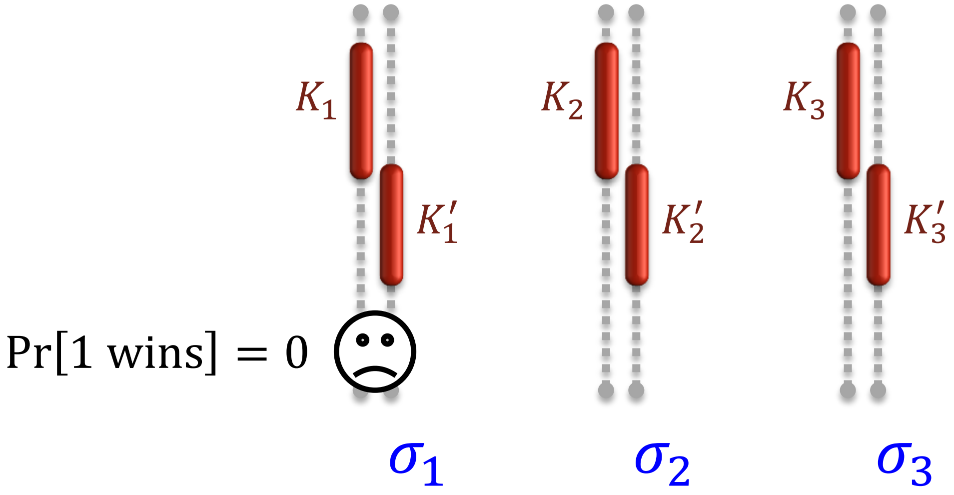

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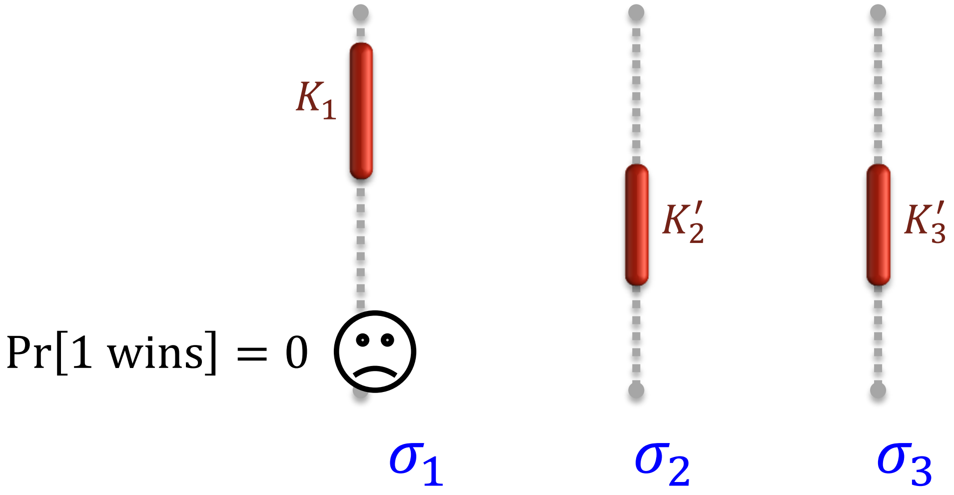
5. Choose the “world” of (K_1, K_2', K_3')… This is the hard instance!



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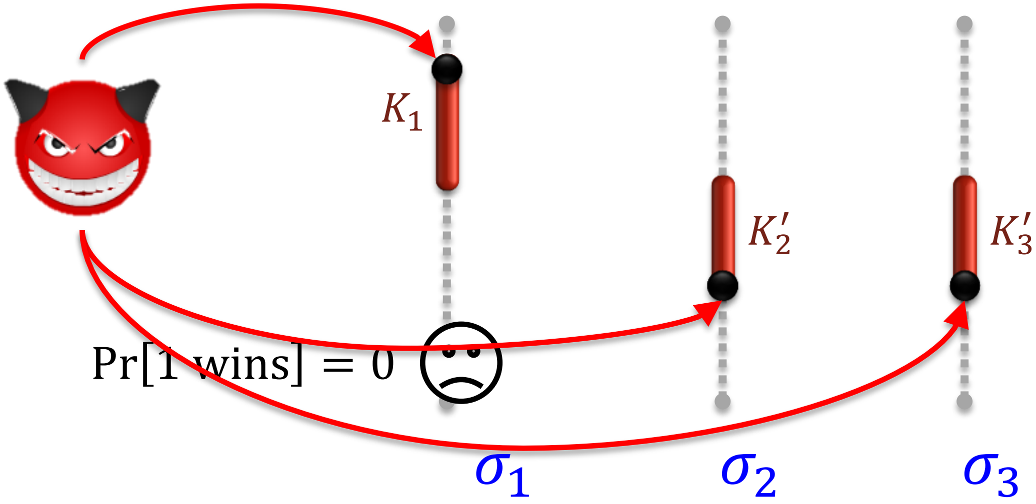

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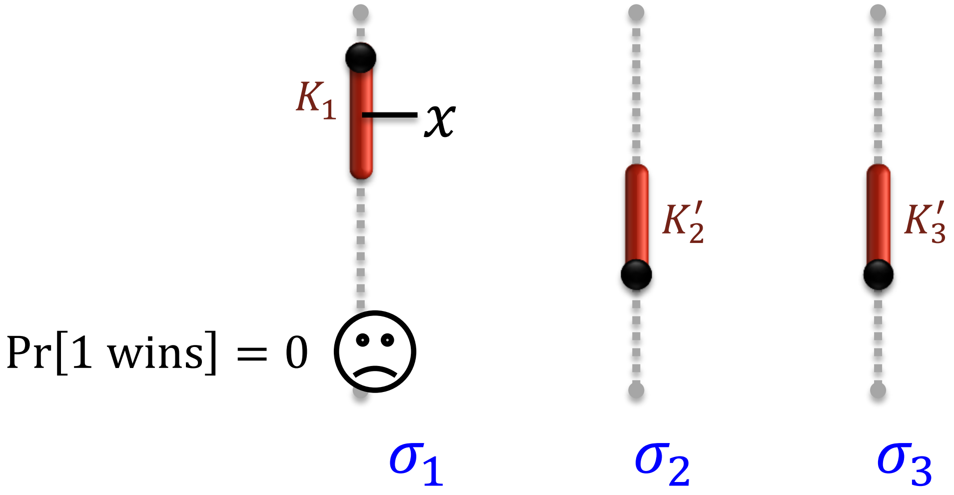


