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算法博弈论中的两个均衡问题
Two Topics on Nash Equilibrium in Algorithmic Game Theory

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Submitted to the Department of Physics
in partial fulfillment of the requirements for the degree of

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at the

TSINGHUA UNIVERSITY

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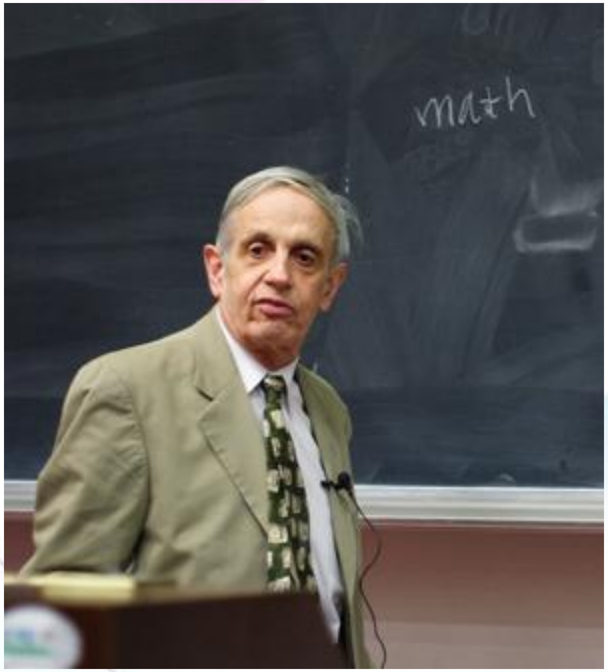
Thesis Supervisor

Prof. Wei Chen

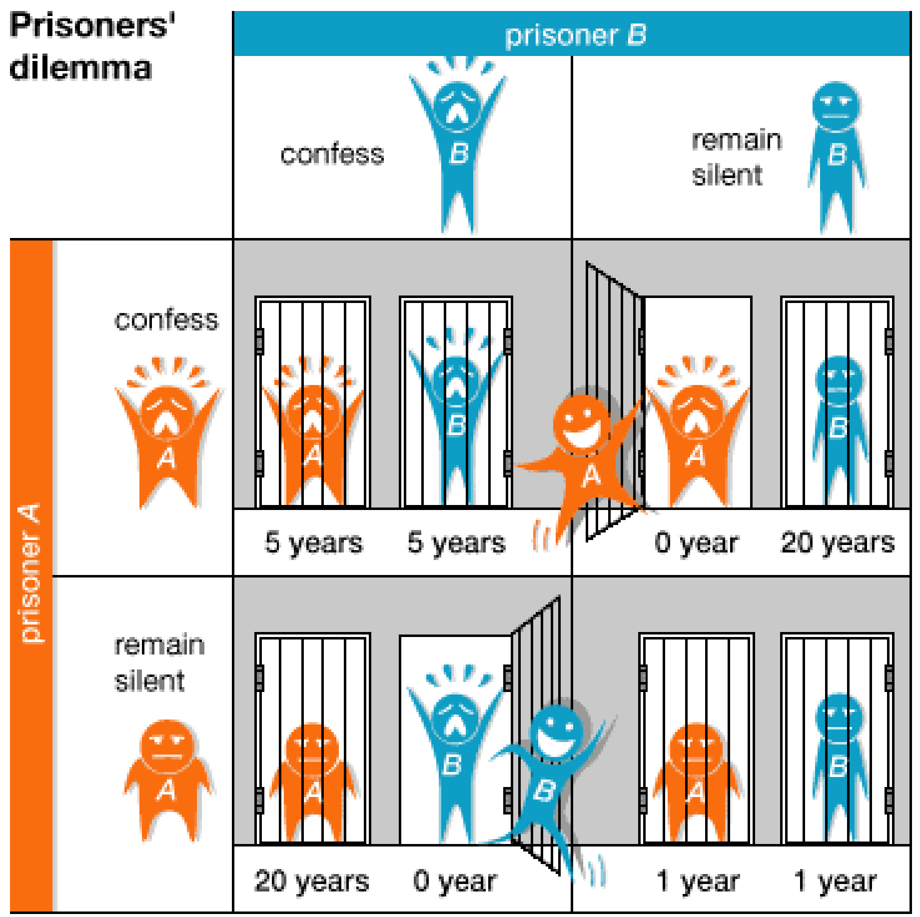
Adjunct Professor
Tsinghua University
Thesis Reviewer

Nash Equilibrium 纳什均衡



Nash Equilibrium 纳什均衡

- 囚徒问题(Prisoner's Dilemma)
  - 对嫌疑犯A和B独立审讯，提出惩罚规则：
    - 若二者都声称无罪，则同时被判入狱1年
    - 若二者都声称有罪，则同时被判入狱5年
    - 若其中一人认罪，则认罪者立即获释，否认有罪者获刑20年。



Nash Equilibrium 纳什均衡

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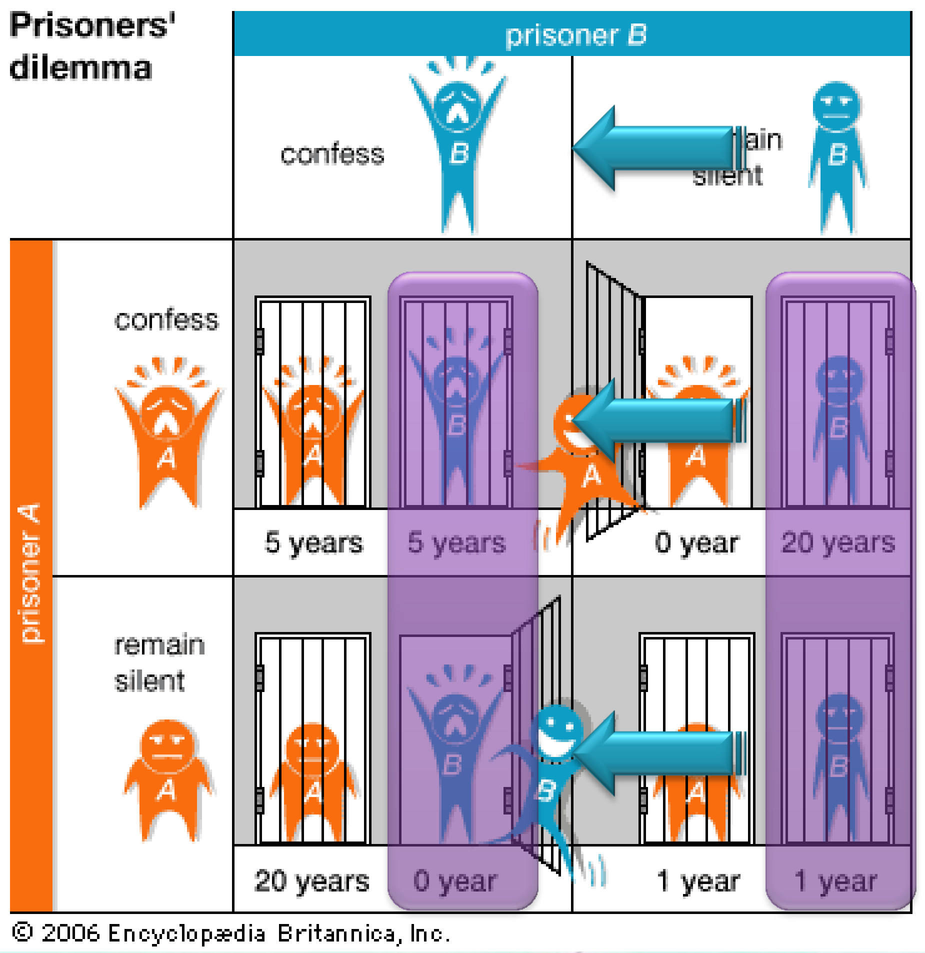

