Mechanism Design with Approximate Valuations

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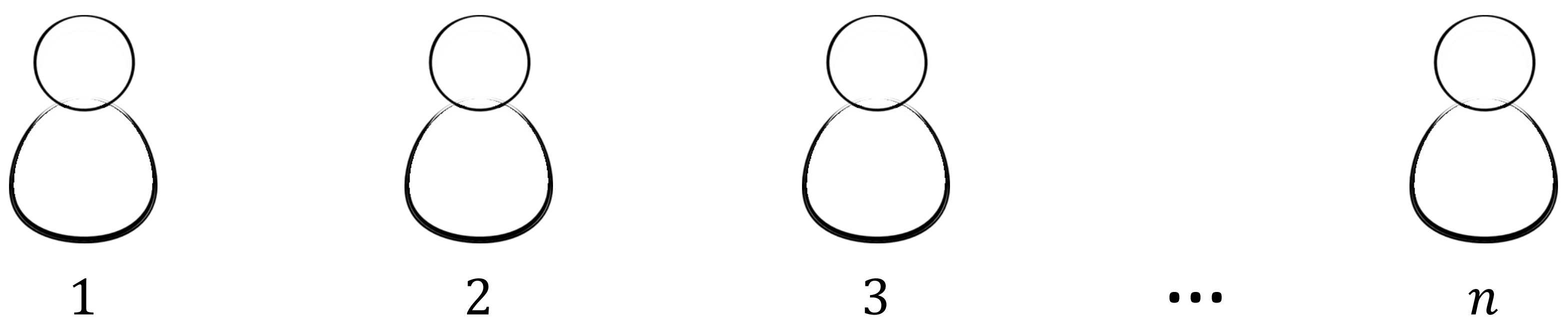
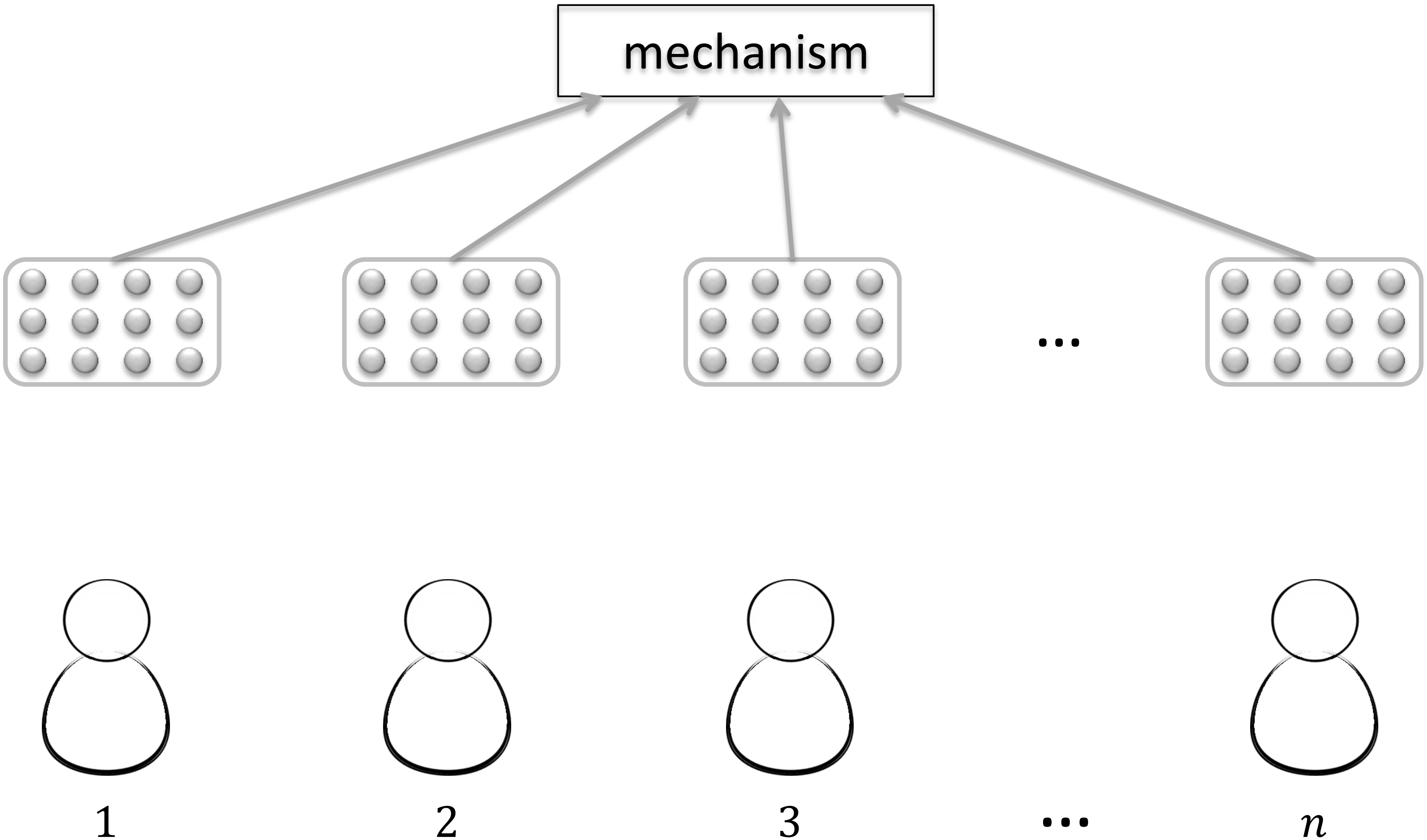
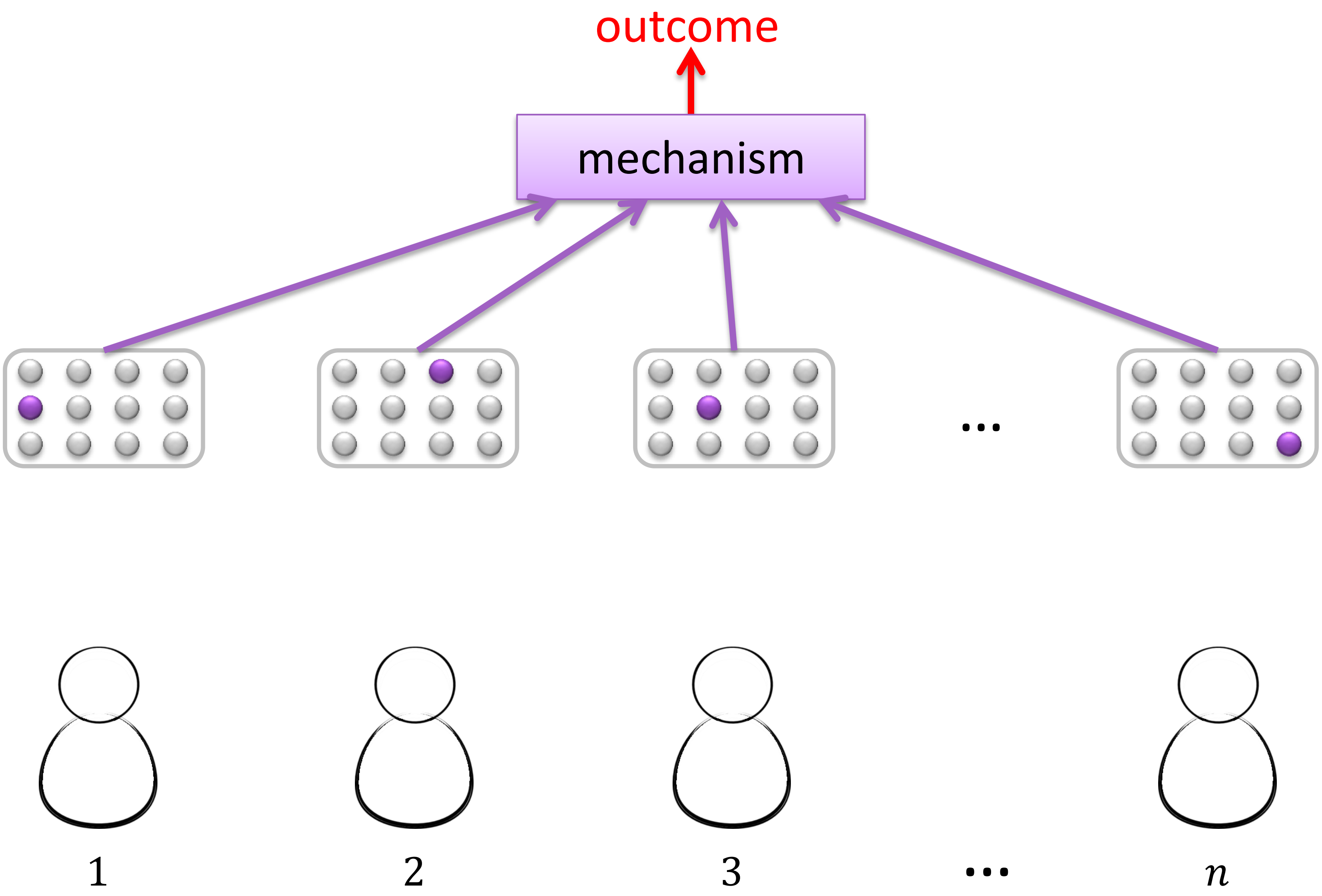
Recall...