$$MSW = (1+\delta)x$$

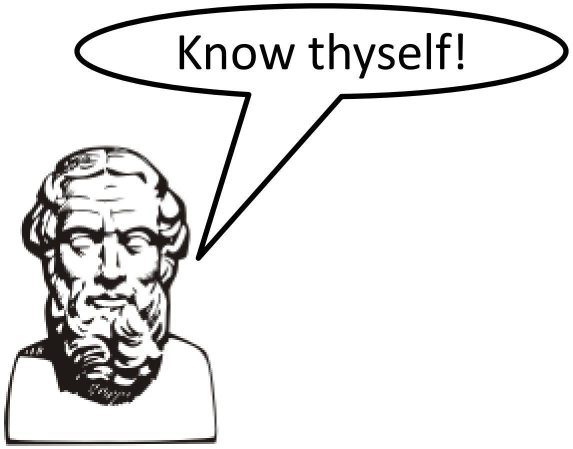
$$SW = \frac{(1-\delta)^2}{1+\delta}x$$

$$\Rightarrow \boxed{\varepsilon \le \left(\frac{1-\delta}{1+\delta}\right)^2}$$

Deterministic: QED

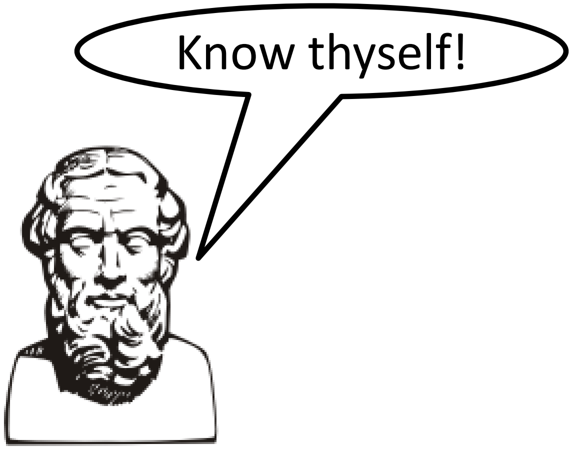
6. Conclusion

Conclusion



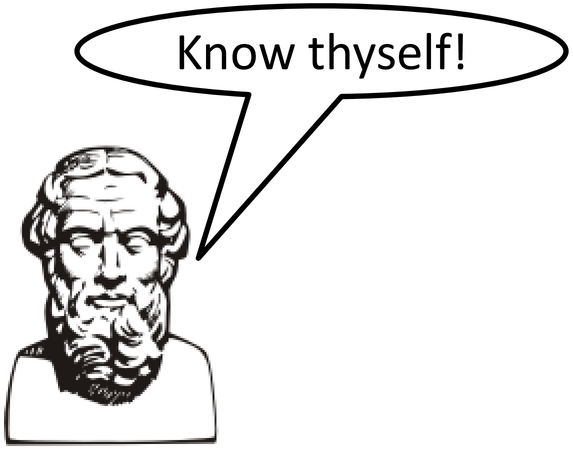
mechanism design =

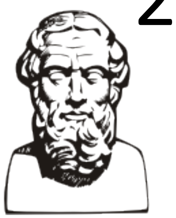
Conclusion



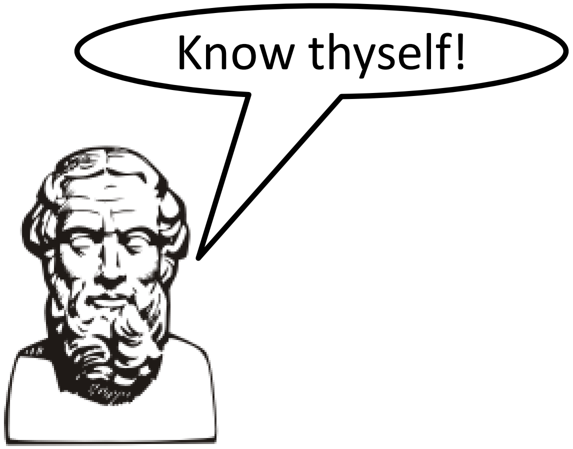
mechanism design $=$ $+\, n \times$ 

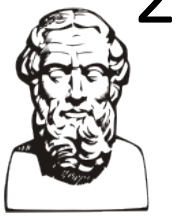
Conclusion



mechanism design =  $^2 + n \times$ 

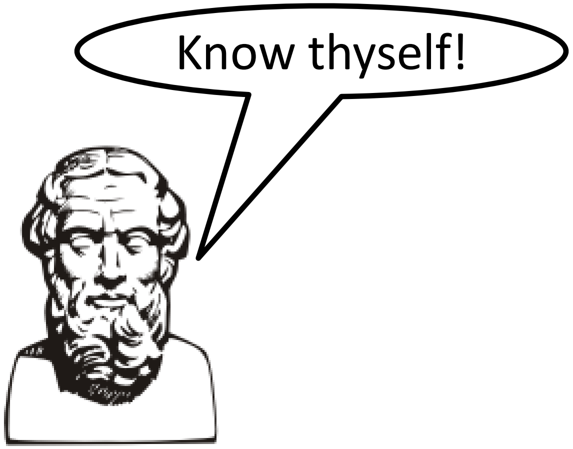
Conclusion

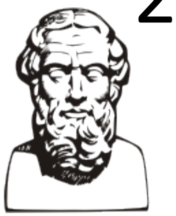


mechanism design $=$  $^{2} + n \times$ 

GOAL: want to learn about others,
who may not know themselves very well.

Conclusion



mechanism design = ^{2} $+ n \times$ 

GOAL: want to learn about others,
who may not know themselves very well.

The GOAL is desirable and doable!

Our Results

Implementation in …

… Dominant Strategies

- Thm 1:
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… Undominated Strategies

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  - Second-price mechanism guarantees $\left(\frac{1-\delta}{1+\delta}\right)^2 \cdot MSW$
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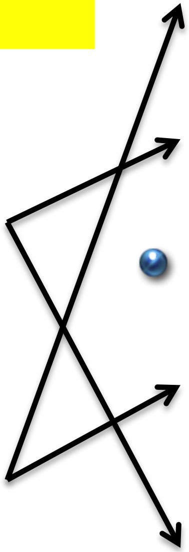

Upper Bound Tool:
Undominated Intersection Lemma

Lower Bound Tool:
Distinguishable Monotonicity Lemma

Impl. in Dominant Strategies

- There is a profile of strategies of players that "cannot be beaten" for which the mechanism M ensures good social welfare.

Classical Model

- $\sigma_i \overset{\mathrm{vw}}{\underset{TV_i}{\geq}} \sigma_i'$ if $\forall \tau_{-i},\ u_i\big(TV_i, M(\sigma_i \sqcup \tau_{-i})\big) \geq u_i\big(TV_i, M({\sigma_i}' \sqcup \tau_{-i})\big)$

- $\mathrm{Dnt}_i(\mathrm{TV}_i) = \left\{\sigma_i : \forall \sigma_i',\ \ \sigma_i \overset{\mathrm{vw}}{\underset{TV_i}{\geq}} \sigma_i'\right\}$ = "unbeatable strategies w.r.t. TV_i"

- A mechanism M implements $\varepsilon \cdot MSW$ if

$$\forall TV \quad , \ \exists \sigma \in \mathrm{Dnt}(TV)$$

$$\mathop{\mathbb{E}}_{M,\sigma}\big[SW\big(TV, M(\sigma)\big)\big] \geq \varepsilon \cdot MSW(TV)$$

Impl. in Dominant Strategies

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Our Model

- $\sigma_i \underset{\underset{K_i}{\cancel{TV_i}}}{\overset{\mathrm{vw}}{\geq}} \sigma_i'$ if $\forall TV_i \in K_i, \forall \tau_{-i},\ u_i\big(TV_i, M(\sigma_i \sqcup \tau_{-i})\big) \geq u_i\big(TV_i, M(\sigma_i{}' \sqcup \tau_{-i})\big)$