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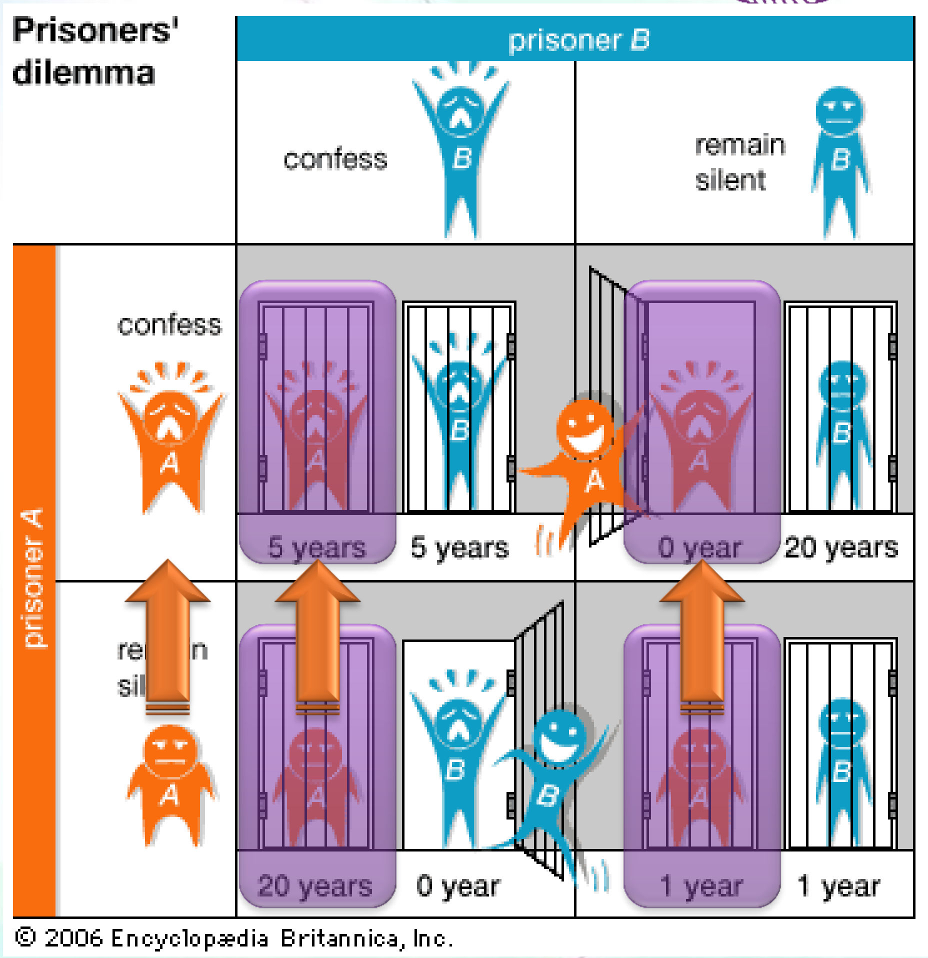

Nash Equilibrium 纳什均衡

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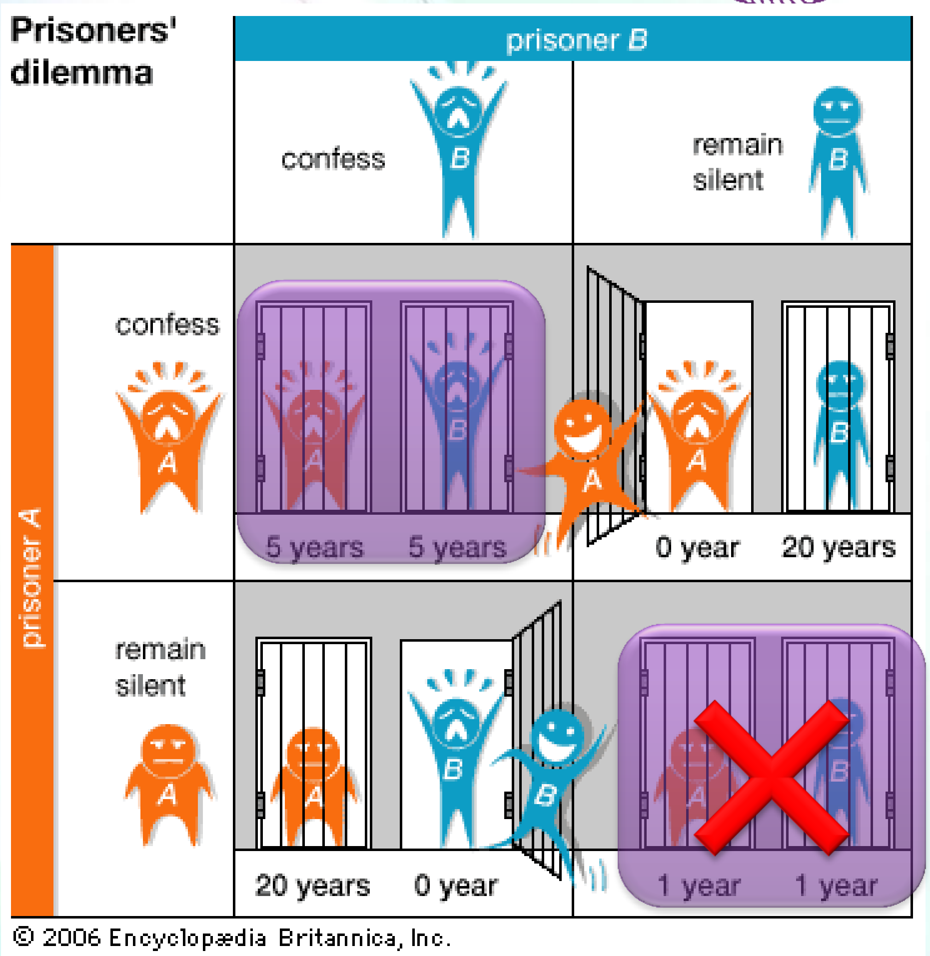

Nash Equilibrium 纳什均衡

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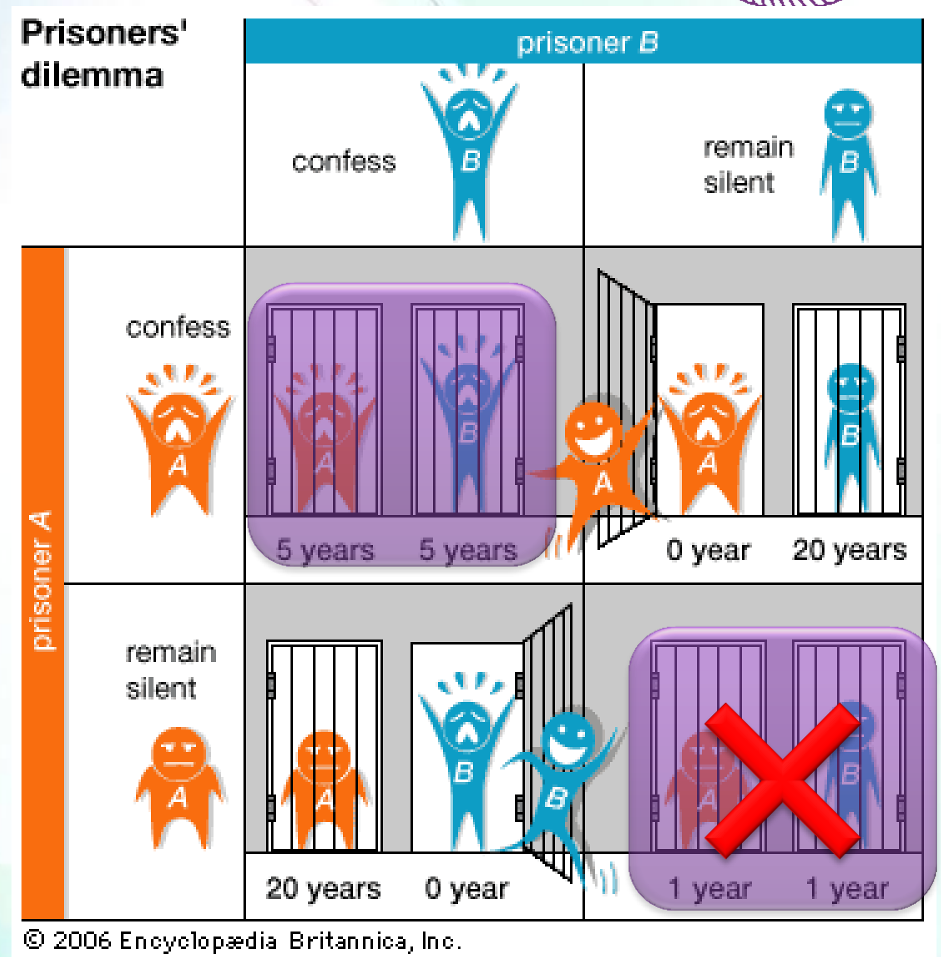
- 囚徒问题(Prisoner’s Dilemma)
  - 参与者不能通过独自行动而增加收益的策略集合，称之为纳什均衡
  - **Nash Equilibrium** is a strategy profile in which every participant cannot benefit through changing her own strategy unilaterally.