Recall...

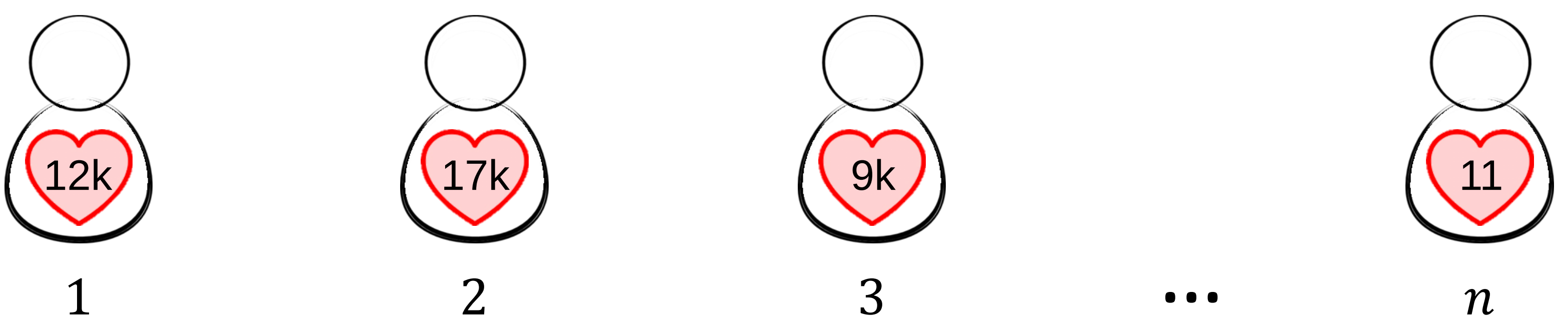


Recall…



Rolex Auction

GOAL: maximize *social welfare*
(valuation of the player who wins)



Rolex Auction

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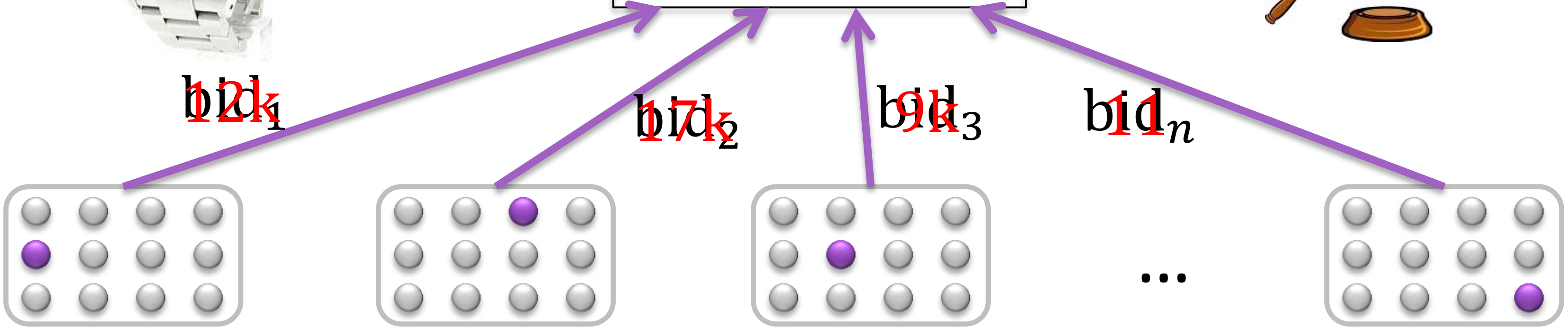
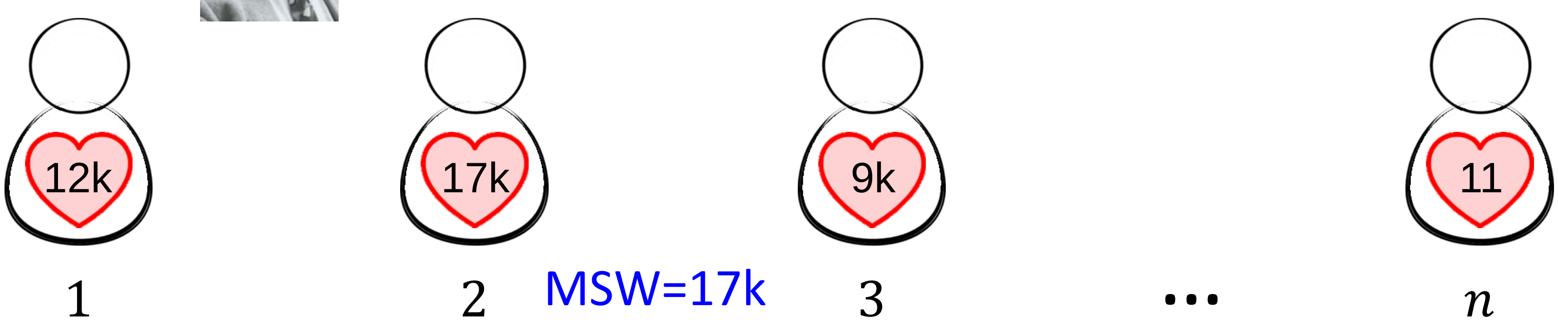


[Vickrey 61]: "run my second-price mechanism"
"highest bidder wins, pays the second bid…"