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- A mechanism M δ-implements $\varepsilon \cdot MSW$ if

$$\forall K \in [\delta], \forall TV \in K,\ \exists \sigma \in \mathrm{Dnt}(\underset{K}{\cancel{TV}})$$

$$\mathop{\mathbb{E}}_{M,\sigma}\left[SW\big(TV, M(\sigma)\big)\right] \geq \varepsilon \cdot MSW(TV)$$

Impl. in Dominant Strategies

- Thm: in dominant strategies, every (possibly probabilistic) mechanism cannot guarantee more than $\frac{1}{n} \cdot MSW$

"Revelation Principle" *

- Claim: in dominant strategies, every (possibly probabilistic) direct mechanism cannot guarantee more than $\frac{1}{n} \cdot MSW$

set of strategies is $[\delta]$ *

Impl. in Dominant Strategies

- Claim: in dominant strategies, every (possibly probabilistic) direct mechanism cannot guarantee more than $\frac{1}{n} \cdot MSW$
- Proof:
  1. Lemma: a dominant strategy direct mechanism M gives the same outcome when a player deviates individually: $\forall K,\ \forall i,\ \forall K_i',$

     $$M_i(K) = M_i(K_i' \sqcup K_{-i})$$

     (Proof: play with different worlds)

  2. Consider WORLD$_1$: all players bid low;
     there is some unlucky player, say player 5.
  3. Consider WORLD$_2$: all players bid low except player 5.
  4. Compute and conclude. QED

Our Results

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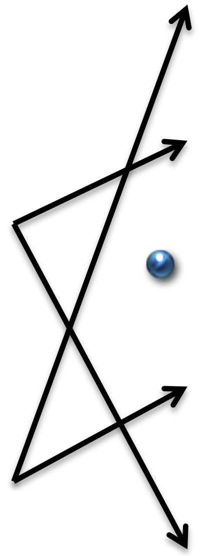

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Impl. in Undominated Strategies

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- “When players use non-stupid strategies, the mechanism M ensures good social welfare”

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Classical Model

- $\sigma_i \underset{TV_i}{\leq} \sigma_i'$ if $\begin{cases} \forall\tau_{-i},\ u_i\big(TV_i, M(\sigma_i \sqcup \tau_{-i})\big) \leq u_i\big(TV_i, M(\sigma_i{}' \sqcup \tau_{-i})\big) \\ \exists\tau_{-i},\ u_i\big(TV_i, M(\sigma_i \sqcup \tau_{-i})\big) < u_i\big(TV_i, M(\sigma_i{}' \sqcup \tau_{-i})\big) \end{cases}$

Impl. in Undominated Strategies

- “When players use non-stupid strategies, the mechanism M ensures good social welfare”

Classical Model

never worse

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better at least once

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- $\mathrm{UDed}_i(\mathrm{TV}_i) = \left\{\sigma_i : \nexists \sigma_i'\ s.t. \sigma_i \underset{TV_i}{\leq} \sigma_i'\right\}$ = “non-stupid strategies w.r.t. TV_i”

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Our Model

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- “When players use non-stupid strategies, the mechanism M ensures good social welfare”

Our Model

- $\sigma_i \underset{\substack{\cancel{TV_i}\\ K_i}}{\leq} \sigma_i'$ if $\begin{cases} \forall TV_i \in K_i, \forall \tau_{-i},\ u_i\big(TV_i, M(\sigma_i \sqcup \tau_{-i})\big) \leq u_i\big(TV_i, M(\sigma_i' \sqcup \tau_{-i})\big) \\ \exists TV_i \in K_i, \exists \tau_{-i},\ u_i\big(TV_i, M(\sigma_i \sqcup \tau_{-i})\big) < u_i\big(TV_i, M(\sigma_i' \sqcup \tau_{-i})\big) \end{cases}$

- $\mathrm{UDed}_i(\underset{K_i}{\cancel{TV_i}}) = \left\{\sigma_i \ :\ \nexists \sigma_i'\ s.t.\ \sigma_i \underset{\substack{\cancel{TV_i}\\ K_i}}{\leq} \sigma_i'\right\}$ = “non-stupid strategies w.r.t. $\underset{K_i}{\cancel{TV_i}}$”

- A mechanism M implements $\varepsilon \cdot MSW$ if

$$\forall TV \qquad , \ \forall \sigma \in \mathrm{UDed}(TV)$$

$$\underset{M,\sigma}{\mathbb{E}}\left[SW\big(TV, M(\sigma)\big)\right] \geq \varepsilon \cdot MSW(TV)$$

Impl. in Undominated Strategies

- “When players use non-stupid strategies, the mechanism M ensures good social welfare”

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- $\sigma_i \underset{\underset{K_i}{\cancel{TV_i}}}{\leq} \sigma_i'$ if $\begin{cases} \forall TV_i \in K_i, \forall \tau_{-i},\ u_i\big(TV_i, M(\sigma_i \sqcup \tau_{-i})\big) \leq u_i\big(TV_i, M({\sigma_i}' \sqcup \tau_{-i})\big) \\ \exists TV_i \in K_i, \exists \tau_{-i},\ u_i\big(TV_i, M(\sigma_i \sqcup \tau_{-i})\big) < u_i\big(TV_i, M({\sigma_i}' \sqcup \tau_{-i})\big) \end{cases}$

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- A mechanism M δ-implements $\varepsilon \cdot MSW$ if

$$\forall K \in [\delta], \forall TV \in K,\ \forall \sigma \in \mathrm{UDed}(\underset{K}{\cancel{TV}})$$

$$\mathop{\mathbb{E}}_{M,\sigma}\big[SW\big(TV, M(\sigma)\big)\big] \geq \varepsilon \cdot MSW(TV)$$

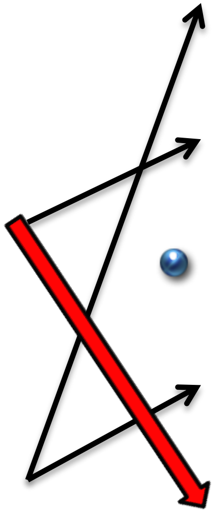
Our Results

Implementation in …

… Dominant Strategies

- Thm 1:

  Cannot get more than $\frac{1}{n} \cdot MSW$



Upper Bound Tool:
Undominated Intersection Lemma

Lower Bound Tool:
Distinguishable Monotonicity Lemma

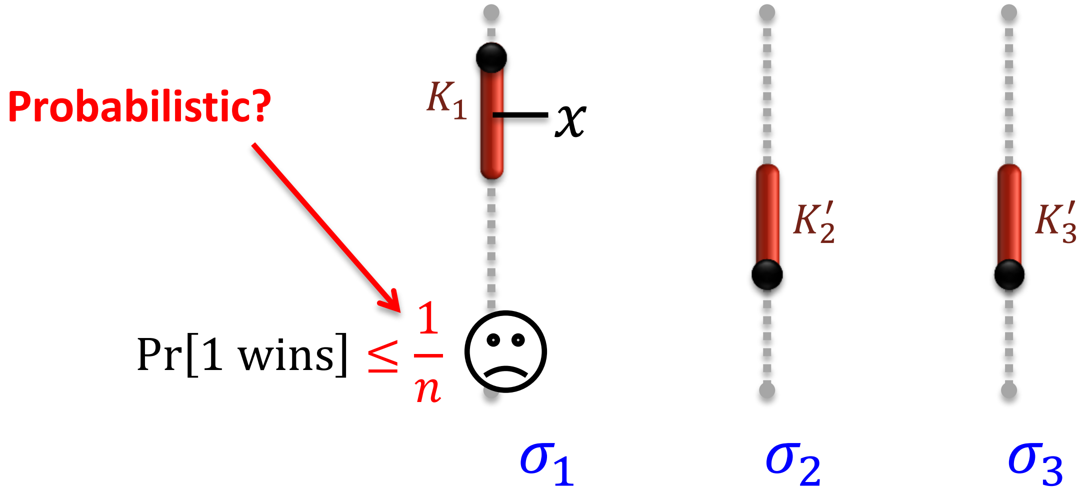
… Undominated Strategies

- Thm 2:
  - Second-price mechanism guarantees $\left(\frac{1-\delta}{1+\delta}\right)^2 \cdot MSW$
  - And it is optimal among deterministic mechanisms
- Thm 3:
  - Our mechanism guarantees $\frac{(1-\delta)^2+\frac{4\delta}{n}}{(1+\delta)^2} \cdot MSW$
  - And it is optimal among probabilistic mechanisms

