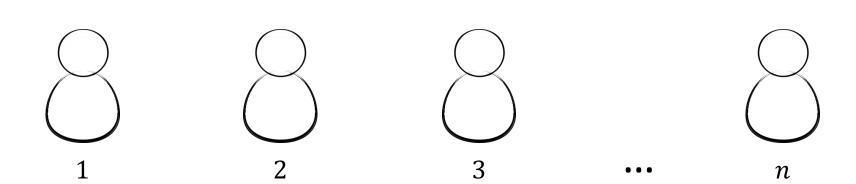
Mechanism Design with Approximate Valuations

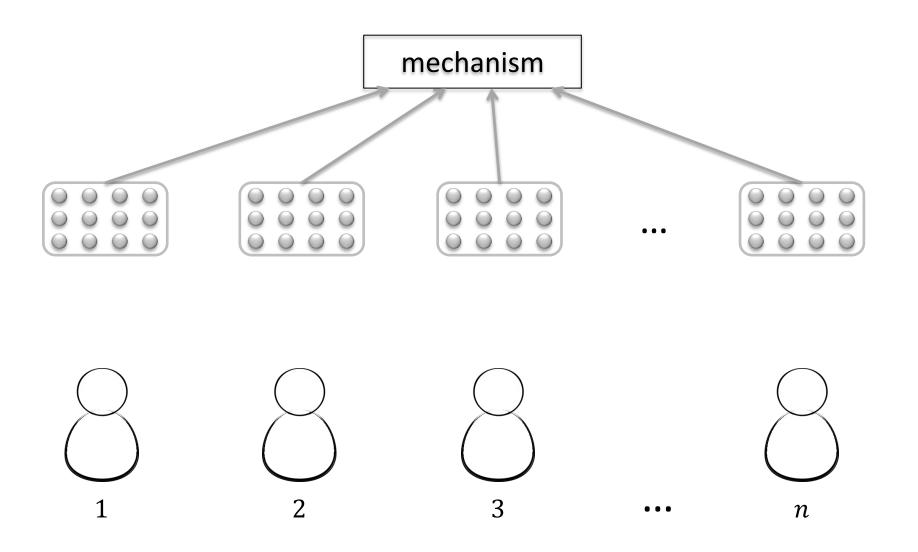
Alessandro Chiesa Silvio Micali Zeyuan Allen Zhu



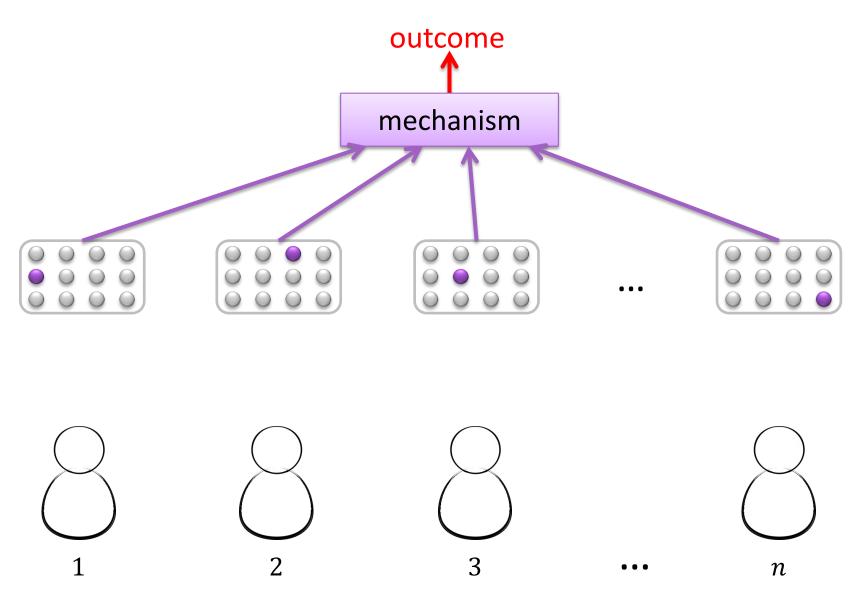
Recall...



Recall...



Recall...

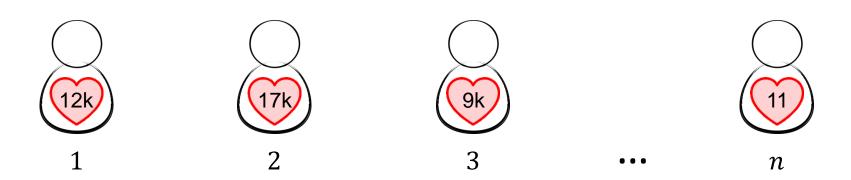


Rolex Auction

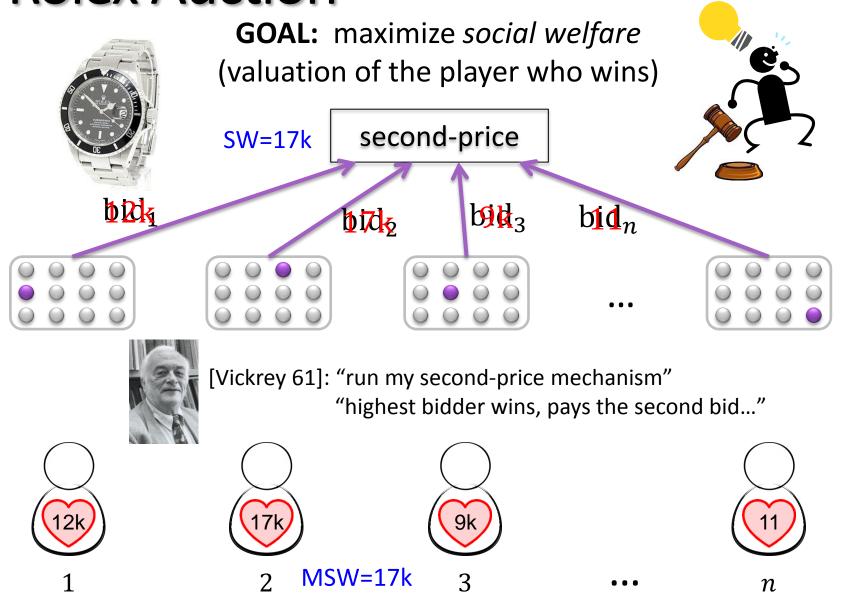


GOAL: maximize *social welfare* (valuation of the player who wins)





Rolex Auction



11,

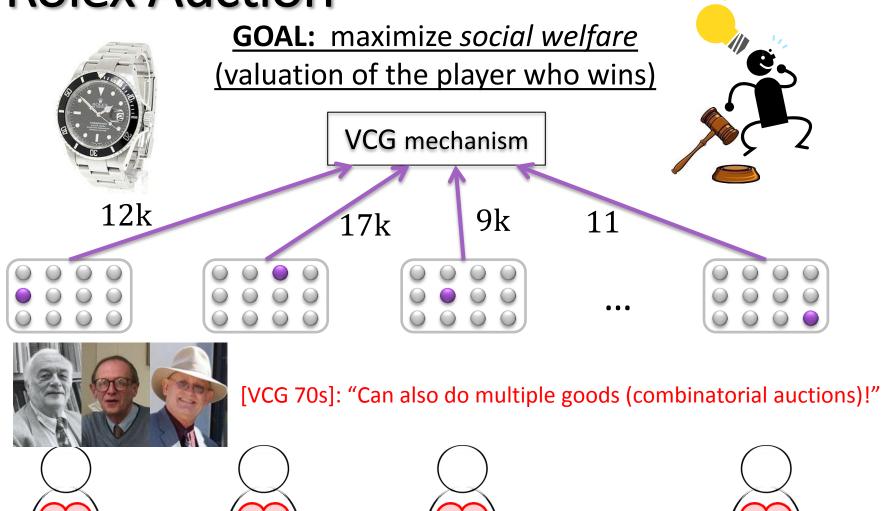
Rolex Auction

|2k|

1

7k

2



9k

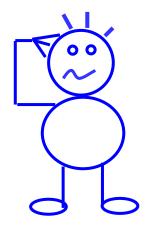
3

11,

n



Two-Line Mechanism
 Two-Line Proof
 Optimal Performance

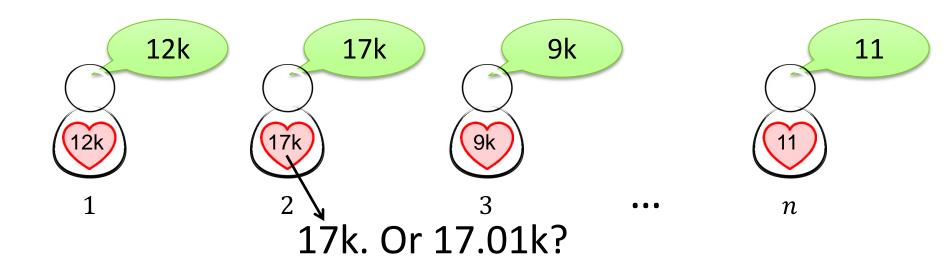


Oversimplified?

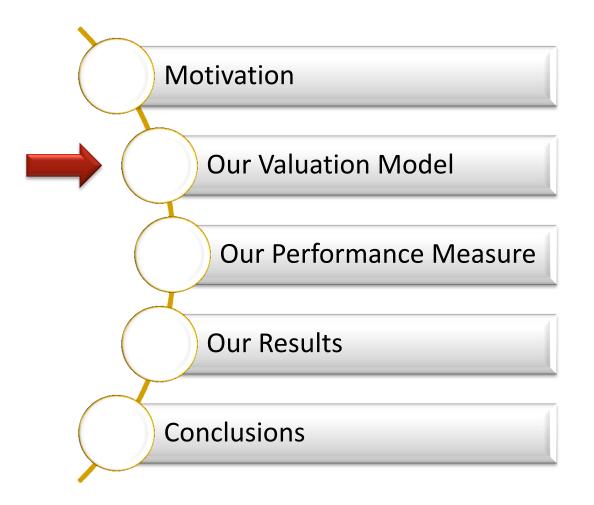
Warning!

optimal performance from an ASSUMPTION:

each player knows his own valuation exactly

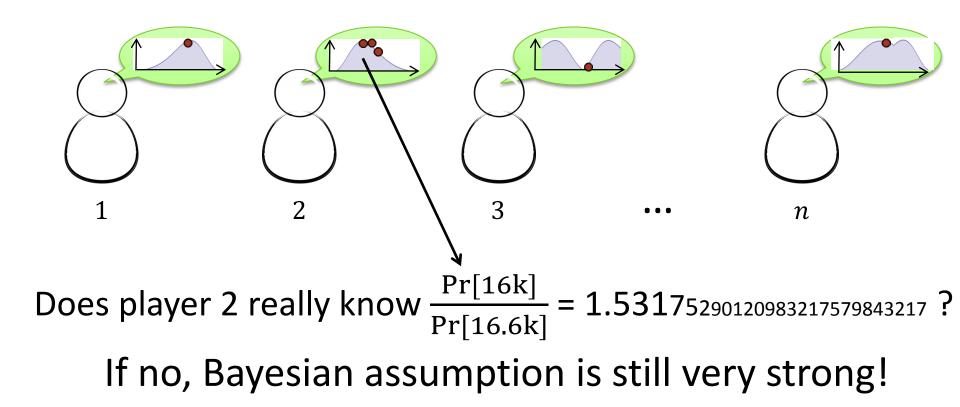


Today's Agenda



First attempt

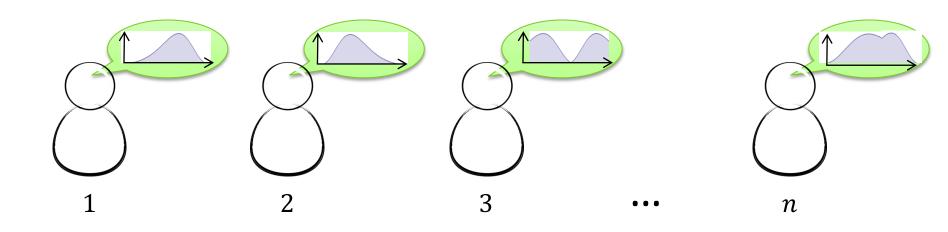
- Weaker assumption: Bayesian?
 - each player knows his own individual Bayesian
 - <u>same</u> second-price mechanism: just truthfully bid your expected value



First attempt

Weaker assumption: Bayesian?