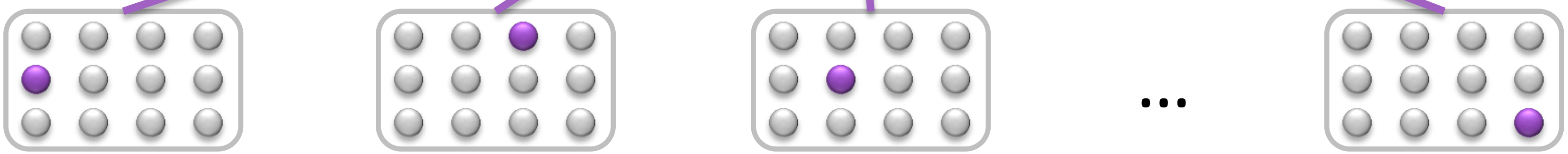


Rolex Auction

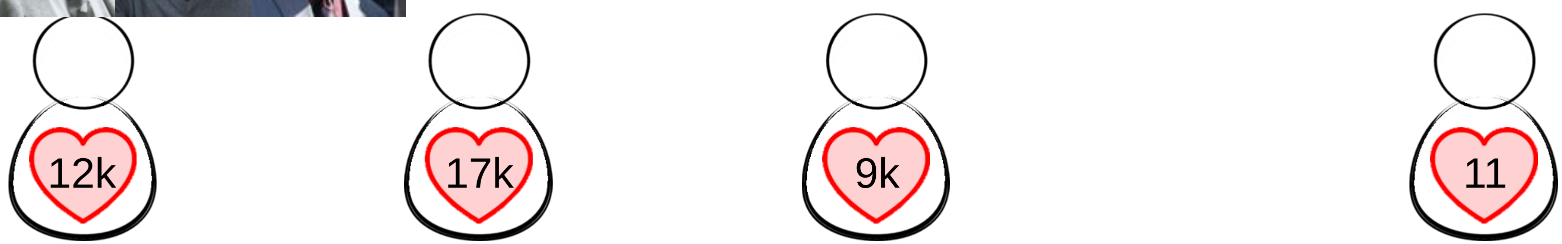
GOAL: maximize *social welfare* (valuation of the player who wins)

VCG mechanism

12k 17k 9k 11



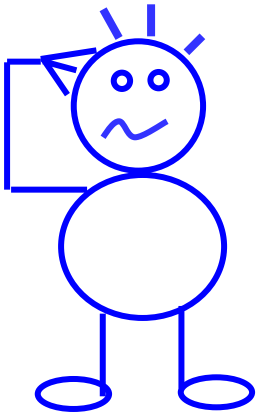
[VCG 70s]: "Can also do multiple goods (combinatorial auctions)!"



1 2 3 … n

: Fantastic!

- Two-Line Mechanism
- Two-Line Proof
- Optimal Performance



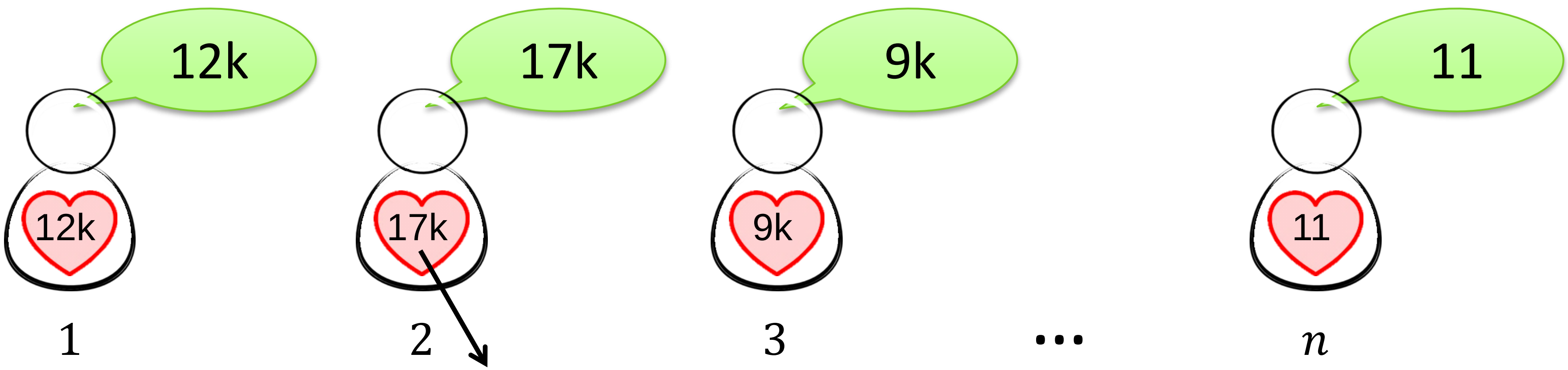
Oversimplified?

Warning!

optimal performance from an

ASSUMPTION:

each player knows his own valuation exactly



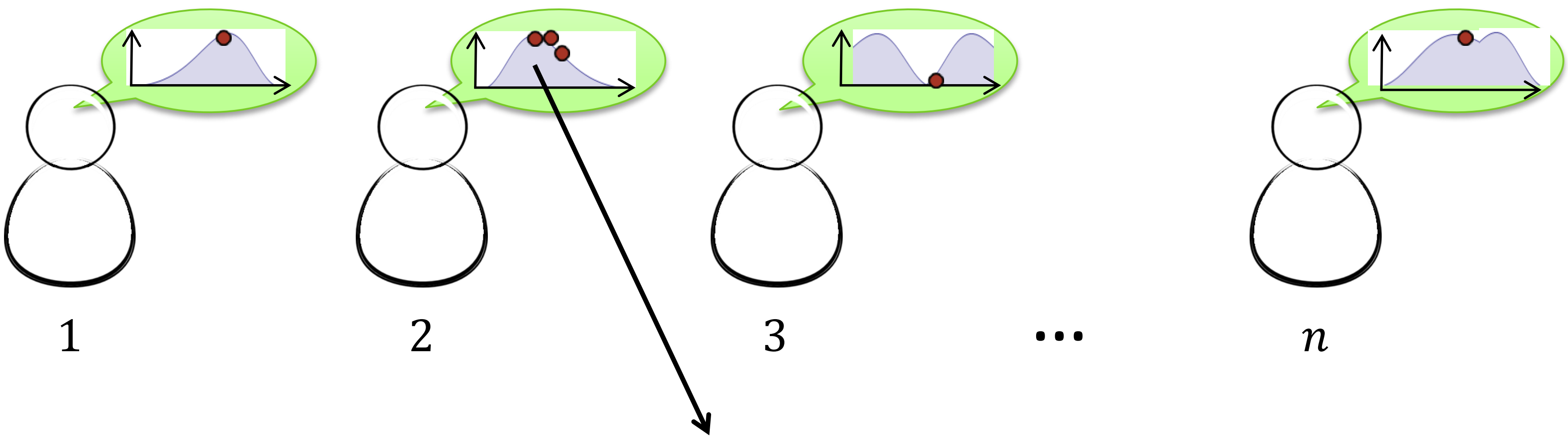
17k. Or 17.01k?