Upper Bounds on ε

Proof:

1. Pick any x and set $K_1 = K_2 = K_3 = [(1-\delta)x, (1+\delta)x]$
2. Set $K_1' = K_2' = K_3'$ to "just touch K_i from below"
3. Apply UIL to obtain $\sigma_i \in \mathrm{UDed}_i(K_i) \cap \mathrm{UDed}_i(K_i')$ for each i
4. When playing $(\sigma_1, \sigma_2, \sigma_3)$, someone is **unlucky**, WLOG player 1
5. Choose the "world" of (K_1, K_2', K_3')… This is the hard instance!



Upper Bounds on ε

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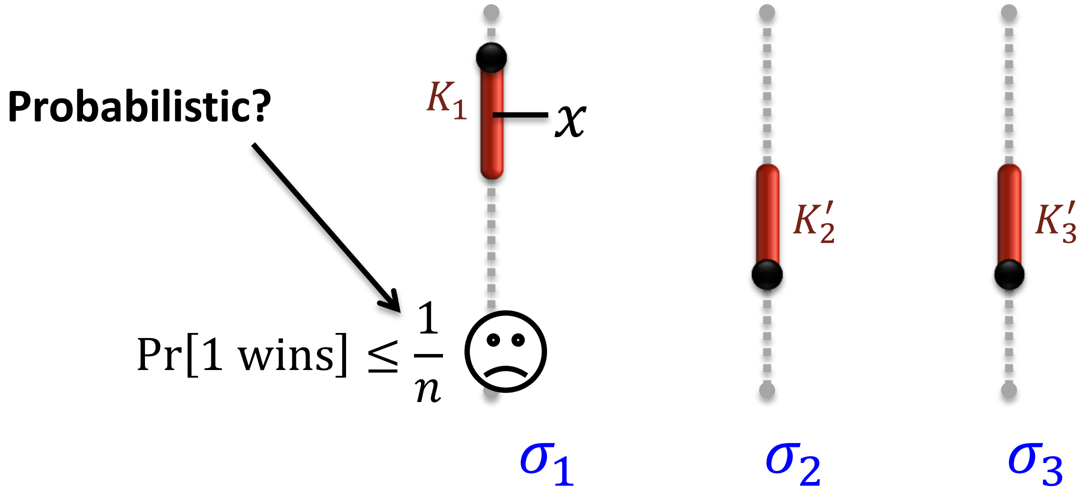

$$\ldots \Rightarrow \boxed{\varepsilon \le \frac{(1-\delta)^2 + \frac{4\delta}{n}}{(1+\delta)^2}}$$

Probabilistic: QED

Our Results

Implementation in …

… Dominant Strategies

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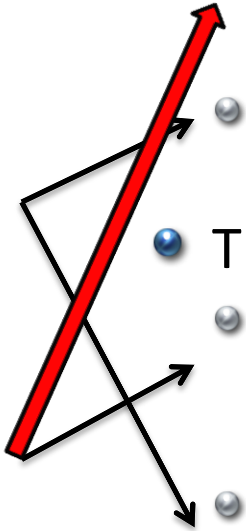

Upper Bound Tool:
Undominated Intersection Lemma

Lower Bound Tool:
Distinguishable Monotonicity Lemma

Lower Bounds on ε

Thm: Second-price mechanism guarantees $\left(\frac{1-\delta}{1+\delta}\right)^2 \cdot MSW$

Proof:

Lower Bounds on ε

Thm: Second-price mechanism guarantees $\left(\frac{1-\delta}{1+\delta}\right)^2 \cdot MSW$

Proof:

1.
2.

Lower Bounds on ε

Thm: Second-price mechanism guarantees $\left(\frac{1-\delta}{1+\delta}\right)^2 \cdot MSW$

Proof:

1.
2. Done!

Lower Bounds on ε

Thm: Second-price mechanism guarantees $\left(\frac{1-\delta}{1+\delta}\right)^2 \cdot MSW$

Proof:

1. $\mathrm{UDed}_i(K_i) \subset [\min K_i , \max K_i]$ for every player i
2. Done!

Lower Bounds on ε

Thm: Second-price mechanism guarantees $\left(\frac{1-\delta}{1+\delta}\right)^2 \cdot MSW$

Proof:

1. $\mathrm{UDed}_i(K_i) \subset [\min K_i, \max K_i]$ for every player i
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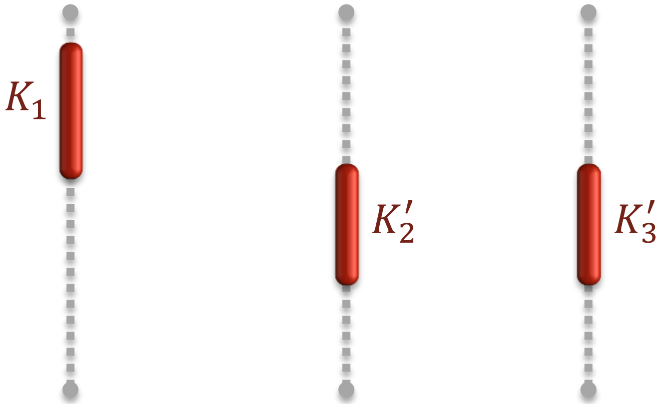

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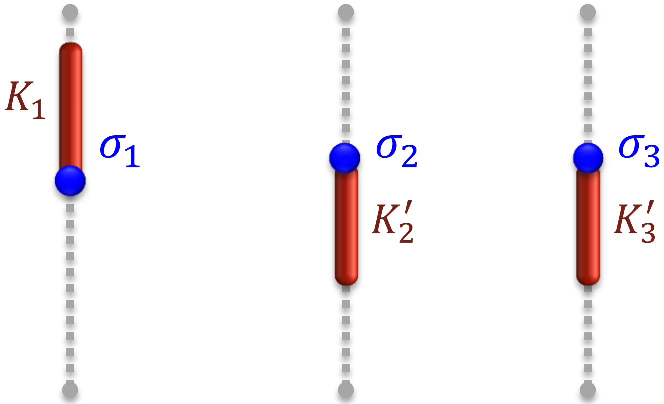