理性 (Rationality) = 纳什均衡 (Nash Equilibrium)

Our Problem 我们的问题

Microsoft® Research

- Chapter 2:
  - 如何计算均衡？How to calculate the equilibrium?
  - 定价问题 A Pricing Problem
  - *In submission to SIAM-SODA 2011.*
- Chapter 3:
  - 构建特定的机制，确保只有较优的均衡存在
    Construct specific mechanism to ensure only the optimal equilibrium is left
  - 没有金钱参与的机制设计
    Mechanism design without money
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社交网络下的定价问题
The Pricing Problem under Social Network

Introduction问题描述

Sells windows 7 at a universal price of p
以统一价p出售

Each buyer has an interval value v_i
每个买家有一个心里价位v_i

$v_1 \sim U(a_1, b_1)$ $v_2 \sim U(a_2, b_2)$ $v_n \sim U(a_n, b_n)$


……

The utility of buyer i： $u_i = v_i - p$
第i个买家的收益：

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第i个买家的收益：

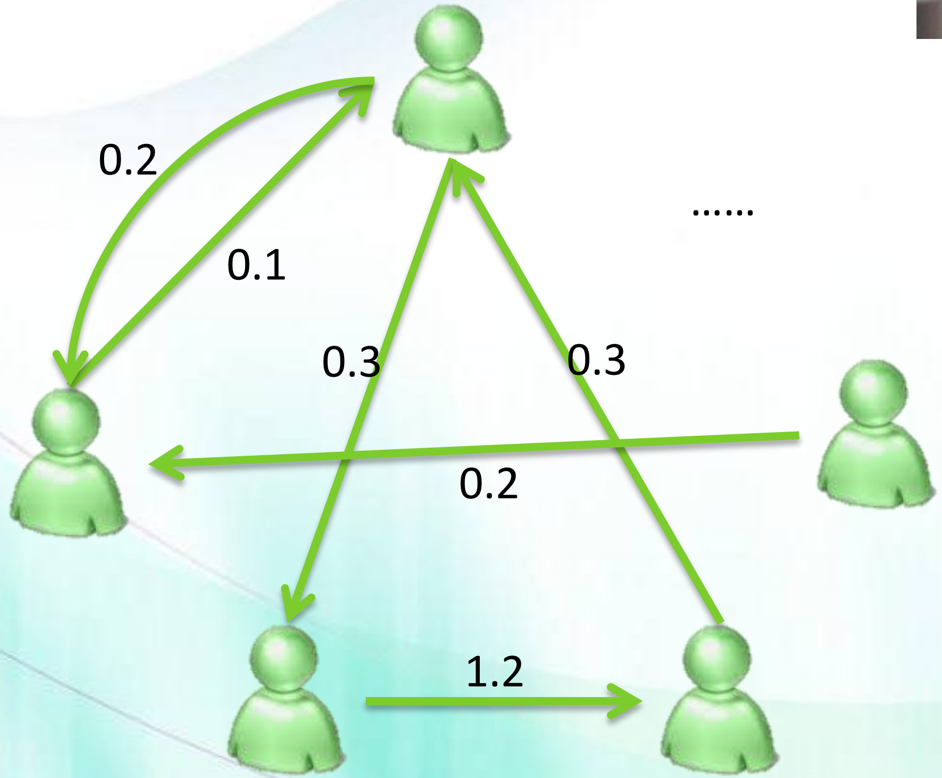
The current best strategy is obvious: buyer i buys Windows 7 if and only if $v_i > p$
此时的最佳策略是显然的：第i个买家购买Windows 7当且仅当$v_i > p$

Introduction问题描述

Microsoft® Research

In the reality, people form a social network, and each potential buyer may have a non-negative influence factor over his acquaintances.

现实生活中，人们组成一个社交网络，每个潜在的买家对他的熟人可以有一个非负的影响因子。



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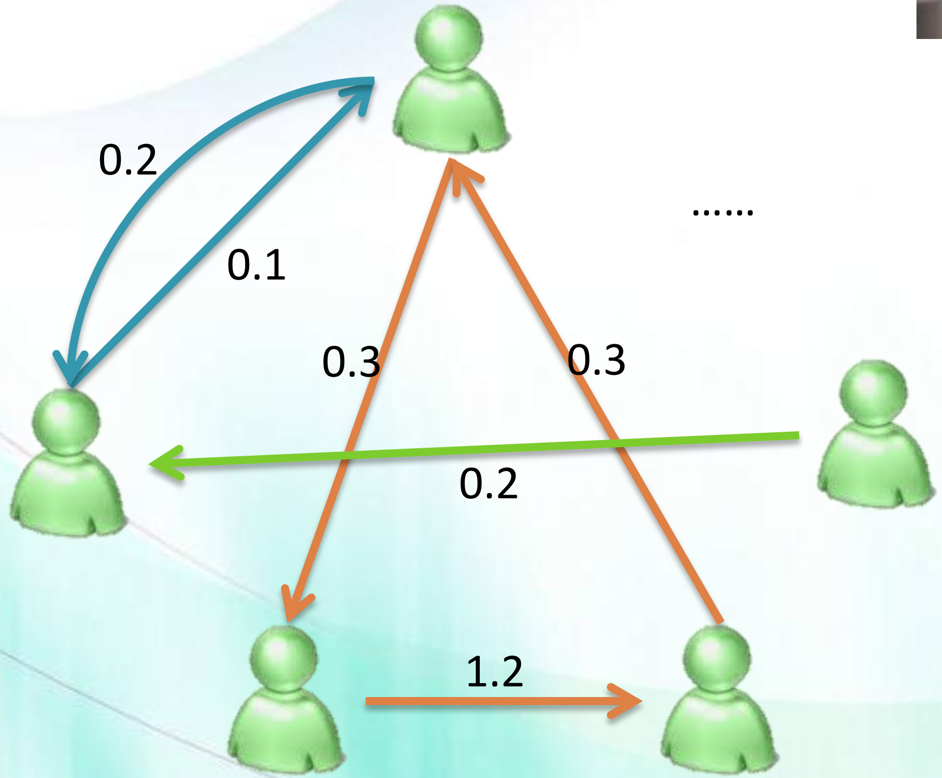

The Model 模型

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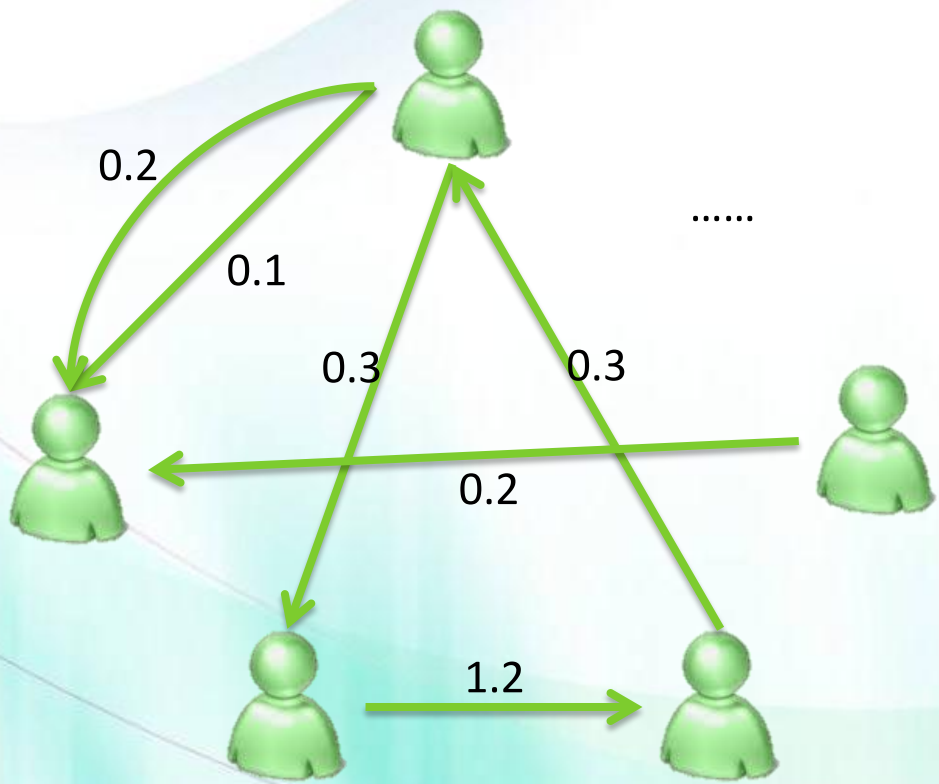