Today’s Agenda

- Motivation
- Our Valuation Model
- Our Performance Measure
- Our Results
- Conclusions

First attempt

- Weaker assumption: Bayesian?
  - each player knows his own individual Bayesian
  - <u>same</u> second-price mechanism: just truthfully bid your expected value

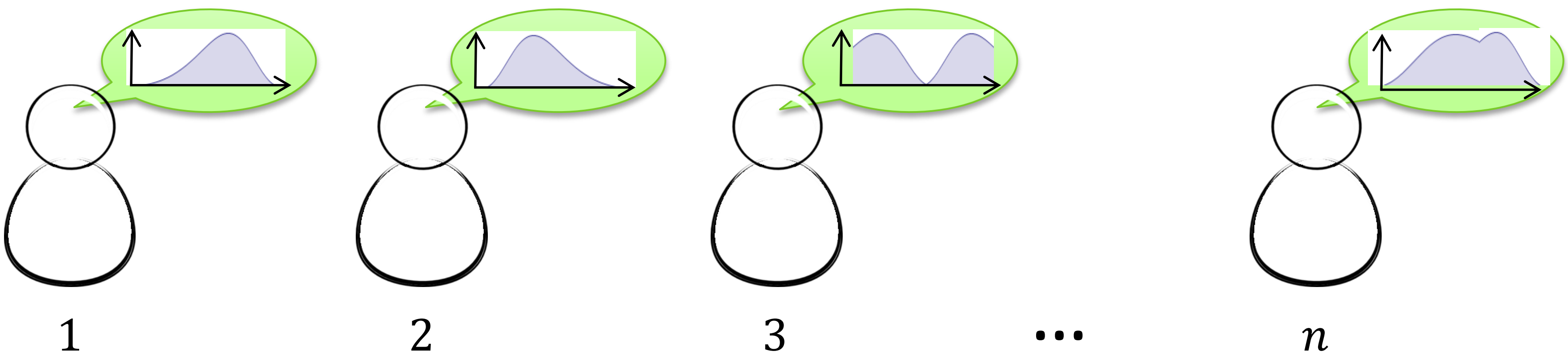


Does player 2 really know $\frac{\Pr[16\text{k}]}{\Pr[16.6\text{k}]} = 1.5317529012098321757984321 7$?

If no, Bayesian assumption is still very strong!

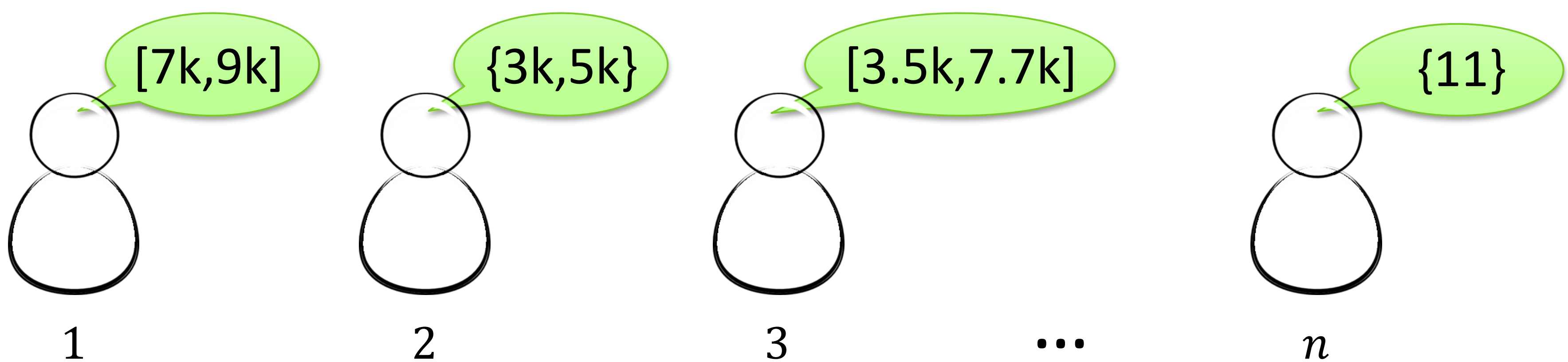
First attempt

- Weaker assumption: Bayesian?



Our Attempt

- Our assumption: "approximate valuation"
  - each player only knows that his valuation is <u>drawn from a set</u>

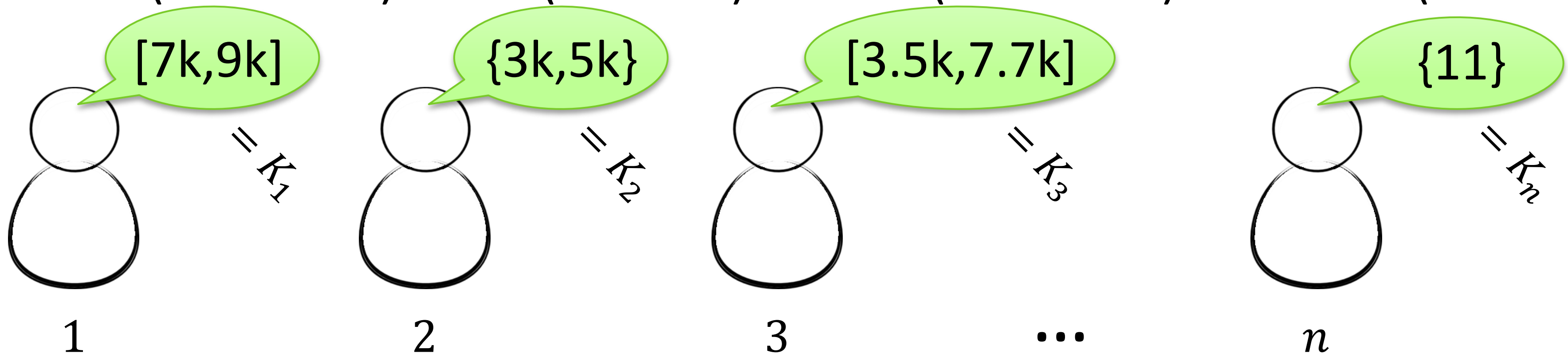


Our Attempt

- Our ~~assumption~~ fact: "approximate valuation"
  - exists some global constant $\delta \in [0,1]$
  - player i has a δ-approximate valuation set K_i
  - player i's true valuation θ_i is guaranteed to be $\in K_i$

Example: $\delta = 40\%$

$\subseteq$8k $(1 \pm \delta)$	$\subseteq$4k $(1 \pm \delta)$	$\subseteq$5.6k $(1 \pm \delta)$	$\subseteq$11 $(1 \pm \delta)$
=8k (1 ± 12.5%)	$\subseteq$4k (1 ± 25%)	=5.6k (1 ± 37.5%)	11 (1 ± 0%)



Our Attempt

- Our assumption: "approximate valuation"
  - exists some global constant $\delta \in [0,1]$
  - player i has a δ-approximate valuation s
  - player i's true valuation θ_i is guaranteed to be $\in K_i$

$\delta = 0 \Rightarrow$ Classical Mechanism Design

$\delta > 0 \Rightarrow$ Mechanism Design with Approximate Valuations

Unrelated work: Knightian decision theory

Uncertainty is modeled as a set, but not studied under mechanism design.