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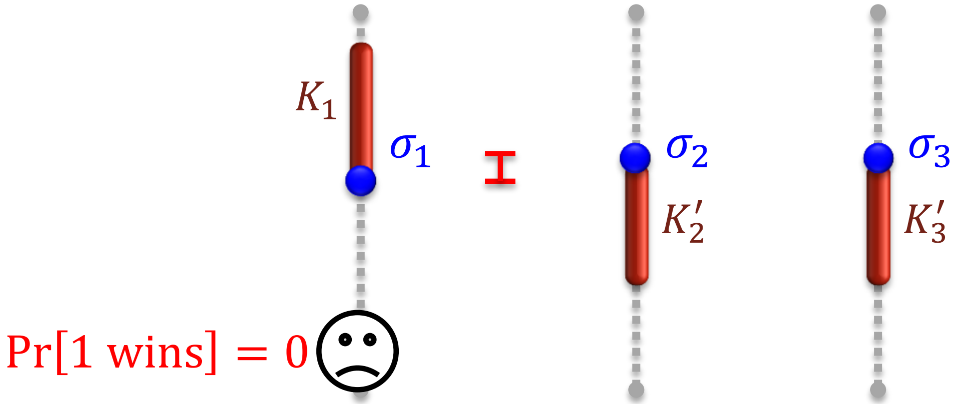

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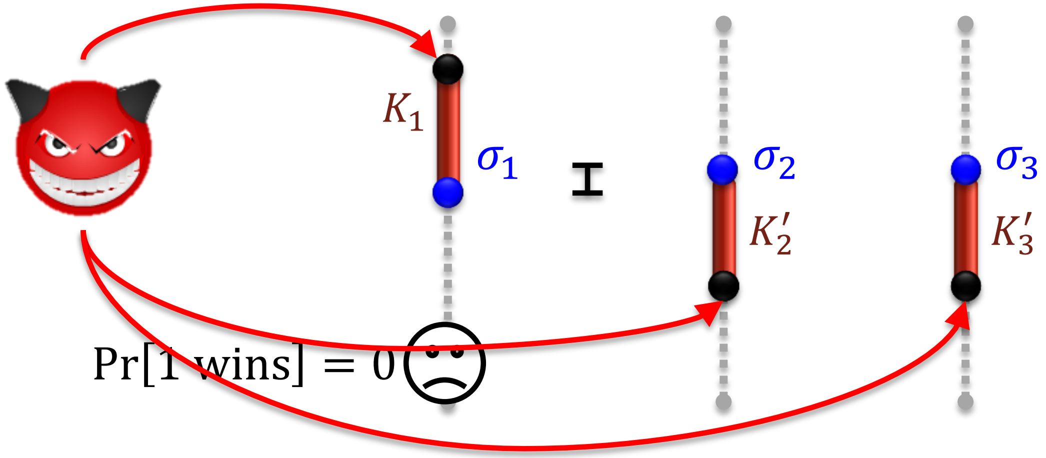

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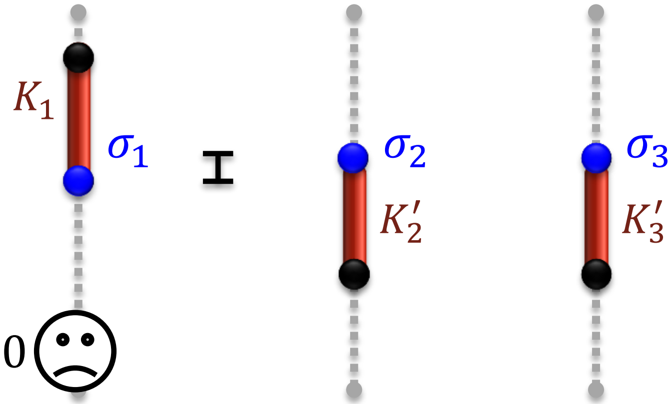

$\Pr[1 \text{ wins}] = 0$

$$MSW = (1+\delta)x$$

$$SW = \frac{(1-\delta)^2}{1+\delta}x$$

$$\Rightarrow \boxed{\varepsilon \geq \left(\frac{1-\delta}{1+\delta}\right)^2}$$

Second-price: QED

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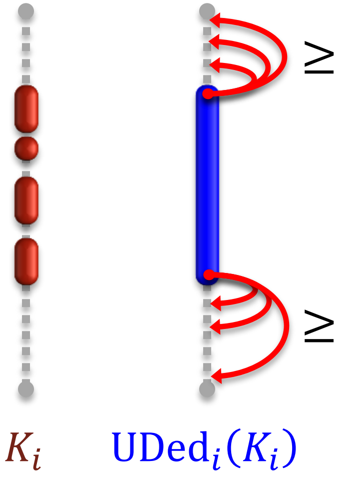

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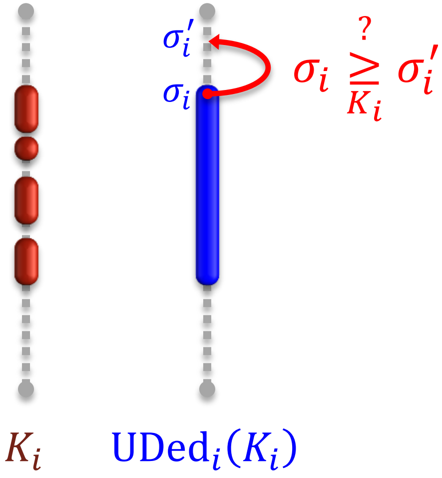

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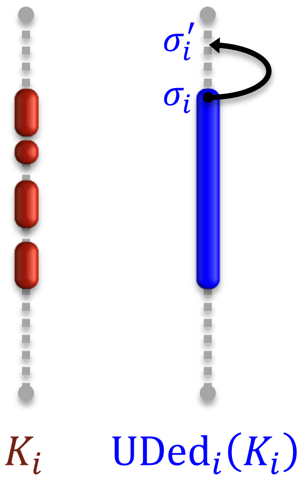

For any τ_{-i}, have three cases:

i loses in $\sigma_i \sqcup \tau_{-i}$; i wins in $\sigma_i' \sqcup \tau_{-i}$

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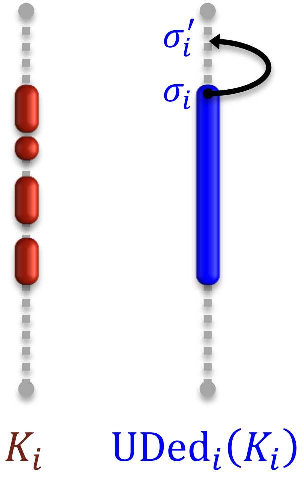

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only interesting one

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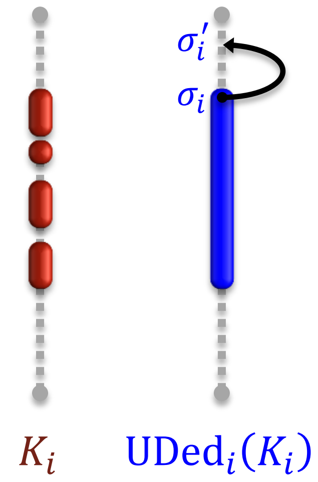