$$u_i = v_i - p + \sum_{j \neq i} T_{ji} d_j$$

$d_j \in \{0,1\}$ - whether j buys

第j个人是否购买

$T_{ji} \in \mathbb{R}$ - the influence factor

影响因子

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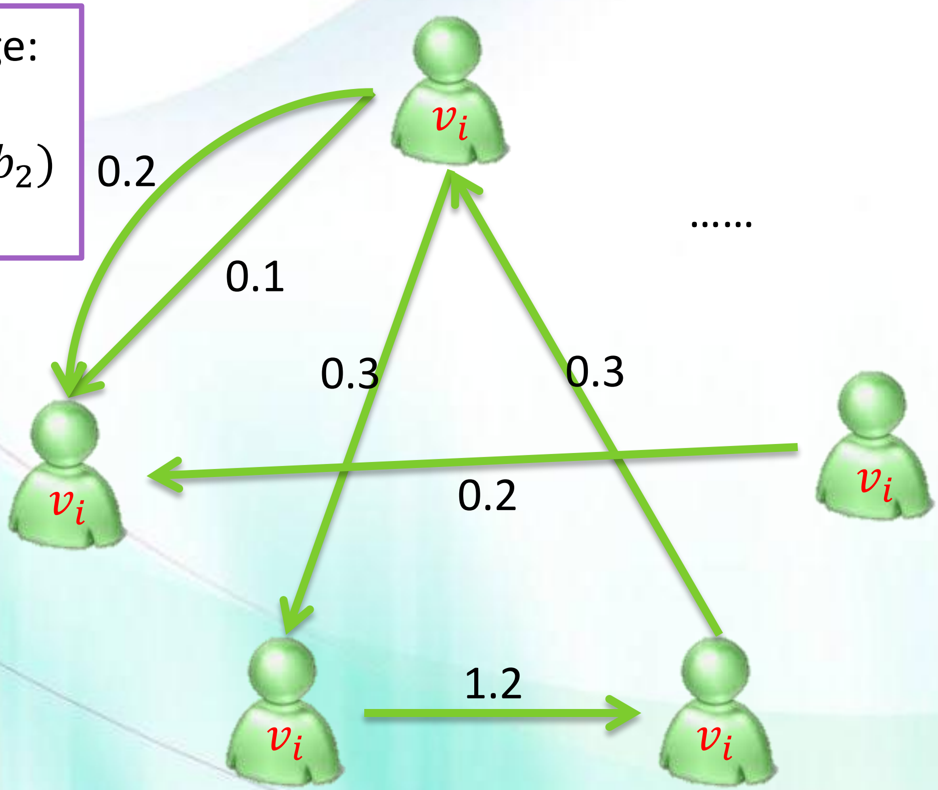
Public Knowledge:
公开信息:
$U(a_1,b_1), U(a_2,b_2)$
$\dots, U(a_n,b_n)$



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Bayesian Nash Equilibrium 贝叶斯纳什均衡

Bayesian Nash Equilibrium for Games with Incomplete Information
不完全信息的博弈，需要考虑**贝叶斯纳什均衡**

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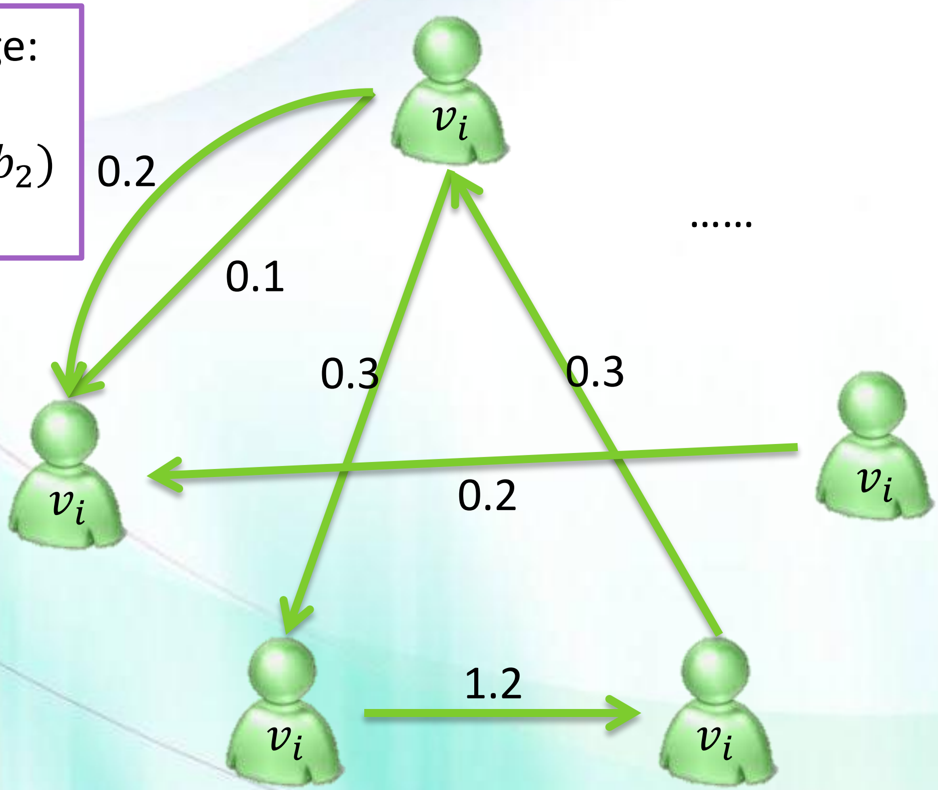


$$u_i = v_i - p + \sum_{j \neq i} T_{ji} d_j$$

$$\tilde{u}_i = v_i - p + \mathbb{E}\left[\sum_{j \neq i} T_{ji} d_j\right]$$

$$= v_i - p + \sum_{j \neq i} T_{ji} q_j$$

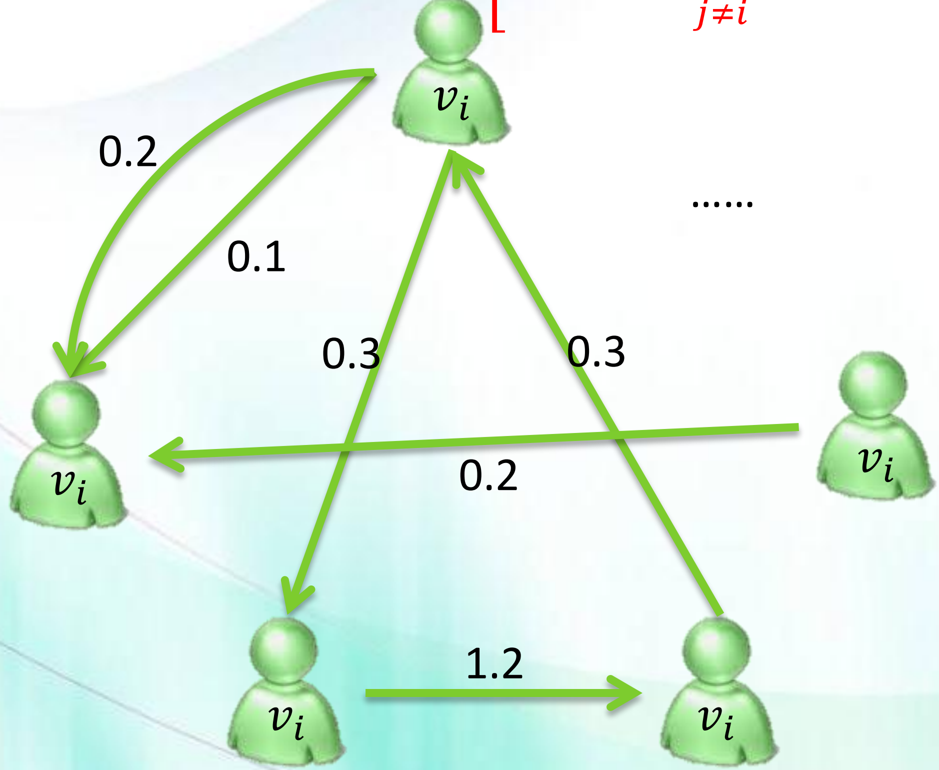
q_j - the probability j buys
第j个买家购买的概率

Bayesian Nash Equilibrium 贝叶斯纳什均衡

Bayesian Nash Equilibrium for Games with Incomplete Information
不完全信息的博弈，需要考虑**贝叶斯纳什均衡**

It can be proved that the equilibrium satisfies
可以证明此均衡满足：

$$q_i = \Pr_{v_i \sim U(a_i,b_i)}[\tilde{u}_i > 0] = \Pr_{v_i \sim U(a_i,b_i)}\left[v_i - p + \sum_{j \neq i} T_{ji} q_j > 0\right]$$



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$$= \operatorname{med}\left\{0, 1, \frac{b_i - p + \sum_{j \neq i} T_{ji} q_j}{b_i - a_i}\right\}$$

1
- Rationality
- 理性

2
- Equilibrium
- 纳什均衡

3
- Equation Set
- 方程组

?