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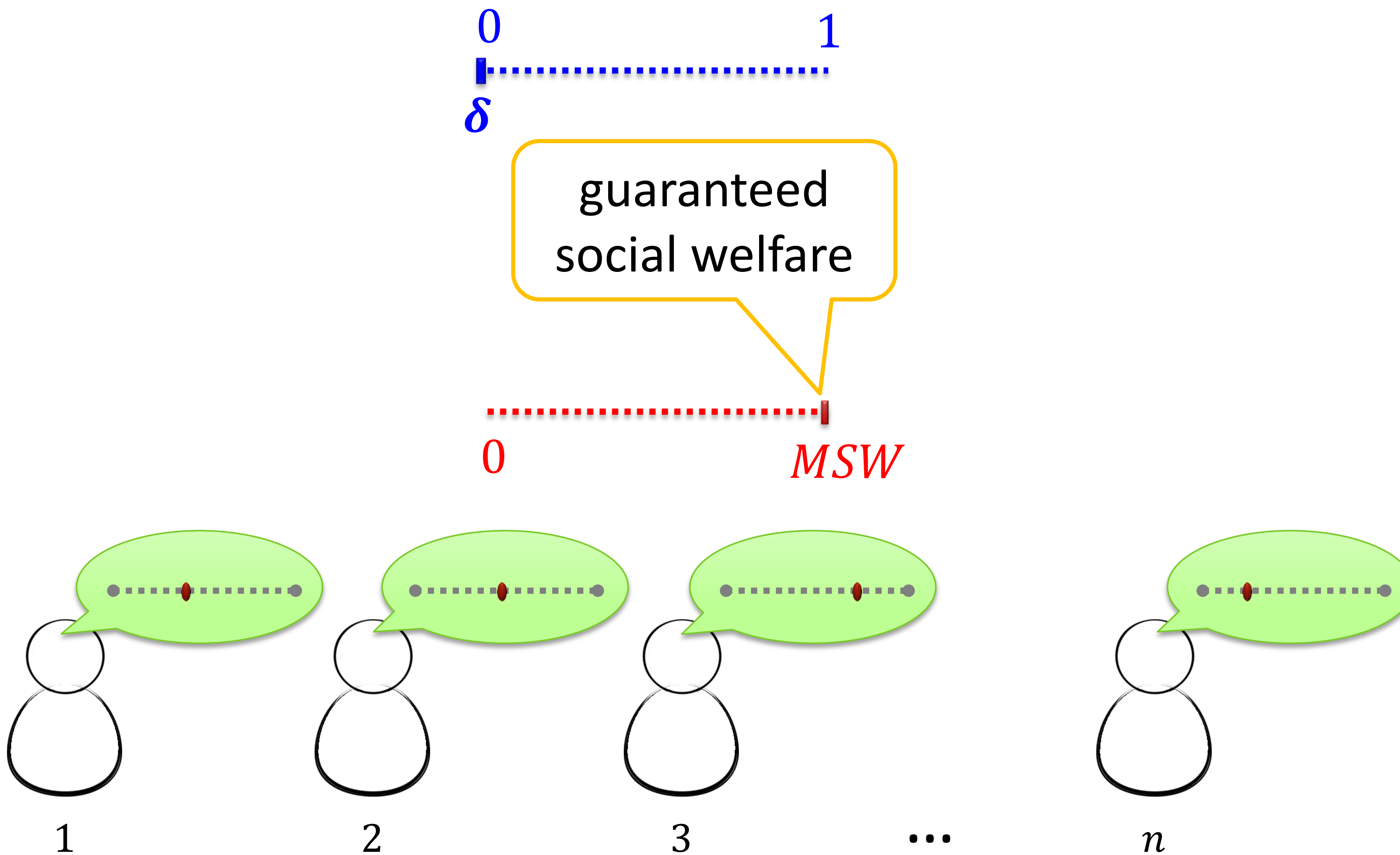
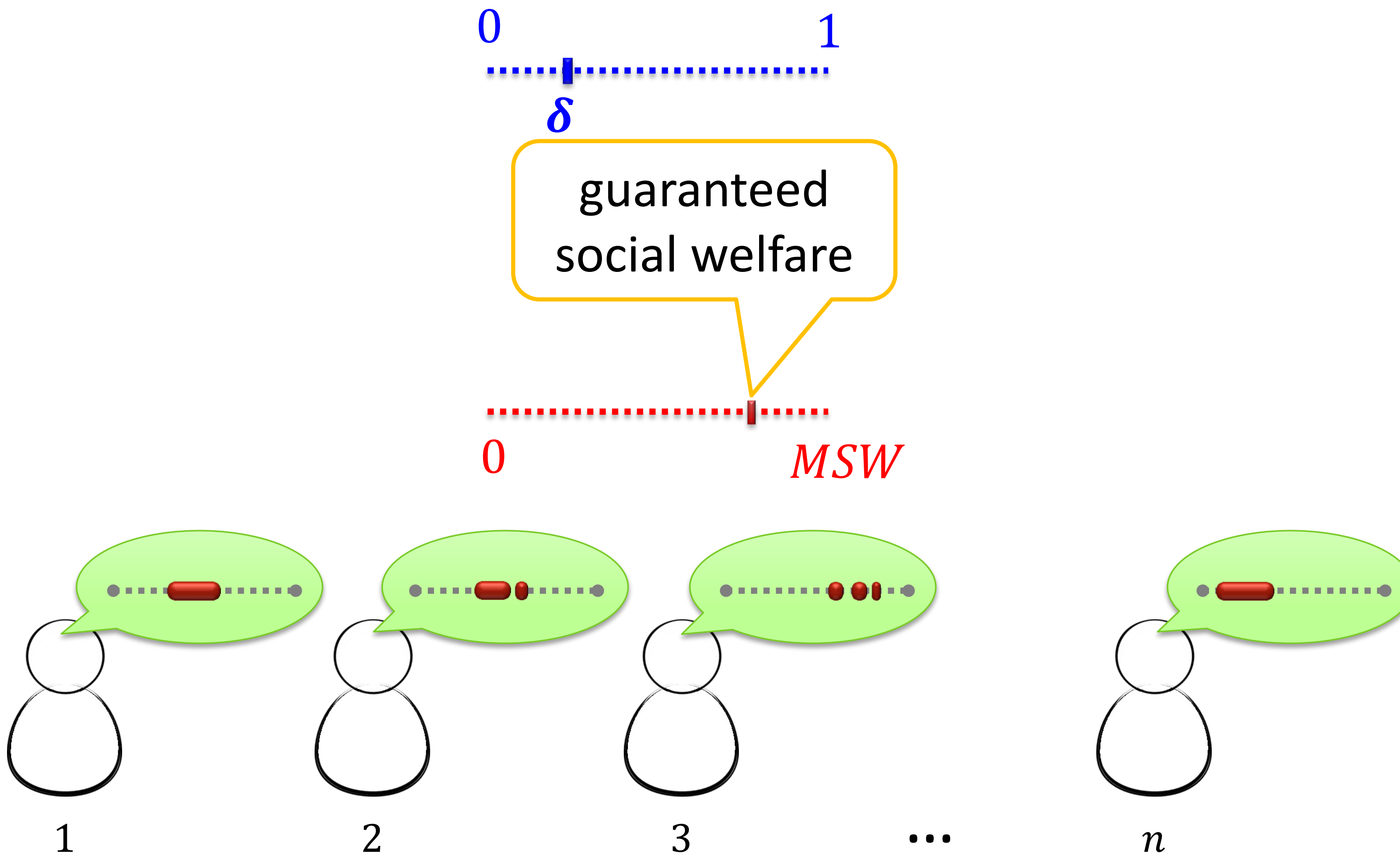
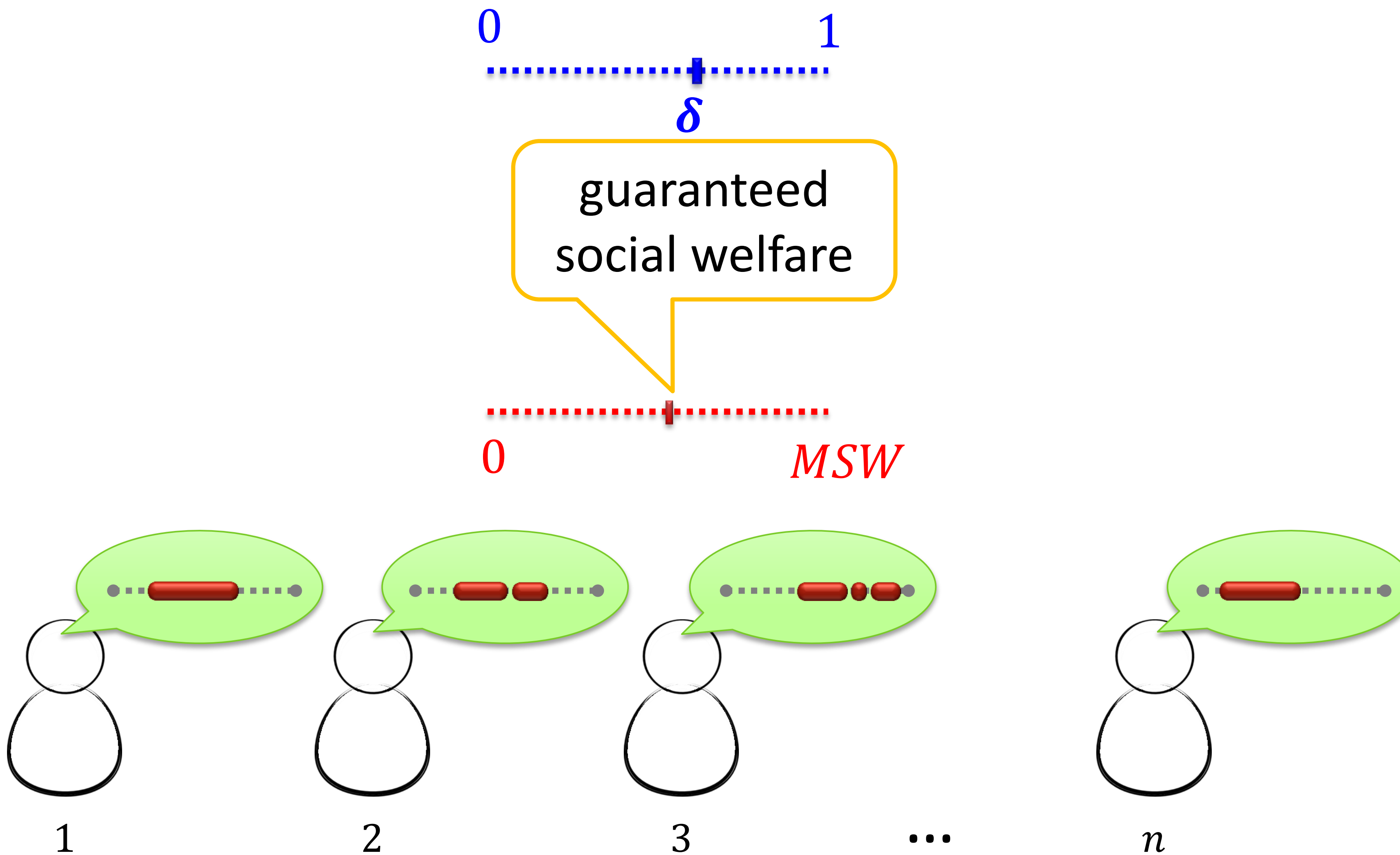
How Much SW Can We Guarantee?