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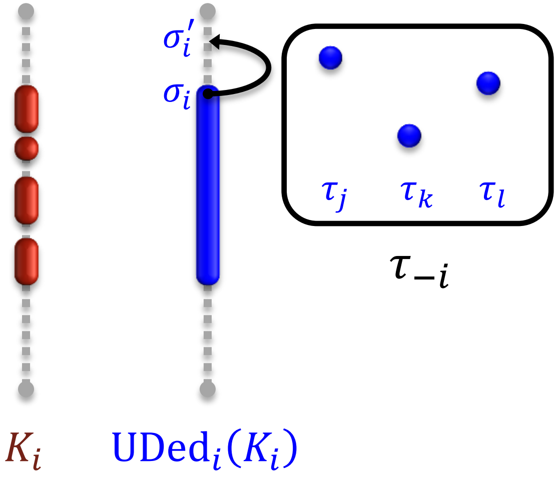

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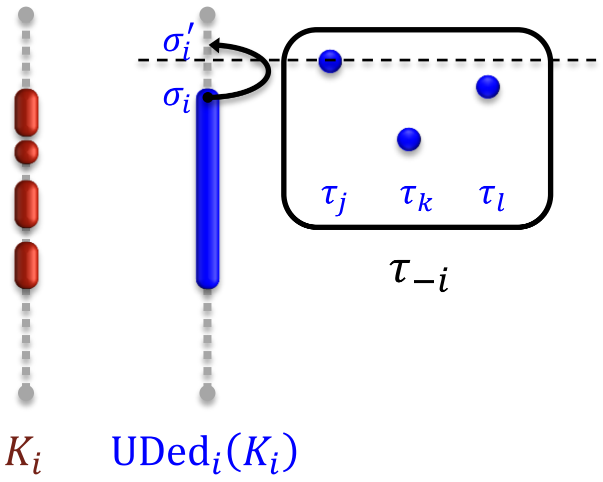

i loses in $\sigma_i \sqcup \tau_{-i}$; i wins in $\sigma_i' \sqcup \tau_{-i}$

Utility of player i:

$$\left.\begin{matrix} \sigma_i' \sqcup \tau_{-i} < 0 \\ \sigma_i \sqcup \tau_{-i} = 0 \end{matrix}\right\} \text{No matter what the devil chooses}$$

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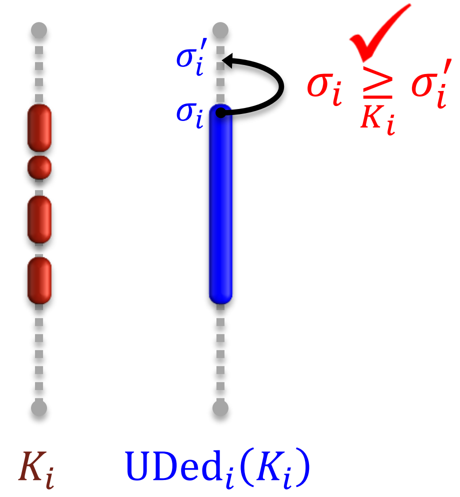

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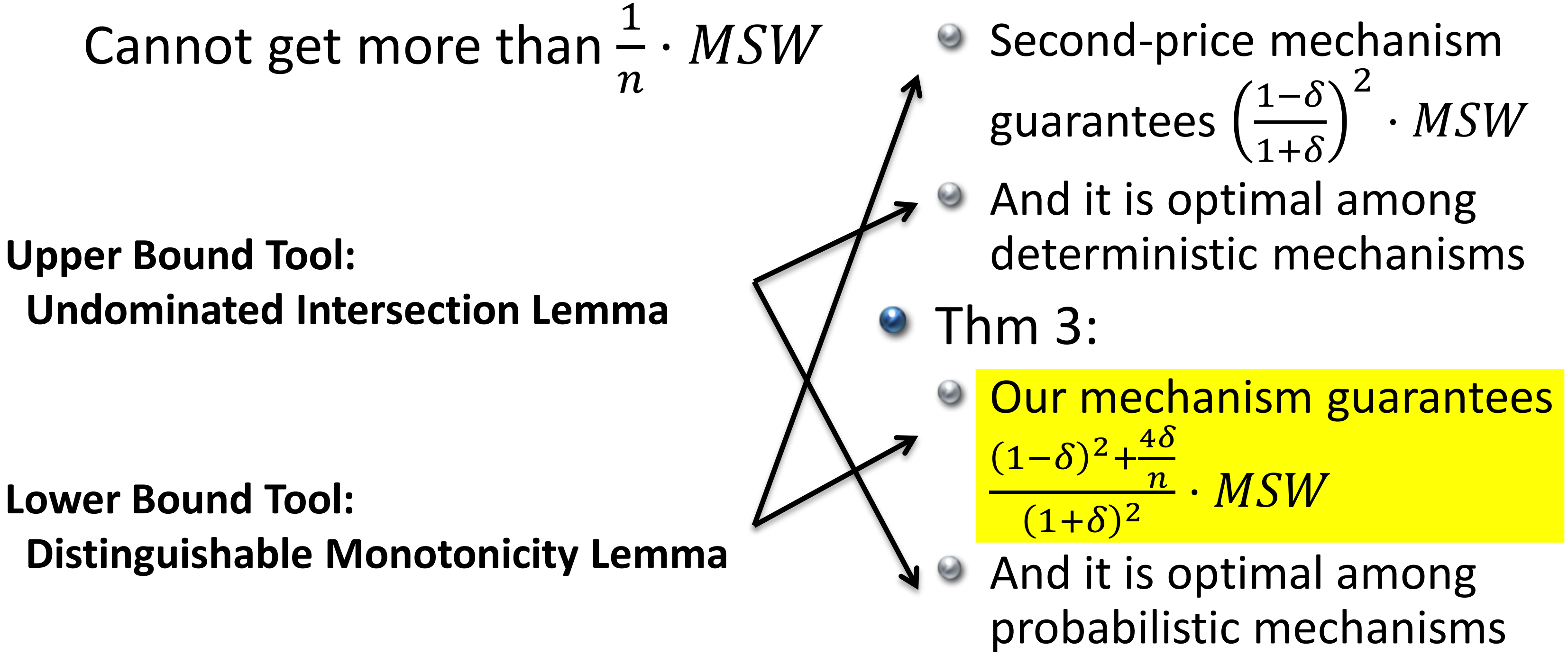
Our Results

Implementation in …

… Dominant Strategies

- Thm 1:
  Cannot get more than $\frac{1}{n} \cdot MSW$



Upper Bound Tool:
Undominated Intersection Lemma

Lower Bound Tool:
Distinguishable Monotonicity Lemma

… Undominated Strategies

- Thm 2:
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Lower Bounds on ε

Thm: Our mechanism guarantees $\frac{(1-\delta)^2+\frac{4\delta}{n}}{(1+\delta)^2} \cdot MSW$

Proof:

Lower Bounds on ε

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Proof:
… where to start?

Lower Bounds on ε

Thm: Our mechanism guarantees $\frac{(1-\delta)^2+\frac{4\delta}{n}}{(1+\delta)^2} \cdot MSW$

Proof:
… where to start?

Lower Bound Tool

Distinguishable Monotonicity Lemma:

$\forall$ monotonic* $f : \mathbb{R}^n \rightarrow [0,1]^n$, M_f satisfies

$\mathrm{UDed}_i(K_i) \subset [\min K_i, \max K_i]$

Lower Bounds on ε

Thm: Our mechanism guarantees $\frac{(1-\delta)^2+\frac{4\delta}{n}}{(1+\delta)^2} \cdot MSW$

Proof:
… where to start?