Bayesian Nash Equilibrium 贝叶斯纳什均衡

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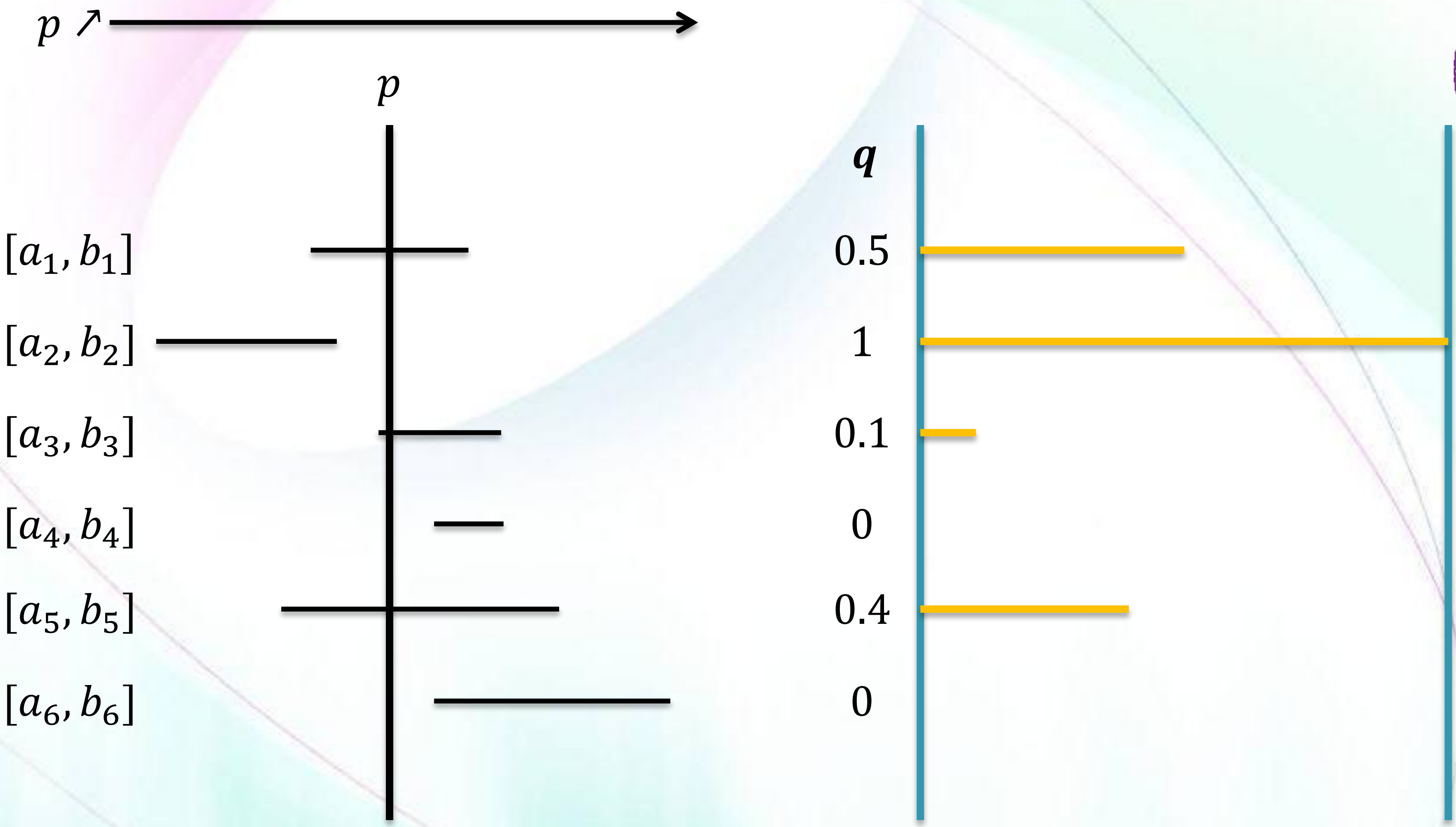
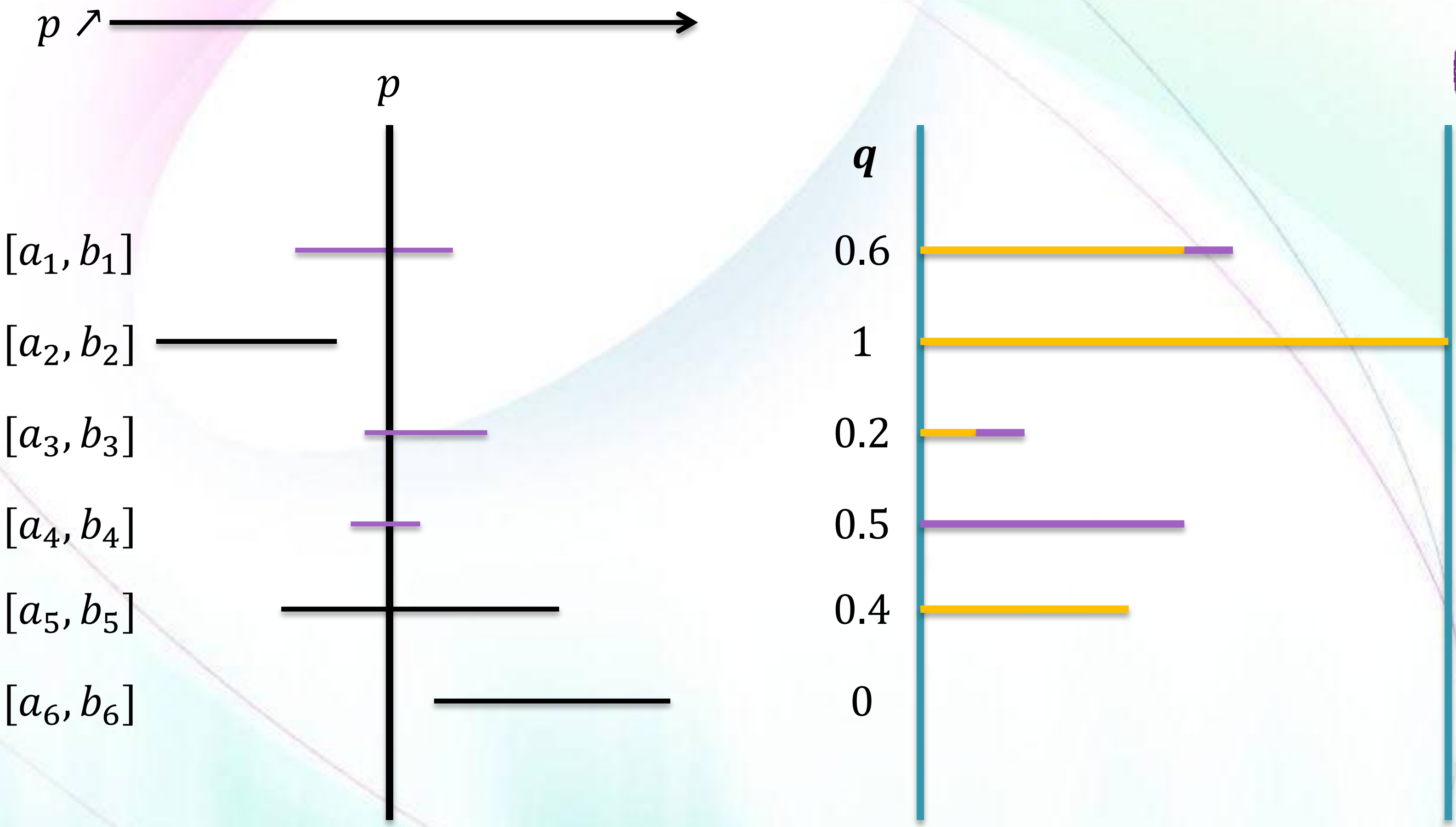
4
- Iterated Function
- 迭代函数