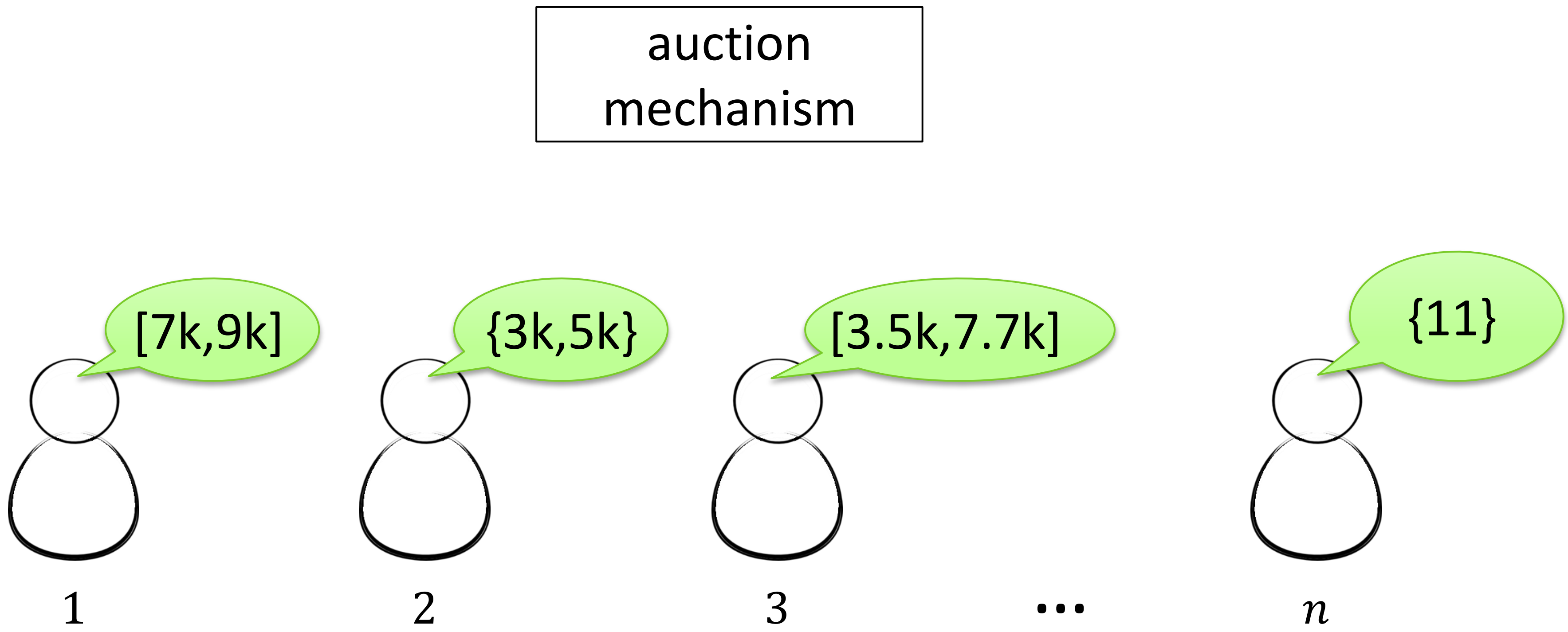
How Much SW Can We Guarantee?