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$\forall$ monotonic* $f: \mathbb{R}^n \rightarrow [0,1]^n$, M_f satisfies

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Task reduces to designing a good f

Lower Bounds on ε

Lower Bound Tool

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- Designing f:
  - When bids are close to each other: give good at random.
  - When there is a “clear winner”: act like second-price.
  - If neither: interpolate in a smart way.

Lower Bounds on ε

Lower Bound Tool

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- Designing f:
  - When bids are close to each other: give good at random.
  - When there is a “clear winner”: act like second-price.
  - If neither: interpolate in a smart way.

This is delicate. In every “intermediate case”, need to:

1) ensure the target social welfare, and
2) ensure distinguishable monotonicity.

Our Optimal Mechanism

Our Optimal Mechanism

“10000m” view:

1. On input bids $(v_1, v_2, \ldots, v_n)$, WLOG $v_1 \geq v_2 \geq \cdots \geq v_n$.

Our Optimal Mechanism

"10000m" view:

$1, 2, 3, \ldots, n^*$ | $n^* + 1, \ldots, n$

1. On input bids $(v_1, v_2, \ldots, v_n)$, WLOG $v_1 \geq v_2 \geq \cdots \geq v_n$.

2. Find "magic" threshold n^*

Our Optimal Mechanism

“10000m” view:

candidate winners | losers

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Our Optimal Mechanism

“10000m” view:

candidate winners
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Our Optimal Mechanism

“100m” view:

candidate winners
$1, 2, 3, \ldots, n^*$

losers
$n^* + 1, \ldots, n$

1. On input bids $(v_1, v_2, \ldots, v_n)$, WLOG $v_1 \geq v_2 \geq \cdots \geq v_n$.

2. Find “magic” threshold n^* s.t. $\begin{cases} v_i > \frac{\sum_{j=1}^{n^*} v_j}{n^*+D(\delta)} & \text{for all } 1 \leq i \leq n^* \\ v_i \leq \frac{\sum_{j=1}^{n^*} v_j}{n^*+D(\delta)} & \text{for all } n^* < i \leq n \end{cases}$

3. Assign good to only “candidate winning” players $1,2, \ldots, n^*$ where player $i \in \{1,2, \ldots, n^*\}$ wins with “magic” probability:

$$f_i(v) = \frac{1}{n} \cdot \frac{n + D(\delta)}{n^* + D(\delta)} \cdot \frac{v_i(n^* + D(\delta)) - \sum_{j=1}^{n^*} v_j}{v_i D(\delta)}$$

Our Optimal Mechanism

“100m” view:

candidate winners | losers
$1, 2, 3, \ldots, n^*$ | $n^* + 1, \ldots, n$

1. On input bids $(v_1, v_2, \ldots, v_n)$, WLOG $v_1 \geq v_2 \geq \cdots \geq v_n$.

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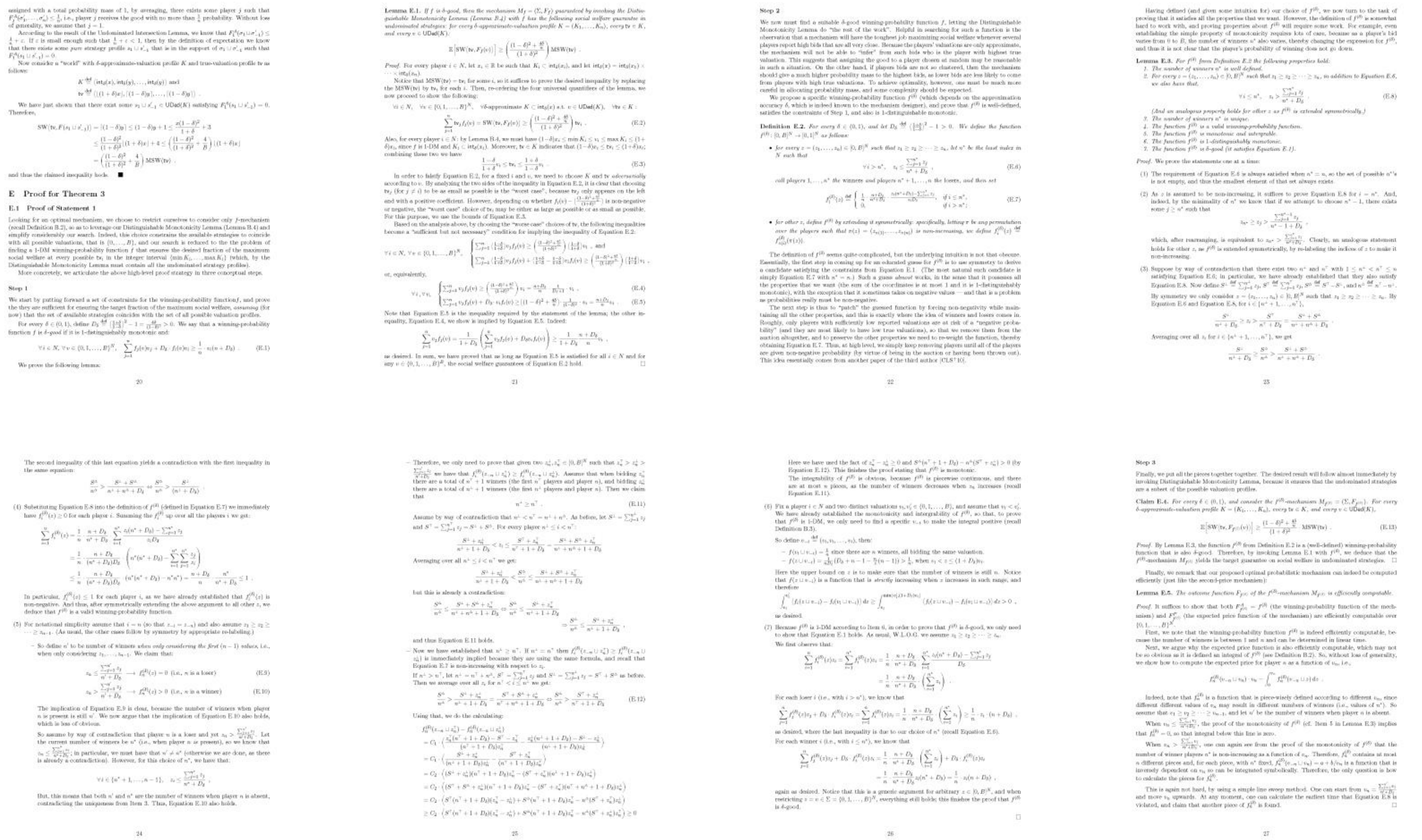
Easy to evaluate, just like the second-price mechanism!