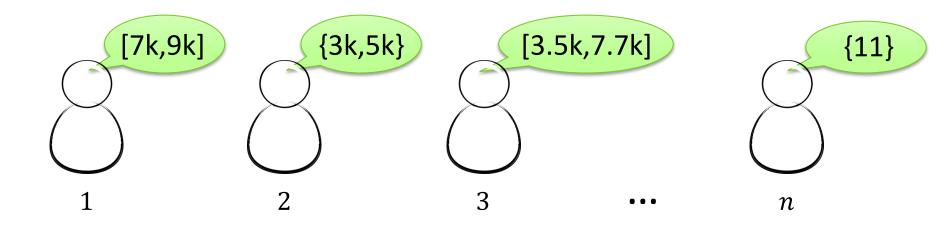




Our Attempt

Our assumption: "approximate valuation"

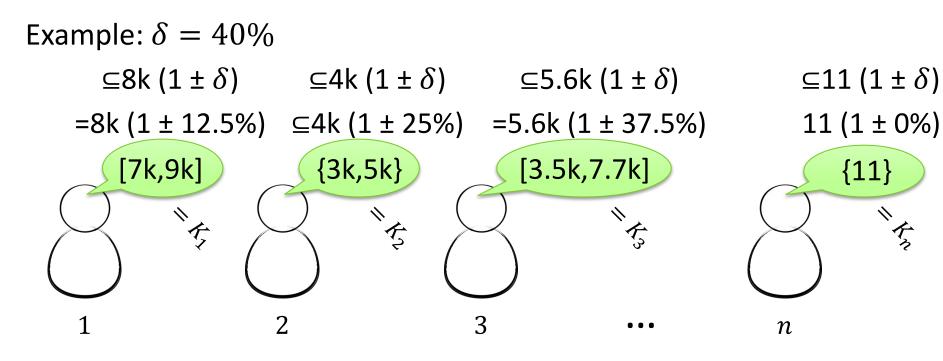
each player only knows that his valuation is <u>drawn</u> <u>from a set</u>



Our Attempt

Our assumption: "approximate valuation"

- Series a series of the se
- In player i has a δ -approximate valuation set K_i
- Solution θ_i is guaranteed to be ∈ K_i



Our Attempt

Our assumption: "approximate valuation"

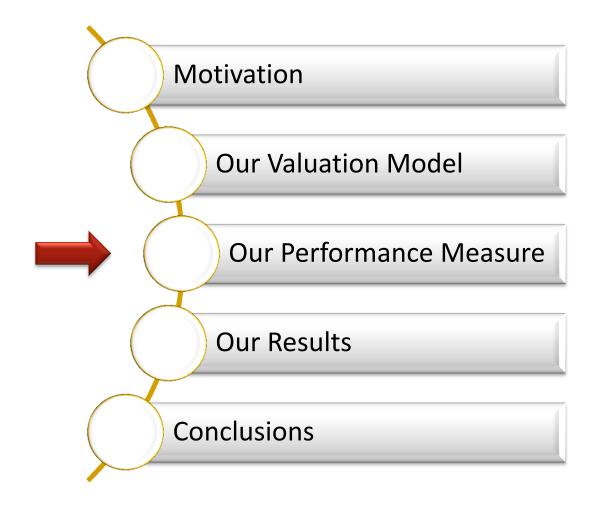
- Series a series of the se
- In player i has a δ -approximate valuation s
- Player *i*'s true valuation θ_i is guaranteed to be $\in K_i$

 $\delta = 0 \Rightarrow$ Classical Mechanism Design $\delta > 0 \Rightarrow$ Mechanism Design with Approximate Valuations

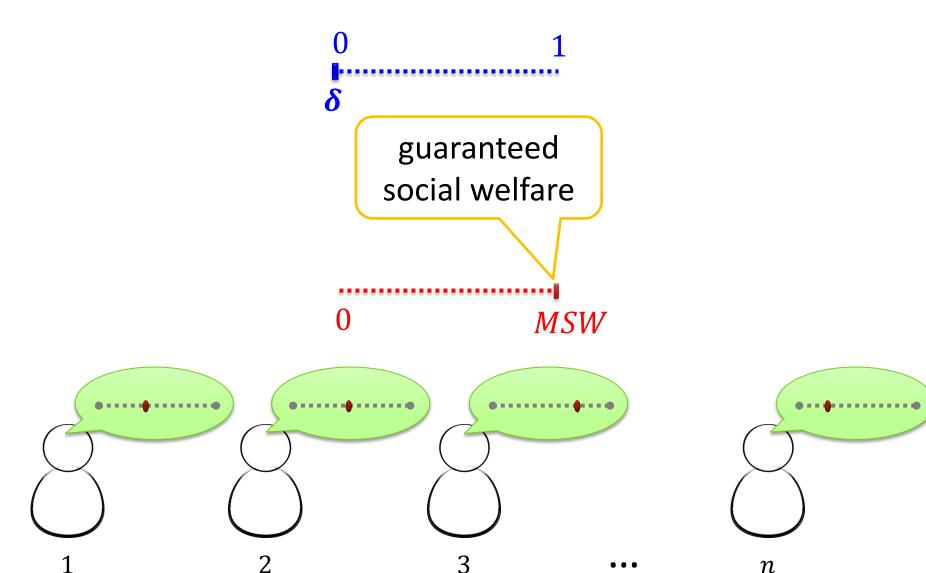
Unrelated work: Knightian decision theory Uncertainty is modeled as a set, but not studied under mechanism design.



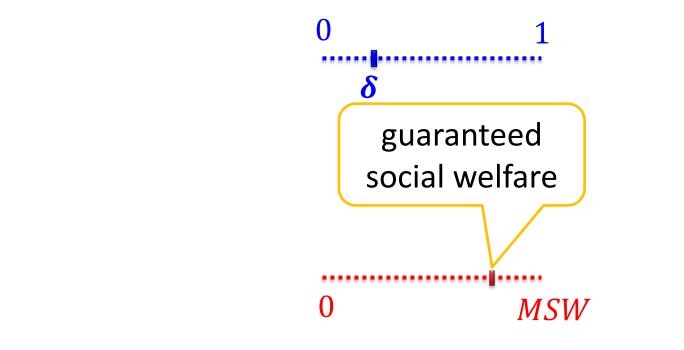
Today's Agenda

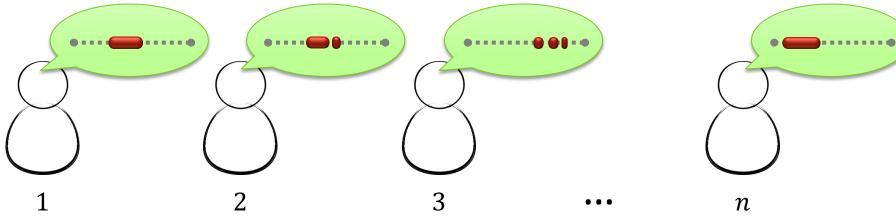


How Much SW Can We Guarantee?

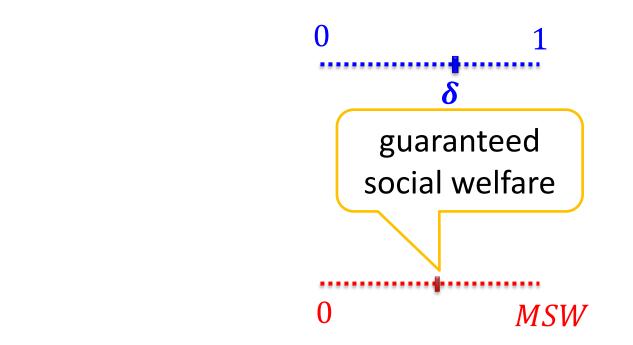


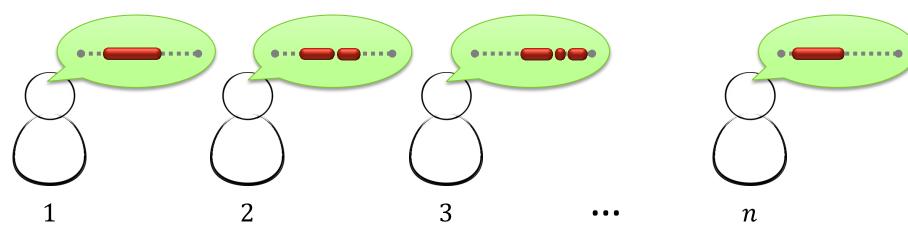
How Much SW Can We Guarantee?





How Much SW Can We Guarantee?

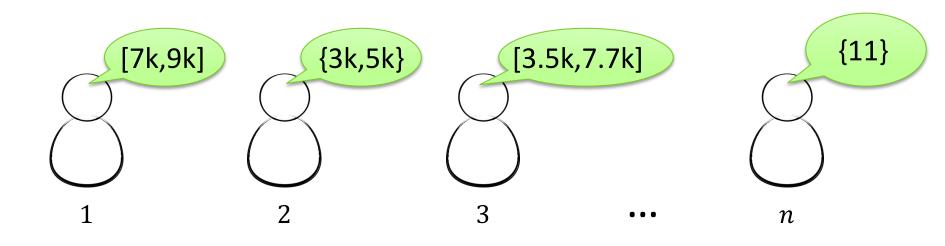


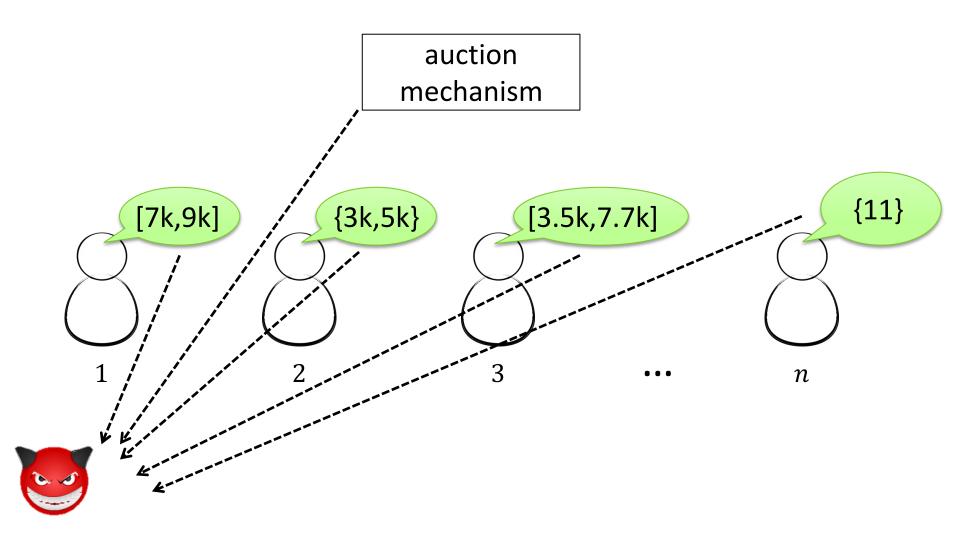


WAIT!!!