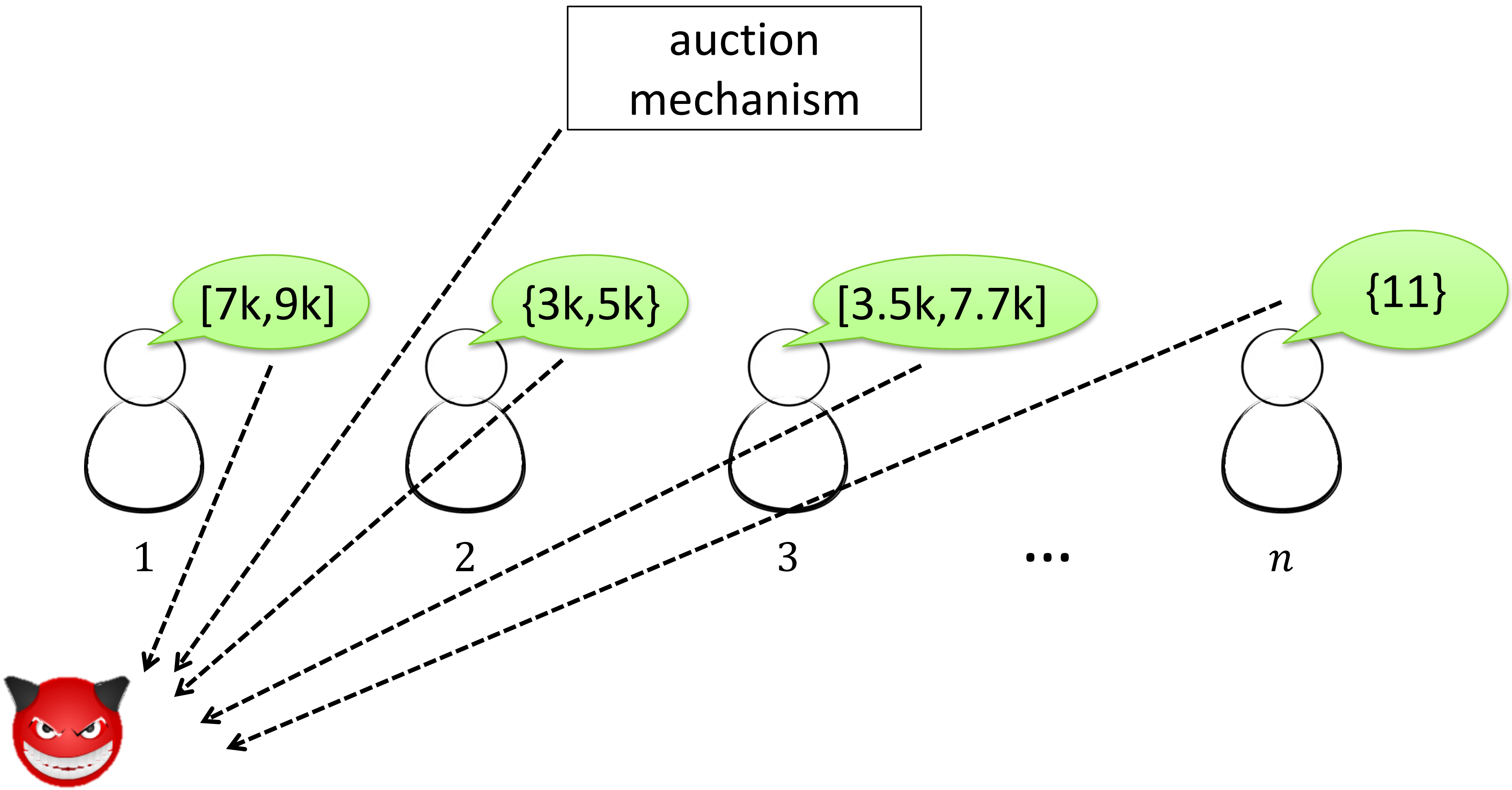
How Much SW Can We Guarantee?



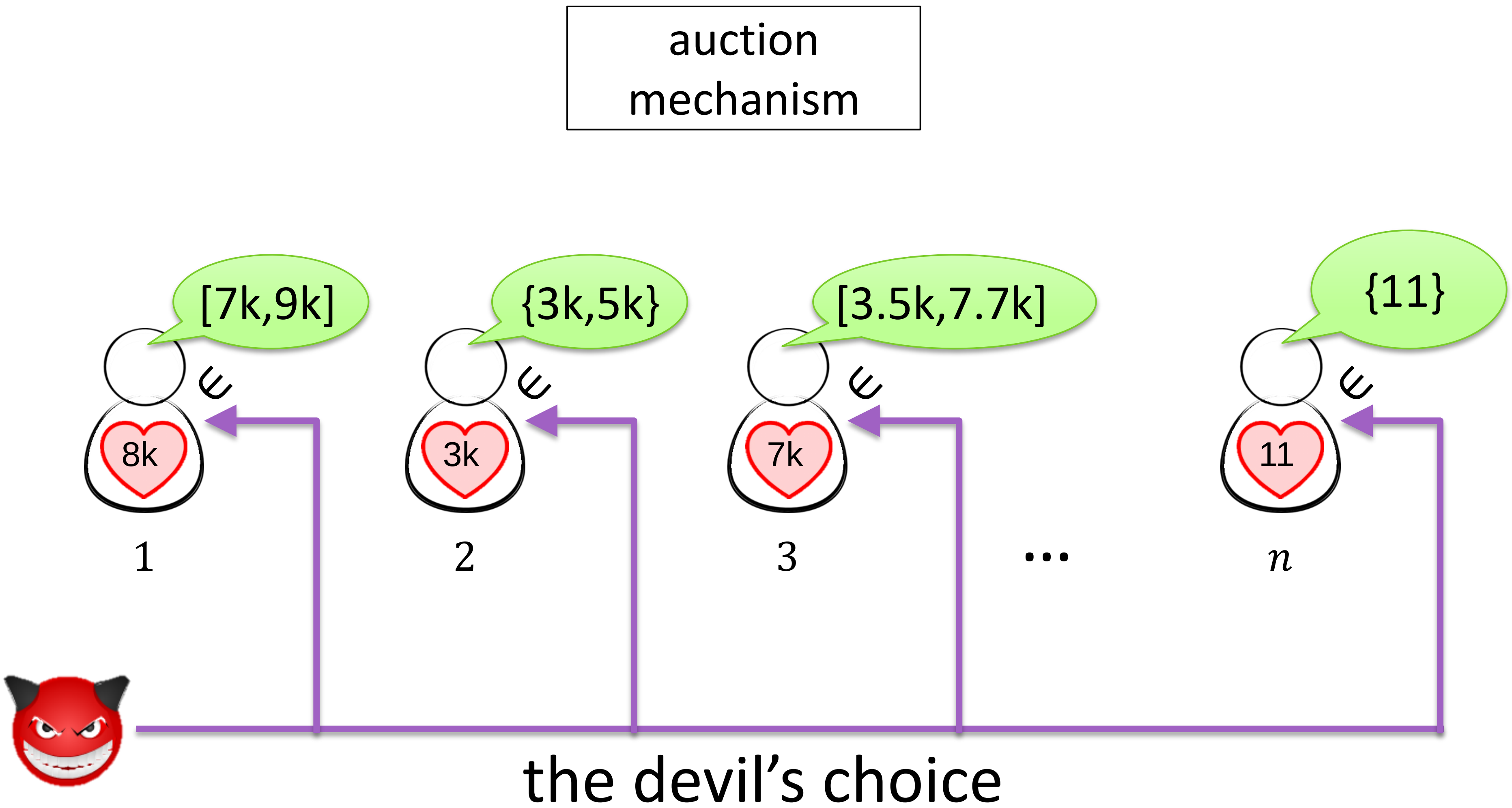
WAIT!!!
How to define SW or MSW when θ_i is unknown?