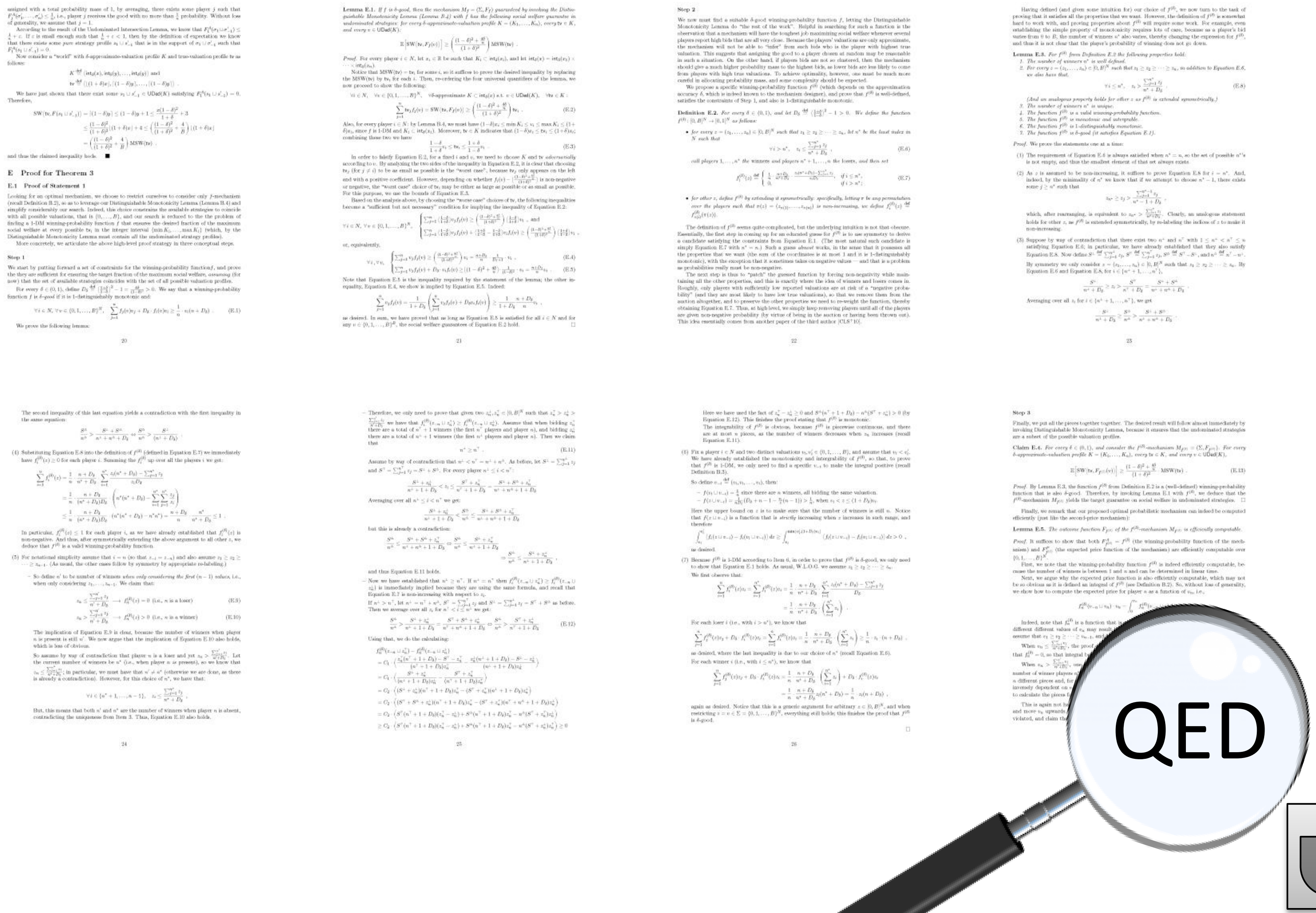
Our Optimal Mechanism

"1m" view:



Our Optimal Mechanism

“1m” view:

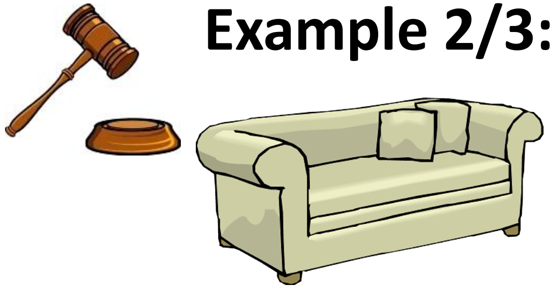


Approximate Valuations

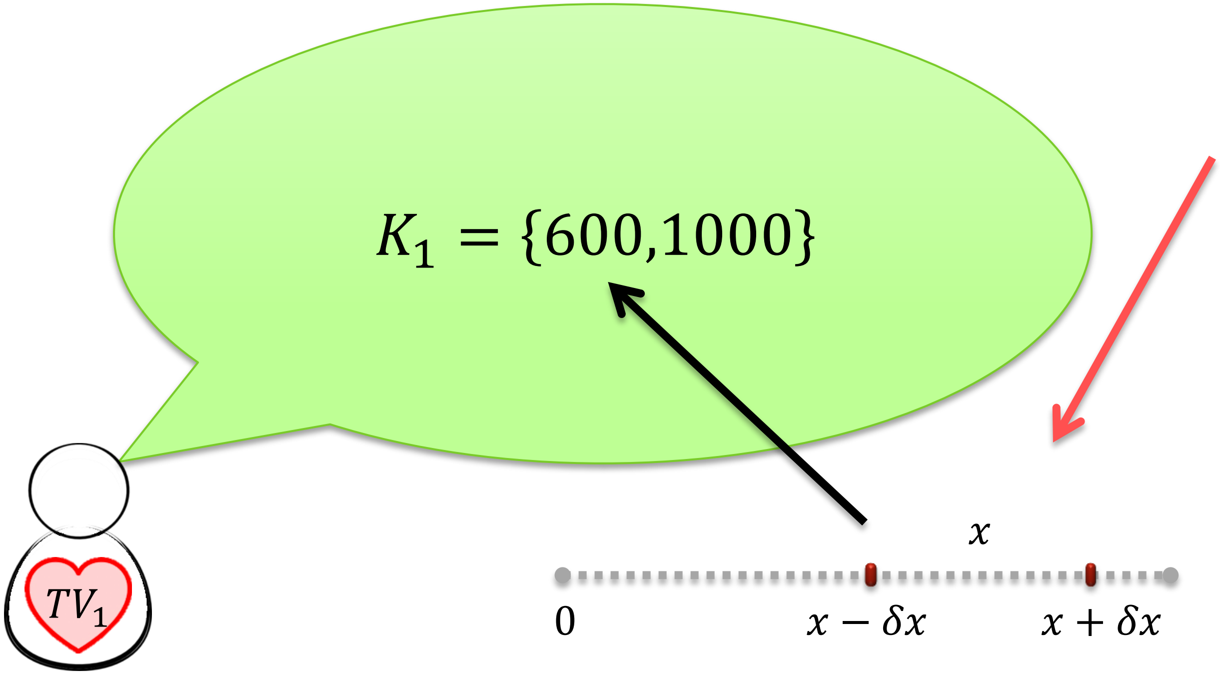
outcome

single-good auction

Example 2/3:



yard sale



$$K_1 = \{600{,}1000\}$$

$$K_1 \subset [(1-\delta)800, (1+\delta)800]$$

different from Bayesian!

$$\Pr\begin{bmatrix}\text{police in another} \\ \text{neighborhood}\end{bmatrix}?$$

$$\Pr\begin{bmatrix}\text{police is on} \\ \text{lunch break}\end{bmatrix}?$$

$$\Pr\begin{bmatrix}\text{police chasing} \\ \text{a thief}\end{bmatrix}?$$

⋮