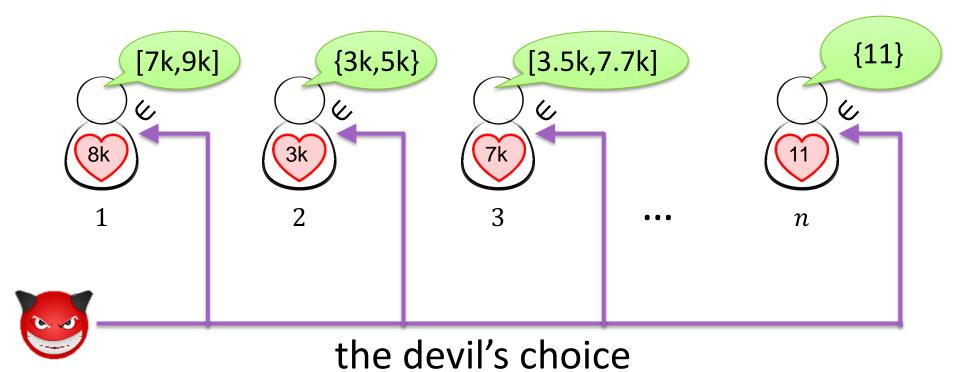
How to define SW or MSW when θ_i is unknown?

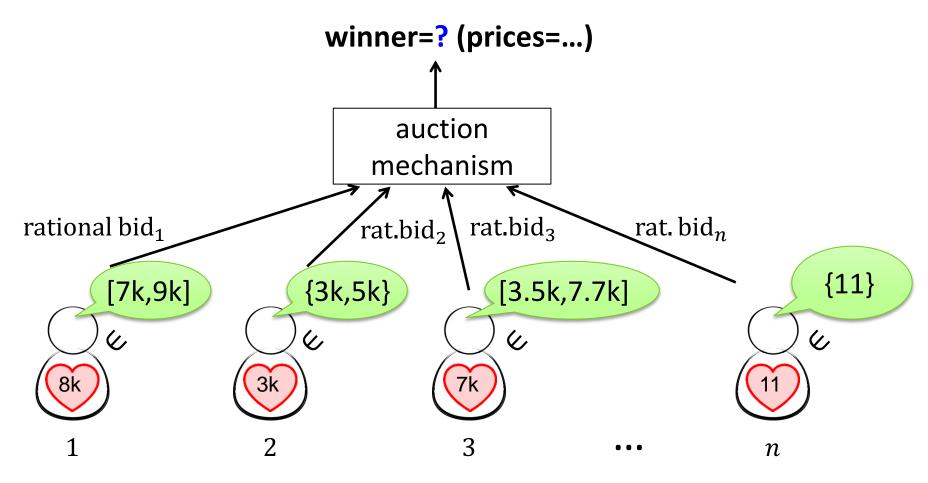
auction mechanism

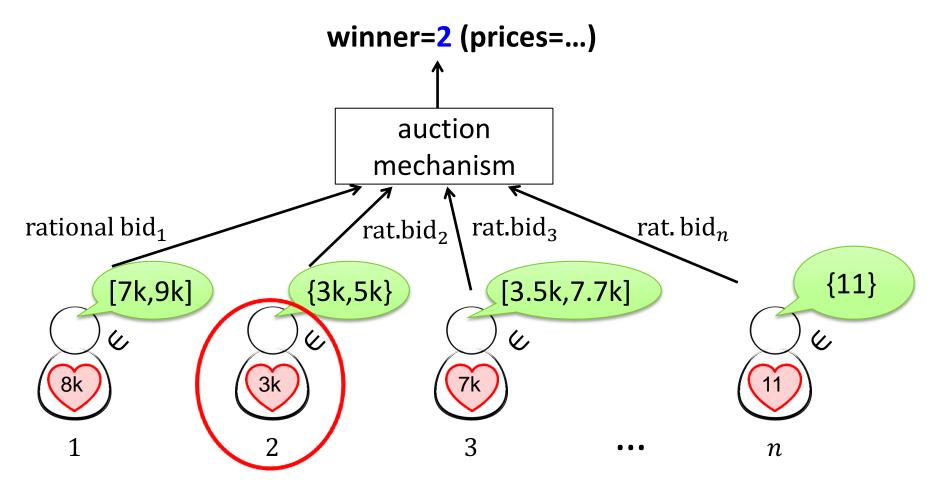


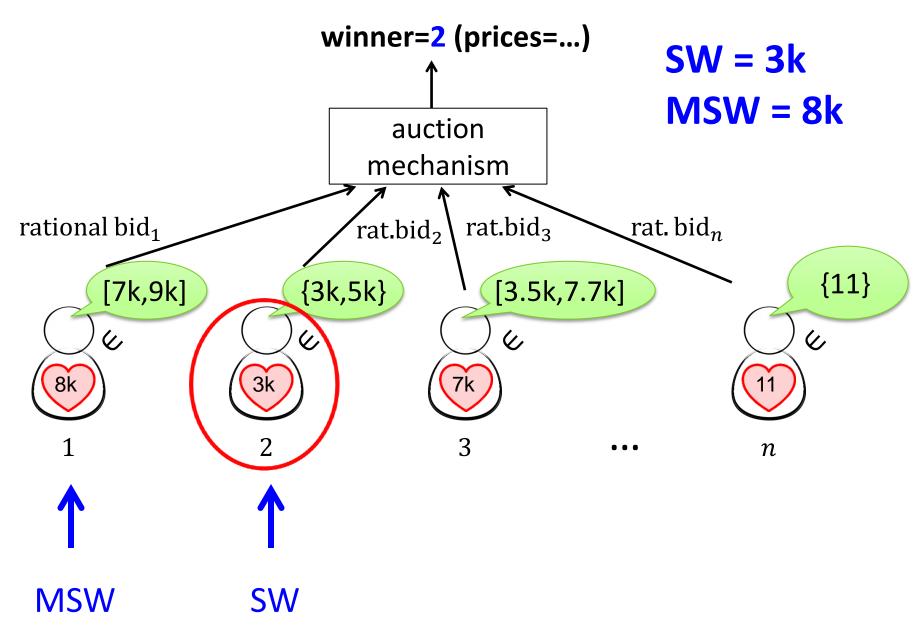


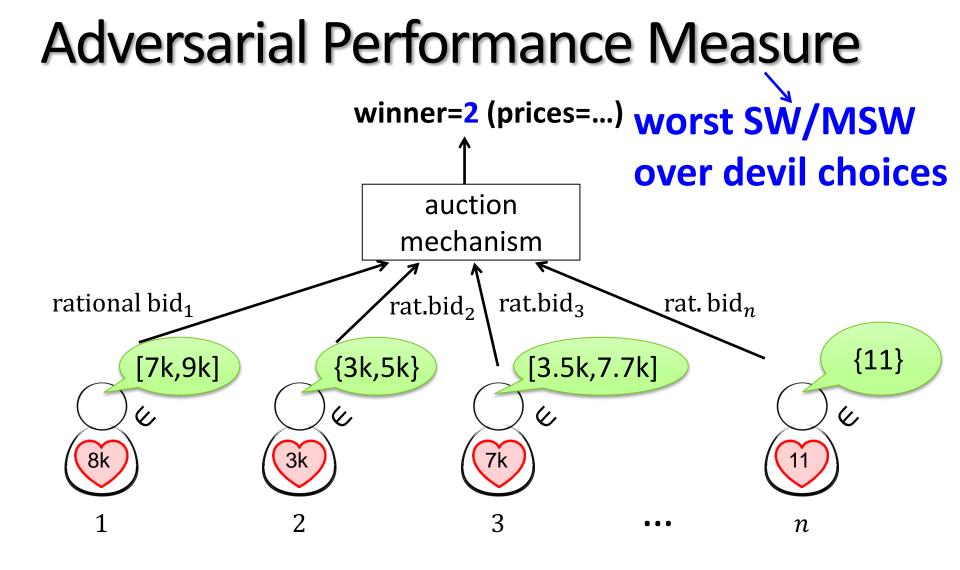
auction mechanism



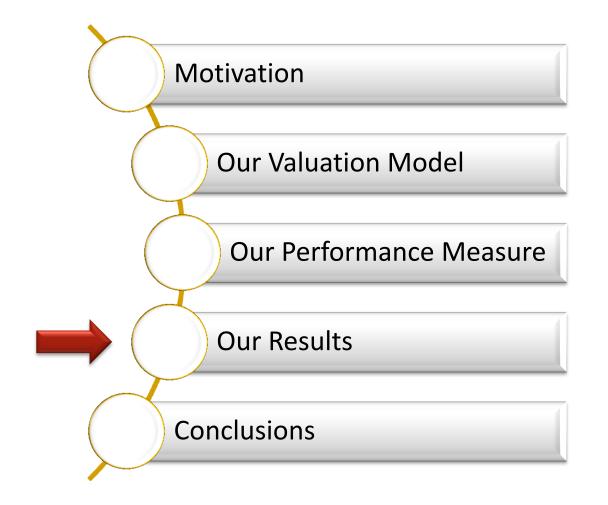








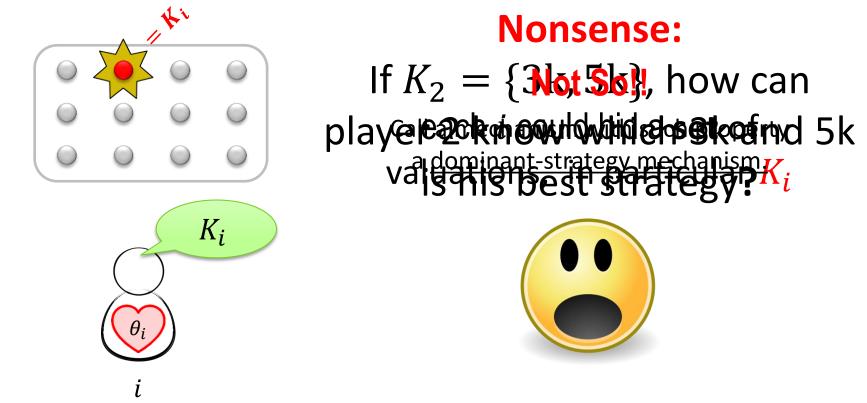
Today's Agenda



	Dominant	Strategies	
Single-good auctions			A classical solution concept, used also by second-price.

Implementation in ...