Iterated Function 迭代函数



Iterated Function 迭代函数



Iterated Function 迭代函数

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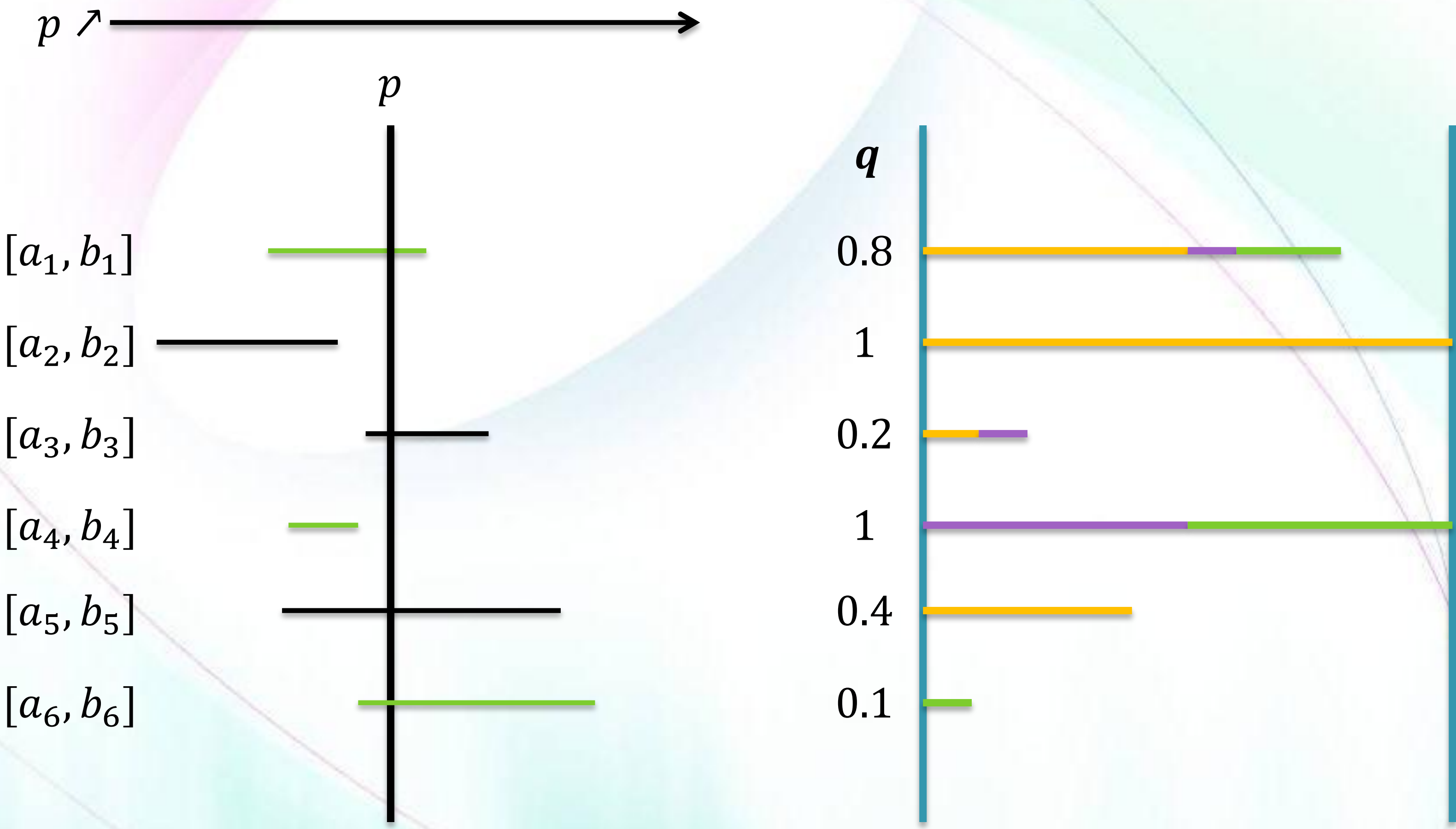

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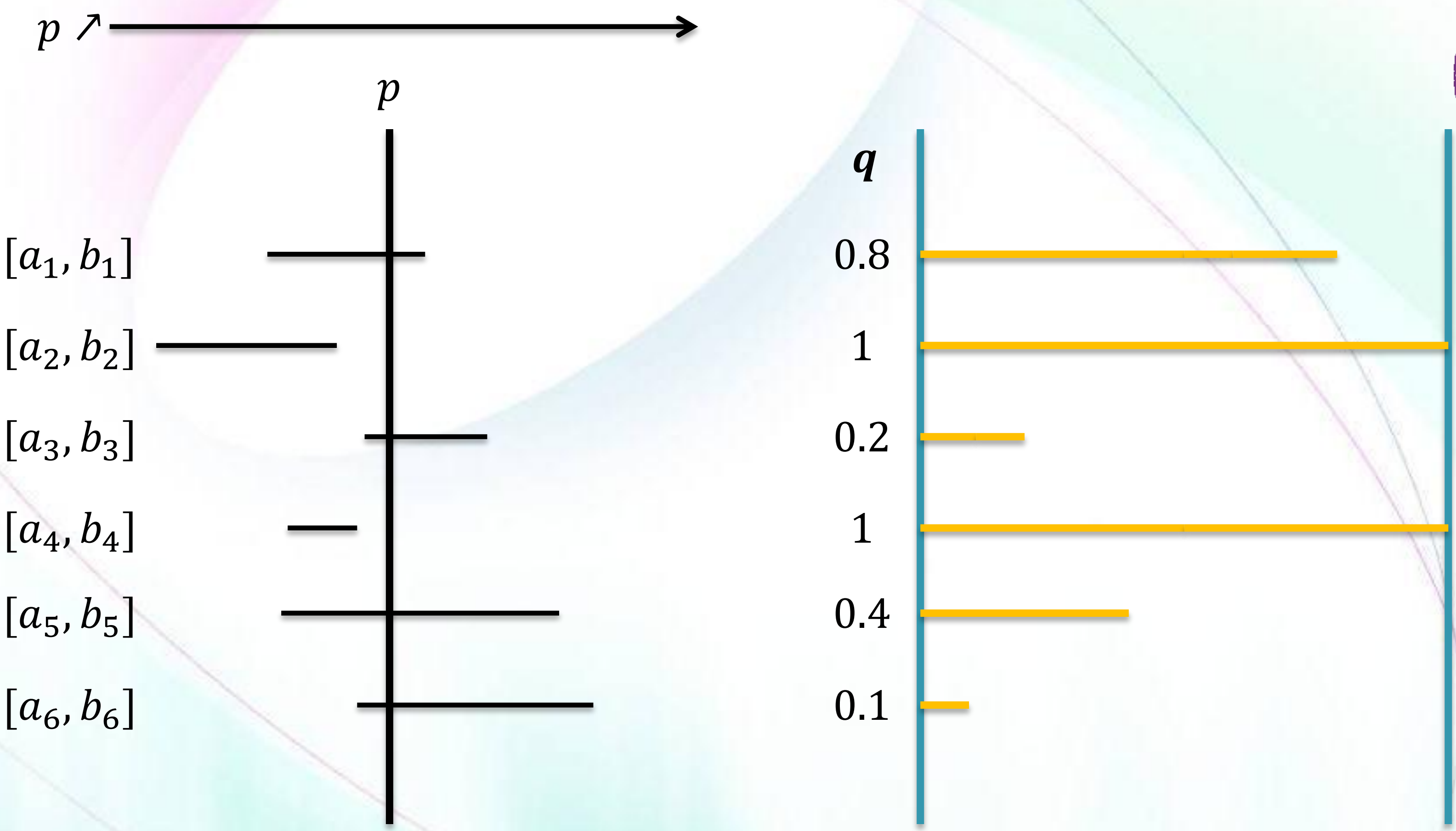