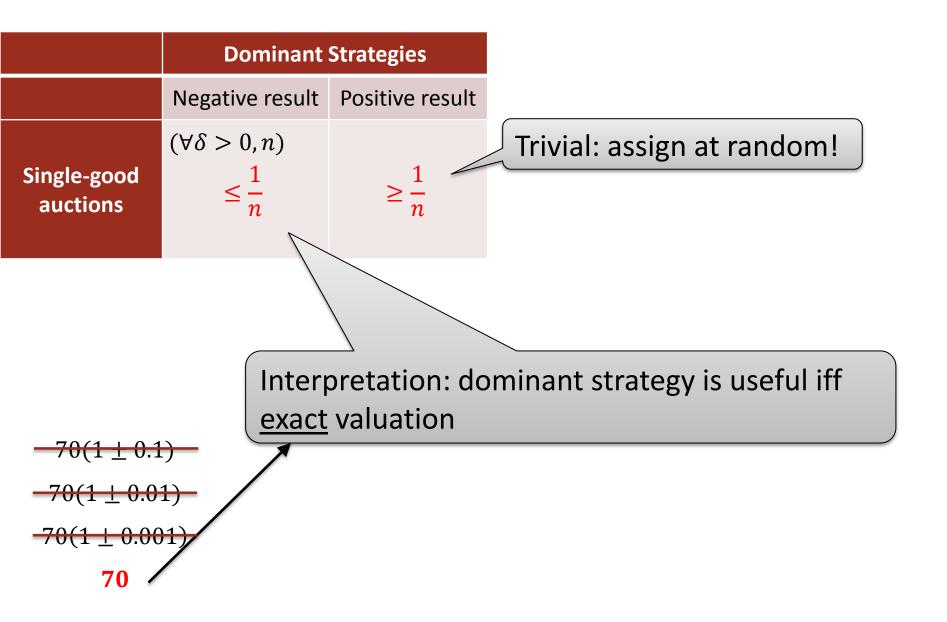
... Dominant Strategies

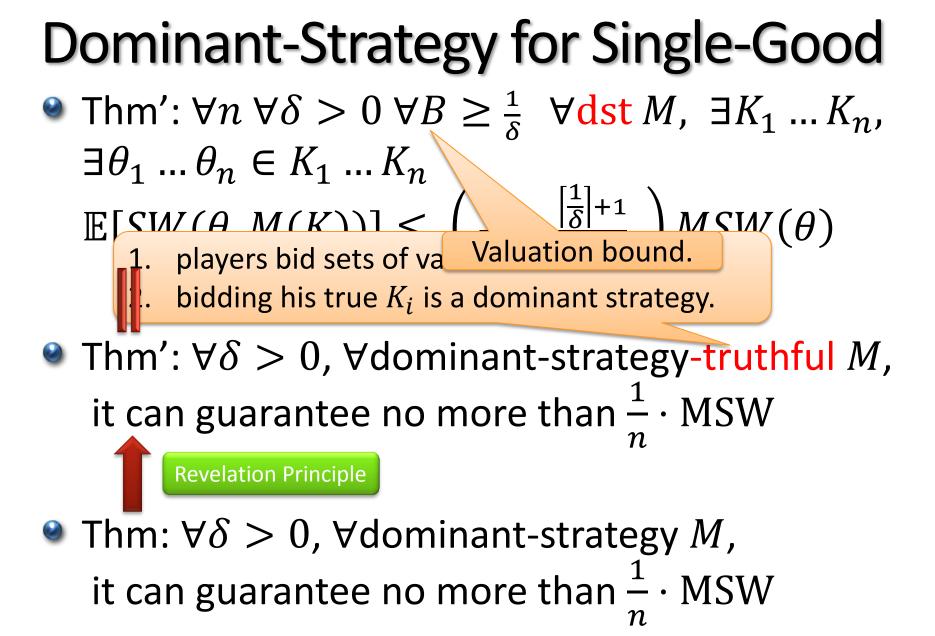


 $s_i \ge s'_i \text{ iff } \forall s_{-i} \quad \forall \theta_i \in K_i \quad u_i(\theta_i; s_i, s_{-i}) \ge u_i(\theta_i; s'_i, s_{-i})$

(Coincides with Knightian decision theory, i.e., 1-player behavioral analysis.)

	Dominant Strategies			
	Negative result	Positive result		
Single-good auctions		$f(\delta)?$ $(1-\delta)?$ $(1-\delta)^2?$		

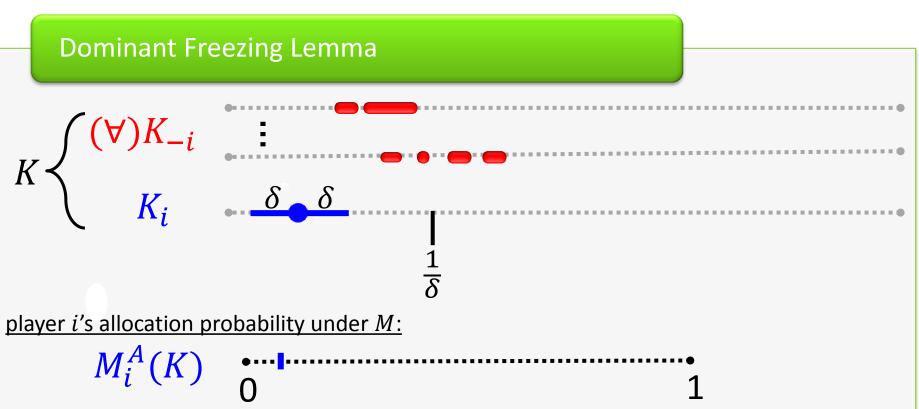






Dominant-Strategy for Single-Good

- Thm': $\forall n \ \forall \delta \ \forall B \ge \frac{1}{\delta} \ \forall \det M, \ \exists K, \exists \theta \in K$ $\mathbb{E}[SW(\theta, M(K))] \le \left(\frac{1}{n} + \frac{\left[\frac{1}{\delta}\right] + 1}{B}\right) MSW(\theta)$
- Proof: $\delta[x] \stackrel{\text{\tiny def}}{=} (x \delta x, x + \delta x) \cap \{0, \dots B\}$

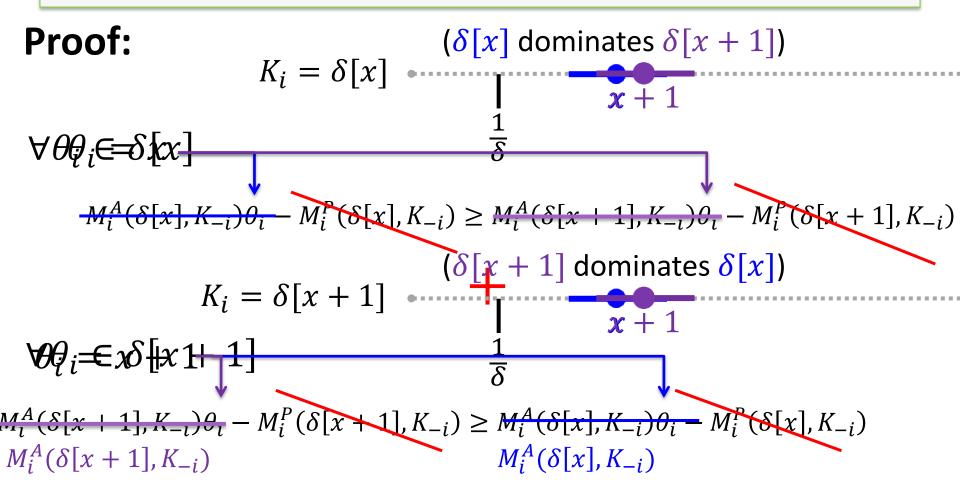


Dominant-Strategy for Single-Good • Thm': $\forall n \ \forall \delta \ \forall B \geq \frac{1}{\delta} \ \forall dst \ M, \ \exists K, \exists \theta \in K$ $\mathbb{E}[SW(\theta, M(K))] \leq \left(\frac{1}{n} + \frac{\left|\frac{1}{\delta}\right| + 1}{B}\right) MSW(\theta)$ • Proof: $\delta[x] \stackrel{\text{\tiny def}}{=} (x - \delta x, x + \delta x) \cap \{0, \dots B\}$ **Dominant Freezing Lemma** • $\forall i, \forall K_{-i}, \forall x \ge \frac{1}{s}$ $M_i^A(\delta[x], K_{-i}) = M_i^A(\delta[x+1], K_{-i})$ δδδ K_n : δδδ *K*₂: δ δ *K*₁: WLOG, M says that player 1 gets the good w.p. $\leq \frac{1}{n}$

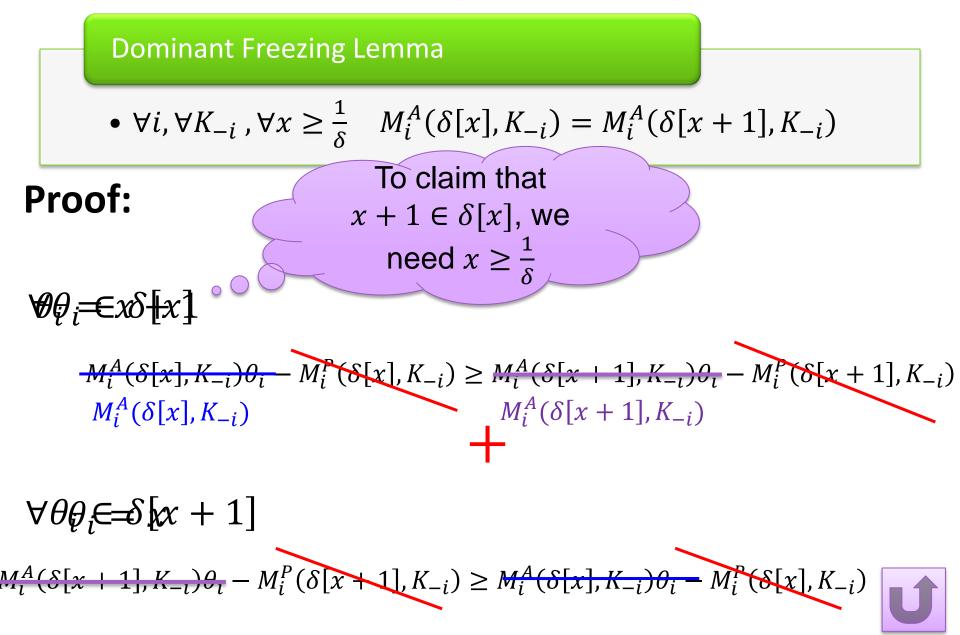
Dominant-Strategy for Single-Good

Dominant Freezing Lemma

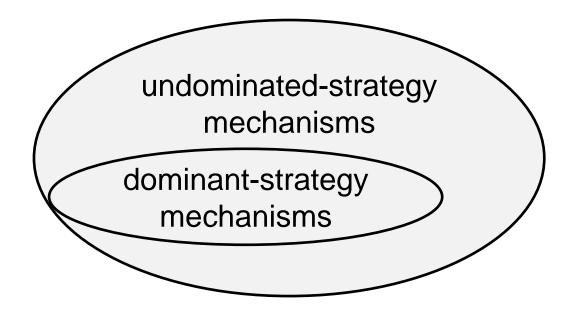
•
$$\forall i, \forall K_{-i}, \forall x \ge \frac{1}{\delta}$$
 $M_i^A(\delta[x], K_{-i}) = M_i^A(\delta[x+1], K_{-i})$



Dominant-Strategy for Single-Good



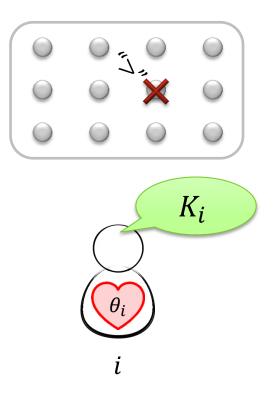
	Dominant Strategies		Undominated Strategies	
	Negative result	Positivo rosult A weaker potio	n than dominant strat	Positive result
Single-good auctions	$(\forall \delta > 0, n) \leq \frac{1}{n}$	A weaker notio $\geq \frac{1}{n}$	n than dominant stra	legies.



Implementation in ...

... Dominant Strategies

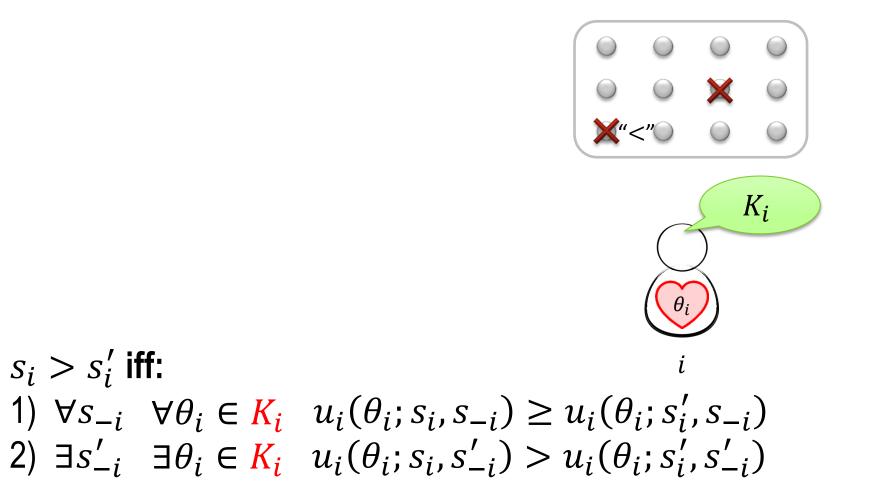
... Undominated Strategies



Implementation in ...

... Dominant Strategies

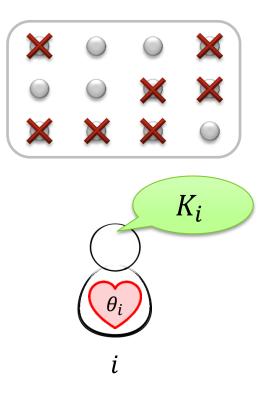
... Undominated Strategies

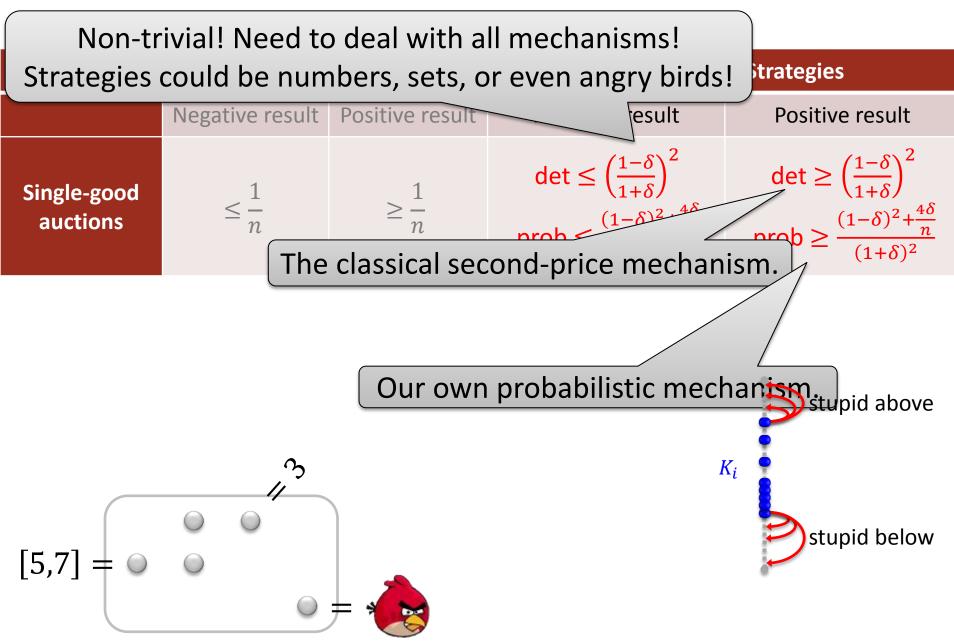


Implementation in ...

... Dominant Strategies

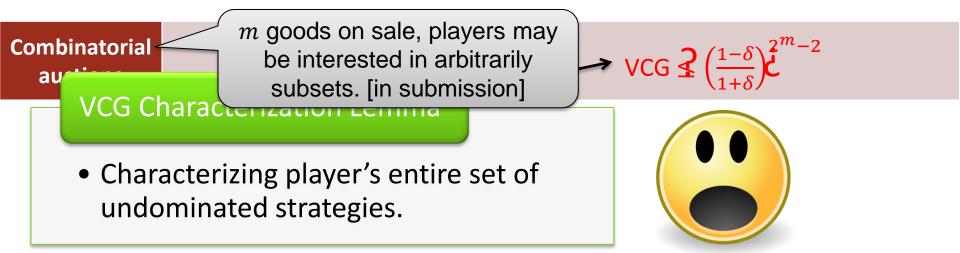
... Undominated Strategies





	Dominant Strategies		Undominated Strategies	
	Negative result	Positive result	Negative result	Positive result
Single-good auctions	$\leq \frac{1}{n}$	$\geq \frac{1}{n}$	$\det \le \left(\frac{1-\delta}{1+\delta}\right)^2$ $\operatorname{prob} \le \frac{(1-\delta)^2 + \frac{4\delta}{n}}{(1+\delta)^2}$	$\det \ge \left(\frac{1-\delta}{1+\delta}\right)^2$ $\text{prob} \ge \frac{(1-\delta)^2 + \frac{4\delta}{n}}{(1+\delta)^2}$

e.g.
$$\theta_i(\{1\}) = 7$$
, $\theta_i(\{2\}) = 10$, $\theta_i(\{1,2\}) = 12$



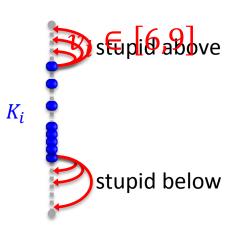
Undom. Strat. in Comb. Auctions

VCG Characterization Lemma

• under the VCG mechanism for combinatorial auctions of m goods, for every player i, his bidding strategy v_i is undominated if and only if...

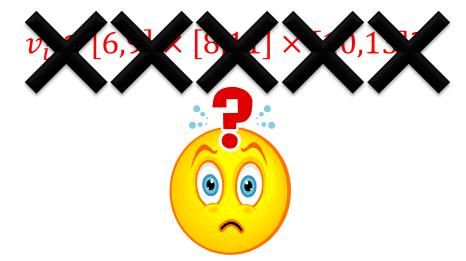
Single-good (2nd price):

- v_i is a number
 - e.g. $v_i = 7$
- K_i is δ -approximate • e.g. $K_i = [6,9]$
- v_i is non-stupid iff:



Combinatorial auction (VCG):

- v_i is a function $2^{[m]} \setminus \{\emptyset\} \to \mathbb{R}_{\geq 0}$
 - e.g. $v_i(\{1\}) = 7$, $v_i(\{2\}) = 10$, $v_i(\{1,2\}) = 12$
- $K_i(S)$ is δ -approximate
 - e.g. $K_i(\{1\}) = [6,9], K_i(\{2\}) = [8,11], K_i(\{1,2\}) = [10,13]$
- v_i is non-stupid iff:





Undom. Strat. in Comb. Auctions

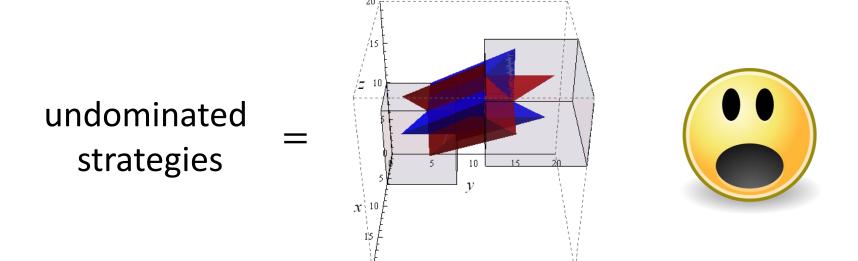
VCG Characterization Lemma

• under the VCG mechanism for combinatorial auctions of m goods, for every player i, his bidding strategy $v_i \in \text{UDed}(K_i)$ if and only if...

" v_i is inside the union of m! triangular cylinders, minus two hypercubes..."