Call this the pessimistic equilibrium.
称之为悲观均衡

Main Idea 主要思想

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1
- Rationality
- 理性

2
- Equilibrium
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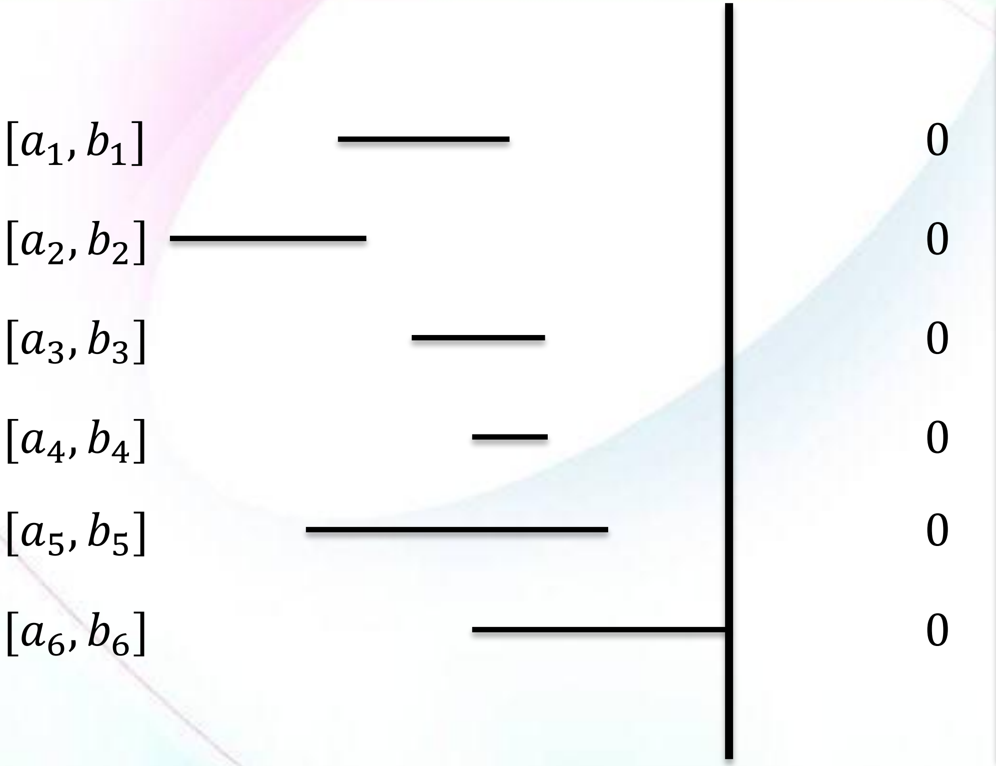
5
- Line Sweep Method
- 扫描法

6
- Eigenvector
- 特征向量

Line Sweep Method 扫描法



Line Sweep Method 扫描法

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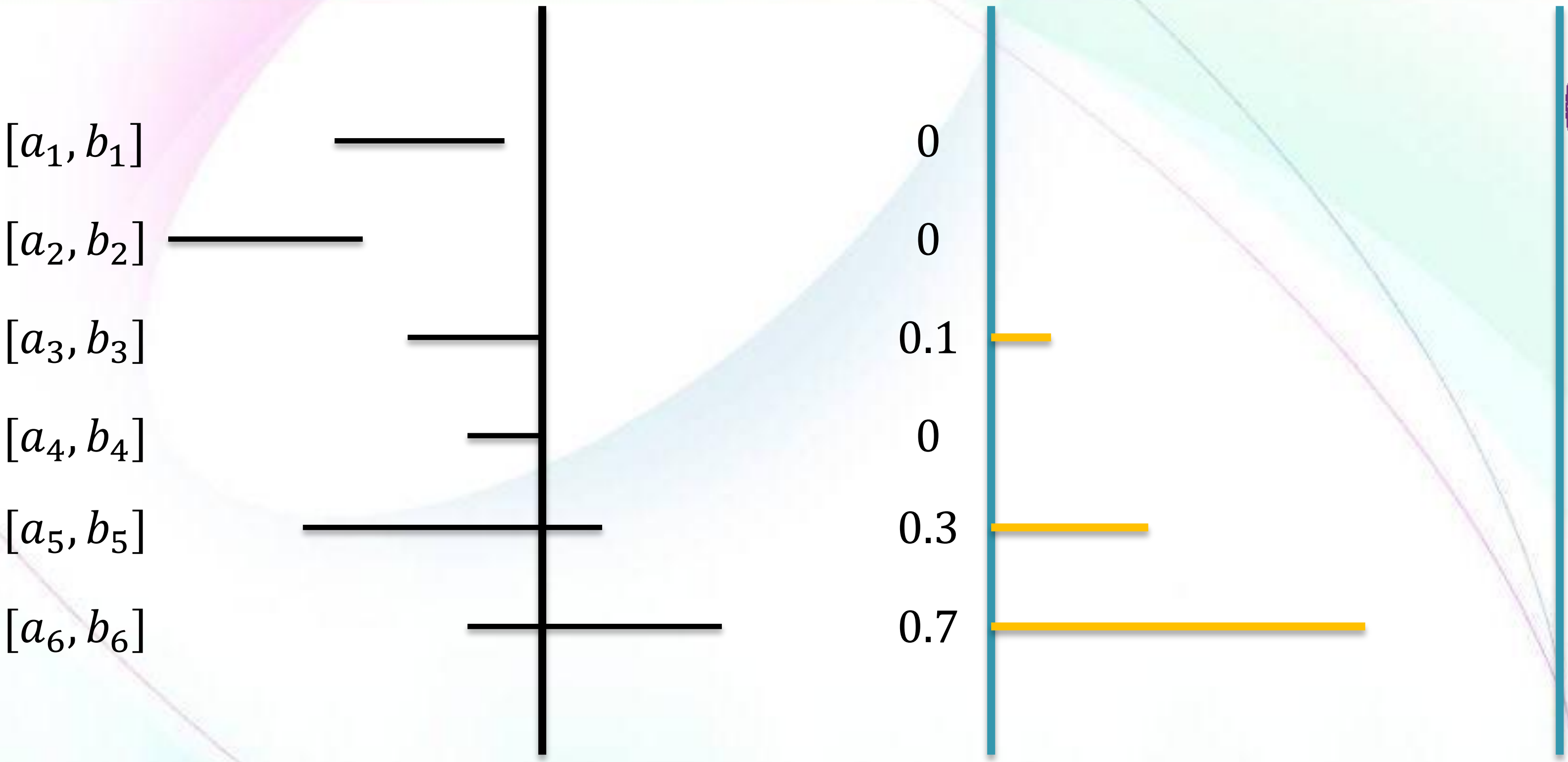

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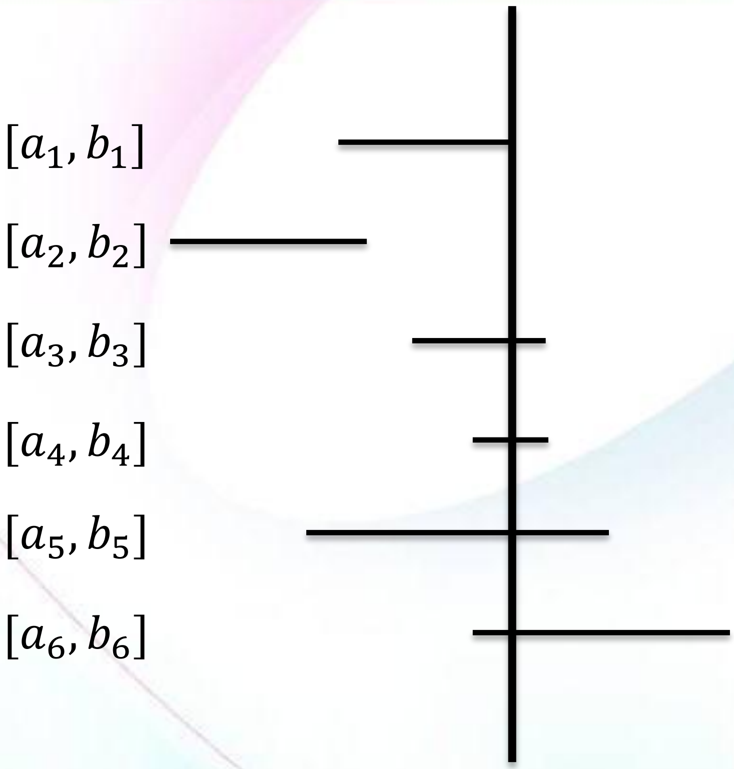
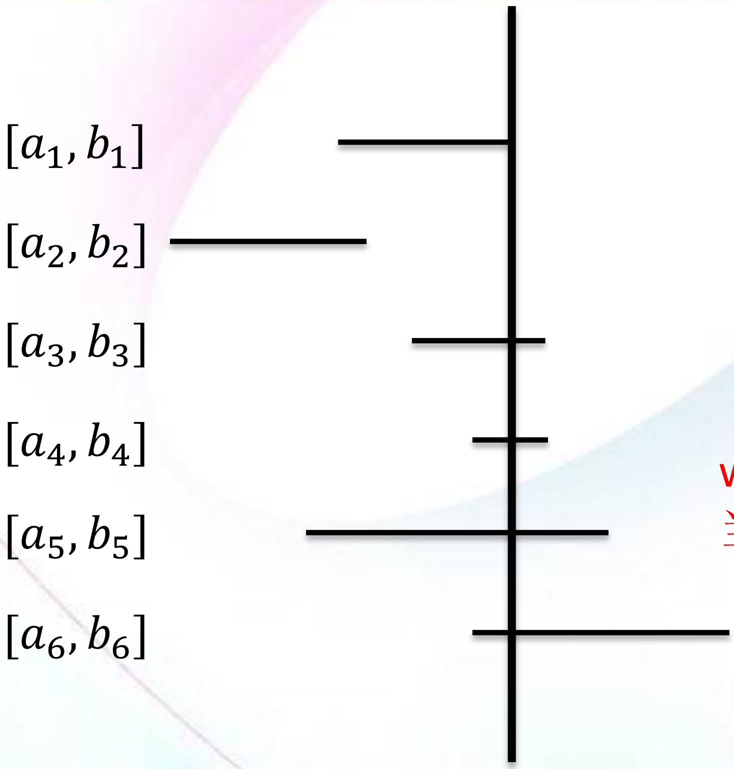