e.g. $K_i(\{1\}) = [6,9], v_i(\{2\}) = [8,11], v_i(\{1,2\}) = [10,13]$

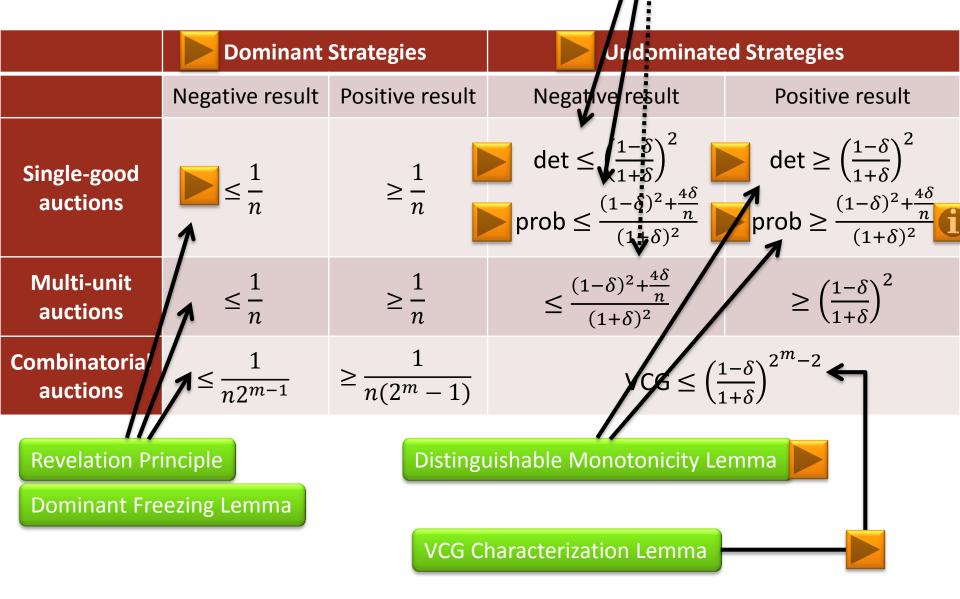
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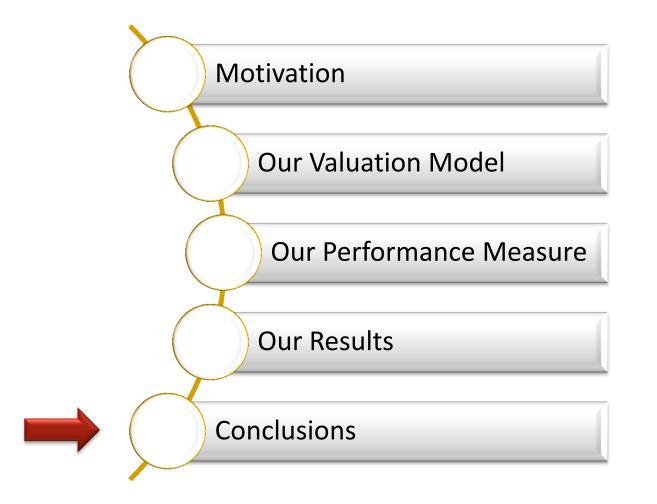
Undom. Strat. in Comb. Auctions • Thm: $\forall n \geq 2, m \geq 2, \delta > 0$, the VCG mechanism guarantees $\left(\frac{1-\delta}{1+\delta}\right)^{2^m-2}$ · MSW. E. al load nor mechanic of at in infere (raps, alone) for averagending upper (raps, lower) Claim F.L. if we share $\delta(m) = K(m)^2$ and $\delta(\ell) = K(R)^2$ for excepting size $\beta(n, R \neq 0)$ and $\beta(n) = 0$. Now define $K_{-}(t) \stackrel{\text{def}}{\longrightarrow} ||f||$ and $K_{0}(t) \stackrel{\text{def}}{\longrightarrow} |f_{0}|$ for all monopoly $T \neq K_{1}$ and $K(t) = \delta[t]$ for all plasmes ||f|| and monopoly $T \leq ||n||$. Invoking the last we proved above embly below with plasme $1 \times K \to 0$ for proof, and $1 = n_{1} < t_{1} ..., n_{N}$ we obtain both Now, the less impacts p we determ that, for each i and j, $v(u_{i_1}, j \cdots (u_i) \leq 2\delta_i^2(u_{i_1}, j \cdots \delta_i^2(u_j))$ for $i \in \{1, \cdots, 2^{n-1}\}$, will have some up in $i \in \{1, \ldots, n\}$. For the solution matching with the right in the less some u_i up to a constant diff. In other work, $\forall X \in [u_i] \text{ with } X \neq u_i \quad \text{will}(X) = u(Y) + \delta_i^2$ for some restant δ_i^2 . i. In this map, $|W|_{2}^{1}| = \frac{2T}{2} (|x_{1}| + N + L)$. For the advantum of $|U|_{2}^{1}$, N much be a value of x_{1} , and therefore $N \in a_{2}$ many, $y_{1} = T/2$ (for a durined, since the VEE mechanism is not. pulsing an advance with the measurement probability of whether. But the durine $M(M)_{1}^{2}$, $|W|_{2}^{2}(|x_{1}|) \leq e^{2T} (|x_{1}| + 2 + e^{-2T} ||x_{1}| + 2 + e^{-2T} ||x_{1}|| + 2 + e^{-2T} ||x_{1}|| + 2 + e^{-2T} ||x_{1}||$. Therefore, the for during the N is the 2-2-2 minute on the of the during the set. Final: Read link, instead of channing near architectic) large R_1 we channe T=0 in the intermediate of the order R is the same R is the same architectic large model with the same R=1 and R=1 and R=1 and R=1 are also as a simulation of the S(R) and R=1 and R=1 are also associated as R=1 and R=1 are also as R=1 and R=1 are also as R=1 and R=1 and R=1 are also as a sociated as R=1 and R=1 are also as a sociated as R=1 and R=1 are also as a sociated as R=1 and R=1 and R=1 are also as a sociated as R=1 and R=1 and R=1 are also as a sociated as R=1 and R=1 and R=1 are also as a sociated as R=1 and R=1 and R=1 are also as a sociated as R=1 and R=1 are also as a sociated as R=1 and R=1 are also as a sociated as R=1 and R=1 are also as a sociated as R=1 and R=1 are also as a sociated as R=1 and R=1 are also as a sociated as a Suppose that the second inequality of Remains F_{ii} does not hold, that is, $v(t') > \min\{v^{(2)}(t')\}$. Residuely as in Case 1, we let $J = \sup_{t \in I} \max_{i \in I} \{v^{(2)}(t')\}$ be the set of minimized, and let $j' \in J$ be more if then. We not always domain more A such that $$\label{eq:solution} \begin{split} & \mathcal{D}[\mathbf{f}(\mathbf{VCE}(\mathbf{v} \cup \mathbf{w})) - K(\mathbf{v}_{\mathbf{u}})^{*} + K - \max_{\mathbf{v}} \mathbf{w}(\mathbf{f})], \end{split}$$ 1.8°C(H), H(P) S.8(P)*, 1.8°C(H), H(P) S.8(P)*; $F_{2,2}^{\dagger}(K) = F_{2,2}^{\dagger}(K) \leq \frac{1}{2^{100-1}}$ b. $D(t|\nabla \Pi(u^{2})\cup u)) \leq K(u_{i})^{2} + K + \Delta - \max_{i} u(X)$ for every $j \in J$, and ng n ng ali nan anggi palanta ng (m) mati ikal, iniling na 😤 ng Substituting the shows data the result of the large disk is summarized, we define that 0.5° , we find that 0.5° and 0.5 $= \widetilde{\alpha}^{2}(u_{1}) + R$. For the advantant of $(R, \overline{u_{1}})$, we have that $\mathbb{P}\mathbb{P}[[R, \overline{u_{1}})]$ $\widetilde{\alpha}^{2}(u_{1}) + R = \mathbb{P}\mathbb{P}[u]$ (using Equation 3.7) in some that the choice of usion $(R, \overline{u_{1}}) + R = \mathbb{P}\mathbb{P}[u]$ (using Equation 3.7) for some that the choice of usion $(R, \overline{u_{1}}) + R = \mathbb{P}\mathbb{P}[u]$ (using Equation 3.7). $n = D(k(N \otimes (n^{(1)} \cup n)) \le K(n)^{2} + K - mail n(N) for any <math>j \notin J$ *(P) = A = +7 (P) ; (7.4) Now, we have more in moting radiust, what units new multi-lie migration by the VCD methods, on input stron, and on hid off two. We new remeric the band part that is to show that <u>Remaining A</u> halds. We have manyous the motilizer is not there mayne the soluted approximate valuation profile of the players in X. Then, its the dual iso profile d = (0, ..., 0) with $\theta_i(X) = X$ and $\theta_i(Y) = x$ for all momentary $T \neq a$ players $i \neq 1$ and moreophy $T \subseteq [n]$, we get the following under others $\forall y \in \{1, \dots, 2^m-1\}, \quad \eta_i(u_j) - \eta_i(u_{j+1}) \geq K_i(u_j)^* - K_i(u_{j+1})^* \ .$ a proved in the first of the (m, \overline{m}) is the only provide allocation in this case, and $m = 0.04 \times 10^{-1} \text{ sc}$ of the $(m, \overline{m}) = K (m, 1' + m / 2) - \text{ scarses}(K) = K (m, 1' + K - m / 2)$ Taken KA (A resident Caren Ka). After shares (KA) - K (K) for recording one staring K $\mathbb{E}\left[TW[d, P(K)]\right] \le \frac{\pi}{\sqrt{2^{n-1}}} + \infty$ Distriction (1 - K(0²)² + K - and with) We now proved in [2010] E.2.5. that (T,\overline{n}) is the only possible elements in this new, we introduce $T \in \mathbf{r}_{n}$, we have $D([|V|D(||^{n-1} - n)] = K(T) + n(\overline{T}) - map w(T) < K(n)^{-1}$ K + A - map w(T). (See some and the weak modulo of K. I. $K(T) < K(n)^{-1}$ - (1 + 7) - a FA Cara b. U.S. WEDLAT [Count) = H + A - maximum (F) for more (* 4 J, and $\leq \left(\frac{1}{\sqrt{2^{n-1}}} + \frac{n}{2}\right) \cdot MOW(0)$ $D(X \setminus CD(x^{(1)} \cup u)) \leq K(X')^{2} + K - constant)$ for every (if a. Here T = it (i.e., n = [n]), at any line (7, 17) is a possible allocation desired in [Same F.3, (7, 8)] for R ⊆ T is not also possible.² The allocation of VCD($v \in w$) for $v = \langle X', \overline{X'} \rangle$ is for all $p \in J$, the allocation of VCD($v \overline{U'} \in w$) for The index that $\{u_1, v_2\}$ is the above in $\{u_1, v_2\}$, we must have that $\{u_1, v_2\}$, we fight the following of u and the product of u_1 and u_2 we have that $\lambda \in V(u_1) = V(u_1) \leq \lambda$. It this may show a long $i \in \mathbb{N}$ is the set of u_1 and u_2 we have that $V(u_1) = V(u_1) \leq \lambda \in V(u_1, v_2)$. This indicates that $\{u_1, v_2\} = V(u_1) \leq \lambda \in V(u_1, v_2)$. $u(x_i)-u(x_i)> \min\{\widehat{u}\widehat{f}(x_i)-\widehat{u}\widehat{f}(x_i)\}\ .$ where hit $u = (2^{2N} - 2) \rightarrow B_{22}$ for the second in first player such that, if U is the stilling basis We have proved in Them F.T. that (P, P) is the only possible allocation in this case, and therefore $O(M \setminus C(P_{1,1}, p)) = O(L(P, P)) = K(P)^{-1} + p(P) - max p(O) = K(P)^{-1} + P$ b. Film $\overline{m} = \omega$ (i.e., m = [m], at any time (7, \overline{m}) is a possible allocation desired in [Main F.3, (7, 8)] for $R \in T$ is now also possible. F Proof of One Side of the Undominated VCG Characterization s. For all $j \notin I_i$ for all radius of VEI($v^{(2)}(m)$ is a $v \in \{T, T\}_i$ where $T \in \operatorname{arguman}_{1, D} v^{(2)}(T)$ (or a probability distribution over them in over of loss). We let $J = \arg \min_{i} \{ \widehat{w^{i}}(\mathbf{x}_{i}) - \widehat{w^{i}}(\mathbf{x}_{i}) \}$ be the set of extensions, and let $j^{i} \in J$ be one of it. We can decrear some A such that $\mathbf{x}_{i}(\mathbf{x}_{i}) - \mathbf{x}(\mathbf{x}_{i}) \times A \times \widehat{w^{i}}(\mathbf{x}_{i}) - \widehat{w^{i}}(\mathbf{x}_{i}),$ (2.15) $\mathbb{E}(\theta, \mathsf{VCD}(\mathsf{w} \cup \mathsf{w})) = \sum p_{ij} \mathbb{E}[\theta, \mathsf{VCD}(\mathsf{w}^{(j)} \cup \mathsf{w}))$ a. We have proved in [2009 Y12]] that $[T_{ij}T_{jj}]$ is the only provide should be formula in this new multi-solution $[S_{ij}(T_{ij})] \in [S_{ij}(T_{ij})] = S_{ij}(T_{ij}) =$ magnets) (in the new stree $\overline{N} = 0$, the alternation might also be (N, R) for more m(R) = 0, and since we have shows R = 0 this stillar remains still holds.) As stated in Netion 4, we shall prove the Understanded VUI Characterical the devices that we used, we give local address conditions on a physic bit schedule) is after for the last in locar andonizated strategy (while with the app From T_{1} for any modulus alteration (X, T) of the VCD methods when the second player biols n_{1} $T \in \mathcal{G}^{-1}([n_{1}]_{1})$, then TW([0, T]) from not module the log term N and is then results then some $TW([n_{1}]_{1})$ is all there mass. Therefore, we may see the results container of the Ferre (X, T) and . We have proved in [25] in [25] (bid (a,[w]) in the only possible allocation in this map, as there is a $[0,VEE](a^{2T},i,w)] = 0 + w([wi] - \max_{i} w(2) = N + A - \max_{i} w(2)).$ The strength and is the photon mathematical of densing more sufficiently large K_{+} denses K = -A. It will make more that $w(x) = w(\nabla_{0}) = 0$ with $w(\nabla_{0}) = -A$. The only plane that we used K being sufficiently large is show we deduce that it and for every $j \notin X$ $\begin{array}{c} \widetilde{cot}(u_{1}) - \widetilde{cot}(u_{2}) = X \\ \end{array} (275)$ Then, we set therefore the first for the start of the start $U_{1} = X + u_{1}^{-1}$ of $U_{1}^{-1} = X = u_{1}^{-1}$ of the start where the static is the same velocity $\overline{u}_{1}^{\prime}=0$ or $\overline{u}_{2}^{\prime}=0$, the alternities might also be (0,1) in some u(0)=0, but one can show that the like many conclusions shift hold, by our choice of \mathbb{R}^{-1} We have present in [Takes $|T_2||$] that $|T_1|^2$ is the only possible allocation in this case, and therefore $D(||V|| \otimes |U||) \le D(|T|)^2 + \alpha(|T|) - \max I \alpha(|T|) = \sum_{i=1}^{n} |T_i|^2 + \alpha(|T|)^2 + \alpha$ is the VCC contraster. The user of promotions, we shall work in the new when such player's set of valuations is regard in Eqs. Use new where such player's set of valuations is equal to $\{0, ..., H\}$ such the proved with a distribution of source neutronous second No. a. In this case, 2W[u] = u(t') + K. If the advantum is of the here $(t, \overline{t'})_i$ by the deteil memory-independence of v_i $(t', \tau') = u$ much be the advantum with the best model without if the advantum is $(u_i | v_i)$ in a model or advantum $2W(u_i) = h + K + u(t') + K - 2W(u'_i)$, using frequency h and h are u = v(t', t'') and h the indexine of the WW methantum t. possible concludes allocation for VCD \rightarrow of the form $(X, \nabla) = 0$, long there are no losse to also consider (X, X) for $X \neq \nabla T$. Research $SW(X, X) = SW(X, \alpha) = SW(X, \nabla_0)$. This means, allocation $(X, X) \neq 0$ of $(X, \nabla) = contributions.$ orders E.A. amand 2.7 is related. a much more concerning, proof. now bijection from all concerning radiants of [m] is $\{1, ..., N^{m-1}\}$ such that [0][m] = histing of all concerning radiants of [m] is a vector $m = \{m_1, ..., m_{m-1}\}$, where the n_i are non-ranging radiants of [m], indicating that radiant of [m] (in respect to relate to the all i From 7. We recall that Denalize E2 (effects that $\eta(u) - \eta(u) \le K(u)^2 - K(u)^2$, but we have $\eta(u) - \eta(u) \le A$ is Denalize F7. This inform that $K(u)^2 \ge K(u)^2 + A$. Now, for every $v \in A$. $f_{0}(X)+m(T)$ is denote the "apparent solid wellaw" of the allocation function when assuming that bits player has the repetiod bits with nin F.J. FTC (and TC (a) b. In this case, $SW[\mu] = H + \Delta$. We the allocation of the form $(K, \overline{F}), SW[K, \overline{F}) \leq \sqrt{2}T(F) + K + \Lambda - SW[\mu]$ (using imposure $T(\mu)$ is wave that the determine of μ . Note the VCD mechanism maximizes actial with an exhibit to the reported bids, we have that $FW(VD(p_1, \omega)) = \max_{g \in \mathcal{T}_1} || \sigma(H) + u(T)||$ where the maximizing in one of $S(T \subseteq [\omega])$ with $F(T = \omega)$ F.I. A lattice and all two starts where of indications of the lattice of T $|\mathbf{v}_{i}(\mathbf{x})\rangle = K(\mathbf{v}_{i})^{2} + K - \max (K) = K(\mathbf{v}_{i})^{2} + K + \Delta - \max (K) \ge D(K)$. In this map, $|W|_{W}|_{W} = \sqrt{2} \langle |W| > N$. For the abundless of $\langle A_{1} | m \rangle$, we have their We $N > 4 \times \sqrt{2} \langle |W| > N = 2 W(m)$ (sing) gravitons E(k) for even thus the choice of a shorthware the transition of N > 400 of N > 400 of $2 \times m_{10} = 200$ (sing) gravitons E(k) (sing) gravitons E(k) (sing) of N > 400 (sing) of $2 \times m_{10} = 200$ (sing) or $2 \times m_{10} = 200$ (sing) or $2 \times m_{10} = 200$ (sing) or $2 \times m_{10} = 200$). See all y < J, the allocation of VCD(vCT.h.m) increases (or a probabilistic distribution over them in our of t pare side of the Undominated VCD (Decoderination Lemma). As the VCD works, side here the are brains, for such physics if the following holds. Require that physics arises \mathcal{K}_1 such that