$$(I - L)^{-1} = \lim_{m \to \infty} \left(I + L + \cdots + L^{m-1} \right)$$

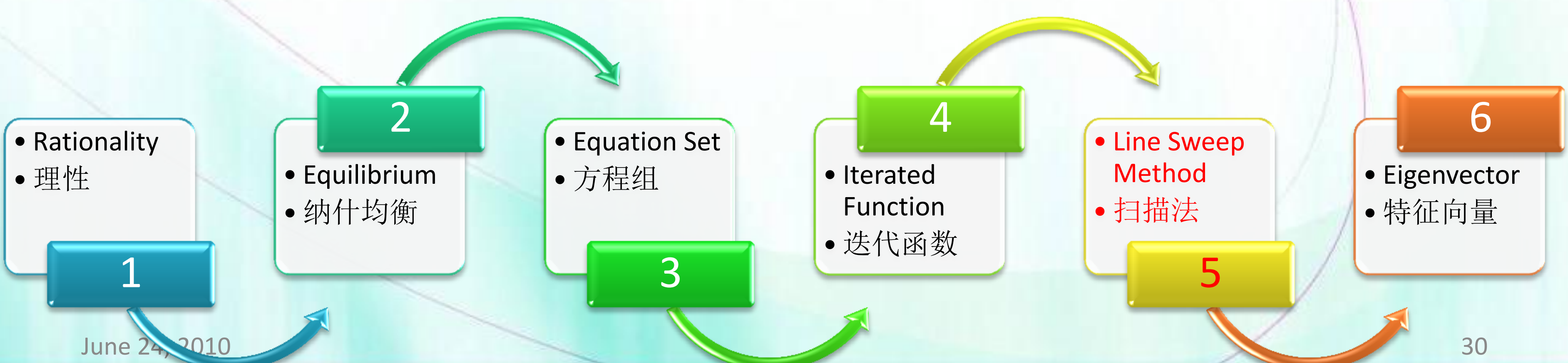
Line Sweep Method 扫描法



$$(I - L)^{-1} \neq \lim_{m \to \infty} \left(I + L + \cdots + L^{m-1}\right)$$

when at least one eigenvalue of L is larger than 1 in norm

当矩阵L至少有一个模大于1的特征值



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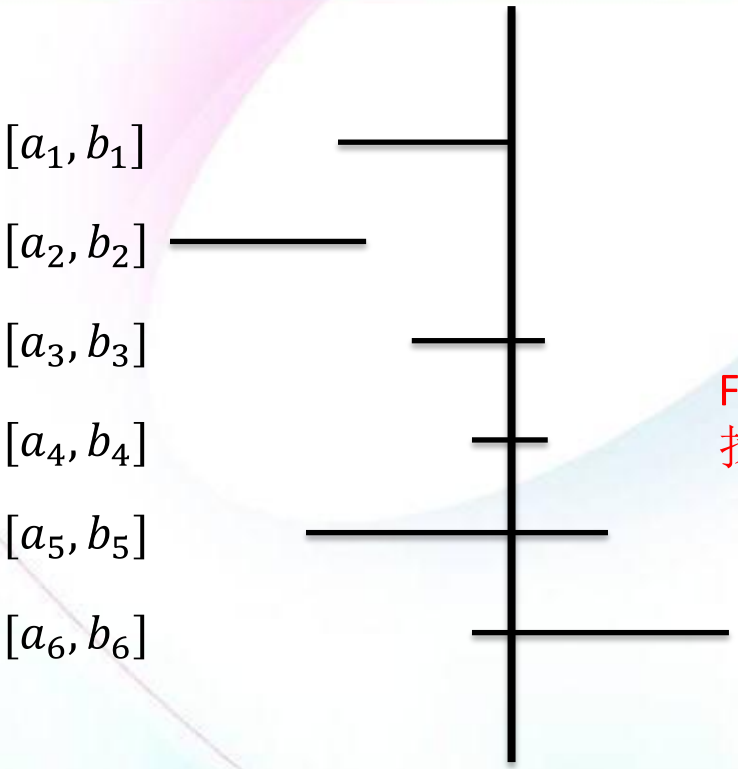

Find one eigenvector with real E.V ≥ 1
找到一个特征值为实数切大于等于1的特征向量

Line Sweep Method 扫描法

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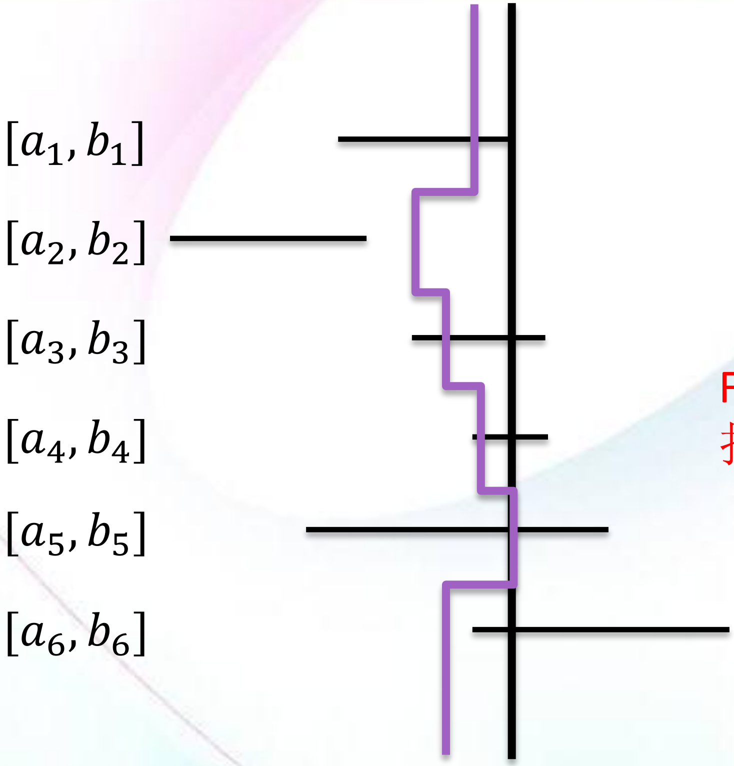

Find one eigenvector with real E.V $\geq$ 1
找到一个特征值为实数切大于等于1的特征向量

Conclusion

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- Formally introduced the rationality of buyers in the social network pricing problem.
  在社交网络定价问题中，定义了买家的理性行为。
- Exactly solved two extreme equilibria for a uniform valuation case in polynomial time.
  在多项式时间内，严格地解出了心理价位均匀分布时，两个极端的均衡。
- Exactly solved the optimum pricing strategy for the seller, assuming the rationality of the buyers.
  假定用户理性行为后，严格地解出了卖家的最优定价策略。
- Further work see our working paper.
  更多的信息参见我们的待投论文。

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Acknowledgement