while he many \$ \$ 4. a. For all j (J, for all makes of VEE($v^{(0)}(x)$) is $x = (T, \overline{X})$ for some $T \in \operatorname{segmap}_{T, \overline{X}} v^{(0)}(T)$ (or a probability distribution new them in our of long). - such K(R) is an interval, i.e., i.e. all non-maps $K \subseteq [m], K(R) = [K(R)^2, K(R)^2]$ for some valifies $(K(R)^2, K(R)^2)$ and many the bilineing inequalities, at least one manuf, both $$\label{eq:states} \begin{split} & \text{tr} \ Y < \gamma, \\ & \mathbb{D}[I_i \, \nabla \mathbb{D}(\mathbf{v} \cup \mathbf{v})] = \mathcal{K}[X^i]^2 + \mathcal{H} = \underset{\mathbf{n} \neq \mathbf{v}}{\text{sp}} \ \mathbf{u}(X) > \mathcal{H} + \Lambda = \underset{\mathbf{n} \neq \mathbf{v}}{\text{sp}} \ \mathbf{u}(X) = \mathbb{D}[I_i \, \nabla \mathbb{D}(\mathbf{v}^{(2^k)} \cup \mathbf{v})] \end{split}$$ $\begin{cases} v(n_{1}) - v(n) \leq \min\left\{v^{(2)}(n_{1}) - v^{(2)}(n)\right\}, & \forall x \in \{1, \dots, 2^{N} - 1\} \\ v(n) \leq \min_{i} \{v^{(2)}(n)\}, & v(n) \leq \max_{i} \{v^{(2)}(n)\} \end{cases}$ indice of these innerstativity implies annual at a Proof. For any conclusion submittion (X, T) of the VAT conclusions when the second plager bids u_i if $T \in \{T_i, T_{i-1}\}$, then XP(|X, T|) does not contain the lag term X and in the second v that any XP(u)is all these sources. Therefore, any other participation of the second v that $u_i \in X$. while the server i of A. • K_1^+ and K_2^- are unadep monotone, i.e., for all $R \in C$ [in] with $R \in T$, $R(R)^+ \leq K(T)^+$ and $R(R)^- \leq K(T)^-$. We read that Employ P.4 gives a residualities and may that it is no estimated at ad the read the second of an increasing without the property and the second of an increasing of the Case 1. Claim F.S. (A variant of $\underline{Cham}(F_i)$. When $\overline{F} = it$ (i.e., $S' = [n_i]$, $\underline{Cham}(F)$ only rep $\mathbb{D}[U_{\mathrm{ACD}}(n;:n;0) = \mathcal{H}\left(h_{i}\right)_{i} \rightarrow \mathcal{H} = \min_{\mathbf{x} \in \mathcal{H}}(h_{i}) \geq \mathbb{D}[U_{\mathrm{ACD}}(n_{i};0]:n;0]$ and $1 \in \mathbb{N}^{2}$ a in this case, $2N[\omega] = u(u_0) + N$. If the allocation is of the form $(N, \overline{u_0})$, by the strict manufacturity of u, by $-u = \omega$ must be the allocation with the balanced section. If the of any line (T,\overline{T}) is a possible allocation dediced in $(\underline{Coint}, E), (T,R)$ for $R \subseteq \overline{T}$ is now where a is any proper kiloling granulated by the hypothesis of t First compute the solution in all theory many measures, computer 1 and an approximate of the design of a bottle solution of anima (a - 2, 4) when 7 is not in any man, which is have 1.9. and a computer of the solution of in of few instellately imples assume to all contains is of the form (X, W), similarly, (m, W) such the the all contains with the basis much solution of the transmission with the basis much solution, however, in this case $u_1(m) + w_1(m) = u_1(m) + K + A + u_1(m) + K - 2W(\omega_1)$, using formulas $Y_{i,i}$ is some, $\omega = (u_1, u_2)$, much the adjustice of the VCC) constants C. Community all fully, inclusion or pay and community and and We recall that assume rising two a contradiction and may that it is an understanded straining and this work the second of the transmission Vice Comparison and provide the formula $v_i(\ell') = v \leq (1+\delta)v = K_i(\ell')^2$ and $v_i(\ell') = v \geq (1-\delta)v = K_i(\ell')^2$. Finally, we say left in only transmitter K_i and we need a "witners biology" for East. We simply chosen the injector v_i for which we have • Let $i = 2^{m} - 1$, by monoing up all the inequalities in <u>Konstan F2</u> for $1 \le i \le 2^{m} - 1$, we C Proof of Theorem 4 The only place that we used if being solicitatly large, is shown we derive this iter only possible considered addition for VGC2 ($\alpha < \beta$) of the file large $(X, \alpha) \in (X, \beta)$. This is a large force as we have to also consider (X, β) for $\beta \neq Z^{-1}$ or $[\alpha]$. However, since $\alpha(X) = 0$, $DV(X, \beta)$ is $DV(X, \alpha) = DV(X, 2)$. $Y_{1}\in\{1,...,2^{N}-2\}\,,\qquad u_{1}(u_{2})-u_{1}(u_{2},j)=-2\delta u=K_{1}(u_{2})^{2}-K_{1}(u_{2},j)^{2}\,,$ and for $i=2^{N}-1,$ equility of Emminer X_{i}^{j} does not hold, that is, $v(X^{2}) \in marg(\pi^{2}(X^{2}))$. 4.2, we let $J = mgmm_{ij} \{v^{(2)}(X^{2})\}$ be the set of maximizers, and let an always shown more \bar{h} such that $0 \geq \sum K_{s}(u_{i}^{*})^{*} - K_{s}(u_{i}^{*})^{*}$ For even of presentation, we shall prove ((**MENDE**)) in the case where such player back of submitting in equal in $\mathbb{R}_{p,0}$ the new where such player's set of submitting in equal in $[0, \dots, R]$ such represent with a structure of such source conference, press. which is christally decays low. However, we have clearly shown that when $1\leq i\leq 2^m-2_i$ the inequality is not only lowe but also light, as the had one of them (which corresponds is $i\in 2^m-1$), then in before Ny deside North an with There is a substantial product of the substantial substantial bid product of the substantial substant a. de. We find compute the utilities in all three name and $F(R) = K (R)^{\gamma}$ for all non-respin t Finally, etc. Co. Barrier Thing Treads Nod, its ing any public contact z (which should be thought of an annual constant), we have R = and = z (inputing on $z_1 z_2$ and z_2 , and show that $z_2 \in Ein(Rz)$. Either G.3. Owner: $\frac{2 \operatorname{NW}(\delta, \alpha)}{\operatorname{MWW}(\delta)} \leq \left(\frac{1-\delta}{1+\delta}\right)^{2^{\alpha}-\delta}$ H Prof a. $D(0, VCD(w_{i}, w_{i})) = H + \Lambda - \max I w(H),$ b. $D(0, VCD(w_{i}^{2}) = w)) \leq H + K(H^{2}) - \max g w(H)$ for every $j^{*} \in J$, and prove the theorem in two stress • S, is in each line $\kappa : \mathbb{D}(0, \nabla \mathbb{D}(\pi^{(1)} \cup \pi)) = \Delta + H - \max_{i} \pi(O) for \operatorname{surg} j \notin I$ Resp. 1 (1995) III is construct a marinal and instance for the VDD methods in figure (i.e., the second state of the second st $$\begin{split} & \mathcal{K}_{k}(\mathbf{x}) = \left[\left(\frac{(1+d)^{2}}{(1-d)^{2}} - \mathbf{x} - \frac{(1+d)^{2-1}}{(1-d)^{2}} \right) \left(1 - d_{1} \left(\frac{(1+d)^{2}}{(1-d)^{2}} - \mathbf{x} - \frac{(1+d)^{2-1}}{(1-d)^{2}} \right) \left(1 + d \right) \right] \\ & \text{for } d \in (1, \dots, \mathbb{T}^{N-2}), \text{ and } f_{k}(\mathbf{x}_{k}, \mathbf{x}_{m-1}) \text{ long } \mathbf{x} \text{ and } d + \text{ and } \text{ node } \\ & \mathcal{K}_{k}(\mathbf{x}_{m-1}) = \mathcal{K}_{k}(\mathbf{x}_{m-1}) + \mathbf{x} \right]. \end{split}$$ all $p \in J$, the allocation of VER($(D^{2})_{i}$ to $i = (T, \overline{D^{2}})_{i}$ where $T \in p$ relative distribution over them in over of local. all $p \notin J$, the allocation of VER($(D_{i} + n)$) is $u = (d_{i}, [m])$. When the plaque that $n=n_1,\ldots,n_d$ distributes $A=\{[n_1],a_1,\ldots,a_d\}$, below the state of A relatives to $n_1([n_1])=n_1(n_{pr},n_d)=(1+2)2^{2k}-2|d|$ as We have proved in the way of the (in (in)) is the only possible allocation in this case, and . We have proved in Chain F.115: Und $(\overline{r},\overline{r'})$ is the only possible closedare in this case, and thereine $D([\sqrt{n}G)(r'):=u))\leq K(T)^1+u(\overline{r'})-\max_{T}u(T)\leq K(T)^1+u(\overline{r'})-\max_{T}u(T)$ Nop 2 (betain 112). We show that if playe 1 has approximate valuation K : and bits vdays 2 has approximate valuation K_{μ} and bits v_{μ} (while silve playes had 0), the back for maximum uncit without had to generated in all work the other stated in the theorem but if the any modulule alimits (0, T) of the VCD contains when the second plage hole n_i $T \notin (T^{(i)}_{ij}(m_i^{(i)})$, then $M^{(i)}_{ij}(T)$ is done not contain the lag invest T and in these results there are $M^{(i)}_{ij}$ is all there mays. Therefore, we only word to consider container of the laws (T_i, T) and • v_0 is in such that $v_0(v_0) = 2i\delta v - s$ (in all $i \in \{1, \dots, 2^m - 2\}$), and $v_0(v_{0^m, 1}) = n + 2(2^m)$. is the other hand, for any allocation giving $n_i \neq i$ is to player 1 and $n_{\mu\nu_{-1},i,j} = \overline{n_i}$ to play the model or the speed to and $w(y) = u_1(y)^{-1} + u_2(w(y))^{-1}$ (here we well for weak monotonicity of $K^{+}_{-}(w_{+}, K[T])^{+} \leq K[H^{+}]^{+}$. In for one where $\overline{H}^{-}_{-} = u_{+}$ for all values and \overline{H} where W(X) = 0, and since we have chosen K = 0 line will be equal to \overline{H} be the T - R between w(X) = 0, and since we have chosen K = 0 line will be equal to \overline{H} be the T- R between W(X) = 0. queral) The s₁ (UDs(K₂). G.1. Construction of The Hard Instance and a series of a first state of the state of a state o The new $|W|_{2}^{i} = K + \Lambda$. Fither allocations is of the form $|X, P'|_{1}$ by the strict momentum by of $v_{1}(P', P') = \omega$ and be the allocation with the best moted voltars. Economy, for mode without $|W|_{2}^{i}P_{1}^{i}|_{2} = \omega(P') + K + K + \Lambda - FW(\omega)$, using Equation 7.11. In man, $(n_{1}(w))$ The constraint many maintive value into K_1 and K_2 and the table η , and η , where, for $(-1,2,K_1)$ and η , log-the unity the hypothesis of an excession with the constraint tensor (-) or finite that for our denses k both that $\eta \in (10\pi k/K_1)$ and $\eta \in (10\pi k/K_1)$. There dense have no modulable large training the table $(-10\pi k/K_1)$ and $\eta \in (10\pi k/K_1)$. First, for sufficiently small a, $E_i[n]^2$ and $E_i[n]^2$ are both particle. Due again it suffices by both the assumptions in the Communicativity Comparison and Lemma 7.2 both ladesd, e_i and e_i are of Leffeldy monotonic $E_i[n]$. Let e_i $E_i[n] = E_i[n] = E_i[n]$. We have proved in (matrix to be like (i.e. (m)) is the only pushfor all values in this map, and therefore $D(g/VE)(d^2 \cup w)) = 0 + w(m) - \max_{i \in U} w(i) = K + A - \max_{i \in U} w(N)$. Use that address by A ; betterman, for any all loss 3. Dependent address address is a second to: $m(m) = m(m^{n} - i) = 0 + 2(2^{m} - 2)i(n - i)$ Final. We recall that the matrix F_{i} and the matrix F_{i} is the state $A=a(P)\geq K(P)^{i}$. Now, for each $P\in A_{i}$ $m \in X \to X$ = m, we have $m(X) = 2m - n \in (2m - n) \bigoplus_{i=1}^{m} - K \cdot q(X_i)^i$ and $m(X_i) = K \cdot q(X_i)^i$. So we let be verify function X_i , and we need a "scheme labeling" for the function M is the function of M is the 19-14 2 err. = [m]is indexed, where = 2 we note by = ([1], (2), (2), (1, 2), (1, 2), (1, 2), (1, 2), (1, 2), (1,البيينينين . In this case, P(0|x) = H + A. For the allocation of the form (R, \overline{P}^{*}) , $P(0|R, \overline{P}) \leq \sqrt{2}(N) + H + A - 2N(|x|)$ (using injugant P(x)) is more than the chain of u. $\mathbb{D}[\delta, \mathrm{VCD}(\mathbf{v} \sqcup \mathbf{u})] = N + \Delta - \mathrm{space}(S) \times N + K(X^2)^2 - \mathrm{space}(S) = \mathbb{D}[\delta, \mathrm{VCD}(\mathbf{v}^{(2)}) \sqcup \mathbf{u})]$ n In 2 ⊂ i ⊂ 2ⁿ − 2 and 16 y = 2ⁿ − 1 − i ∈ [1, 2, . . . , 2ⁿ − 2]). $$\begin{split} & u_{1}^{(n)}(-u_{1}(u_{1,n}^{(n)})-u_{1}(u_{1,n}^{(n)})-u_{2}(u_{2}^{(n)})=2bu\\ & = \left(\frac{(1+d)^{2-1}}{(1-d)^{2-2}}-u_{1}-\frac{(1+d)^{2}}{(1-d)^{2-2}}\right)(1-d) - \left(\frac{(1+d)^{2}}{(1-d)^{2}}-u_{1}-\frac{(1+d)^{2-1}}{(1-d)^{2}}\right)(1+d)\\ & = u_{1}^{(n)}(u_{1}^{(n)})^{2}-u_{1}^{(n)}(u_{1}^{(n)})^{2}-u_{2}^{(n)}(u_{1}^$$ Claim F.H. (A variet of Data F.f.). When $\overline{A}^{*} = a^{*}(A + A^{*} - |a|)$. Claim F.H. only of the bill of the operator and A^{*} $D(\mathbf{0}, \nabla D(\mathbf{v} \cup \mathbf{v})) = \mathbf{N} + \mathbf{A} - \max \mathbf{v}(\mathbf{0}) = D(\mathbf{0}, \nabla D(\mathbf{v}^{(2)} \cup \mathbf{v}))$ $\begin{array}{l} \mbox{Iden U.I. Choose} \\ s \; \mathcal{K}_1 \mbox{is its model had } \mathcal{K}_2(n) = [(1-\ell)s_1(1+\ell)s] \; \mbox{for all } (\in \{1,...,2^m-1\}) \\ s \; s_1 \mbox{is its model had } v_1(n) = (1+2)(-1)\ell v_1 \; \mbox{for all } i \in \{1,...,2^m-1\}. \\ \mbox{for space } v_2(i) \; \mbox{Thes} \; v_1 \in U(\mathcal{K}_1). \end{array}$ providing small distances of any first (T, T) is a possible allowing defined in [Matter Fill], $(T, R) \neq R \subseteq \overline{T}$ is now the restricted. an of these immediately implies assume at a We swall that an end of a provide a contradiction and may that a to an understanded strategy at this such the point of an excession of the conception and the same yet, for Care 3. Proof. Excellence on welly that the assumptions in the Unit minuted Well Characterization Lemma bold. Indeed, K_1^+ and K_2^+ are both weakly monotone because they are constant, or in strictly manipulate intervalved at α with $K \neq \alpha < 0$. If we change $K = K^+ = 0$, we define the The summer will be a lotal thematics along how the invasion when along the $u_{0}(\mathbf{x}_{j}^{1}-u_{0}(\mathbf{x}_{j+1}^{1})-u_{0}(\mathbf{x}_{j+1}^{1})-u_{0}(\mathbf{x}_{j}^{1})=u-K_{0}(\mathbf{x}_{j+1}^{1})^{1}-K_{0}(\mathbf{x}_{j}^{1})^{2}=K_{0}(\mathbf{x}_{j}^{1})-K_{0}(\mathbf{x}_{j+1}^{1})^{1}-K_{0}(\mathbf{x}_{j+1})$

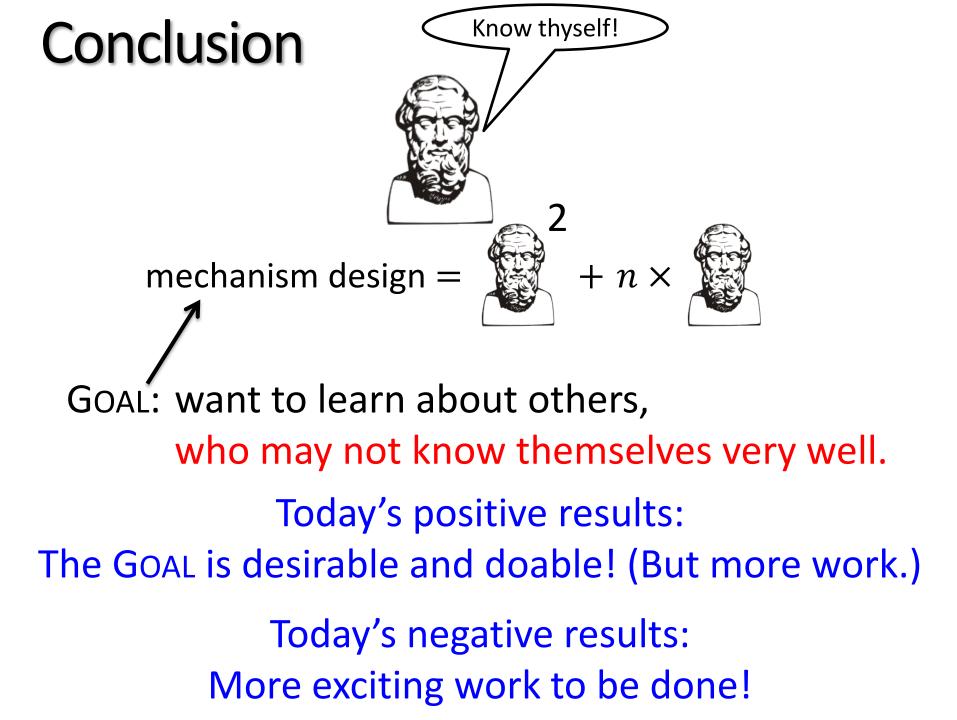
Hyperlink

Undominated Intersection Lemma



Today's Agenda





Thank you!