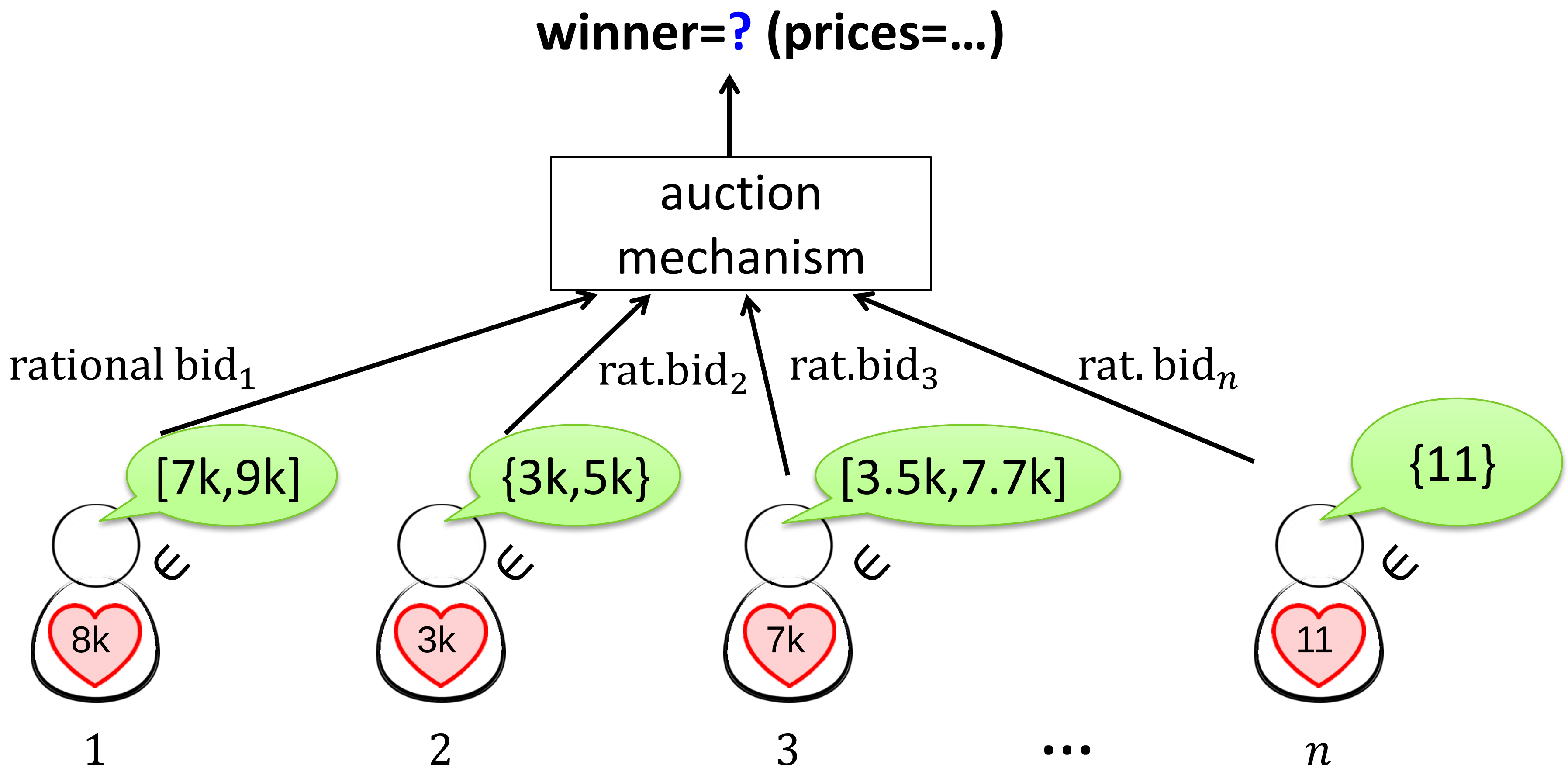
Adversarial Performance Measure



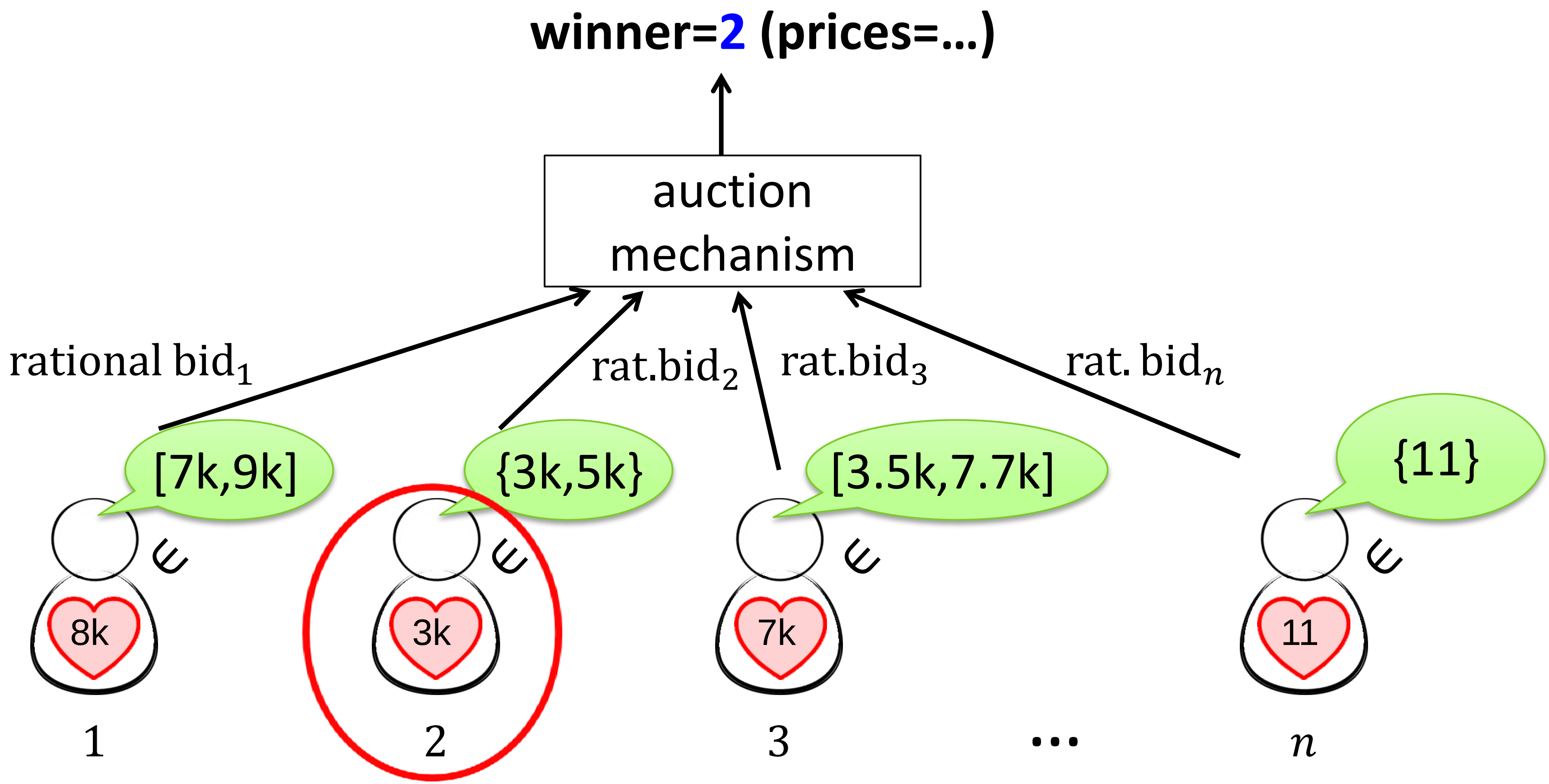
Adversarial Performance Measure



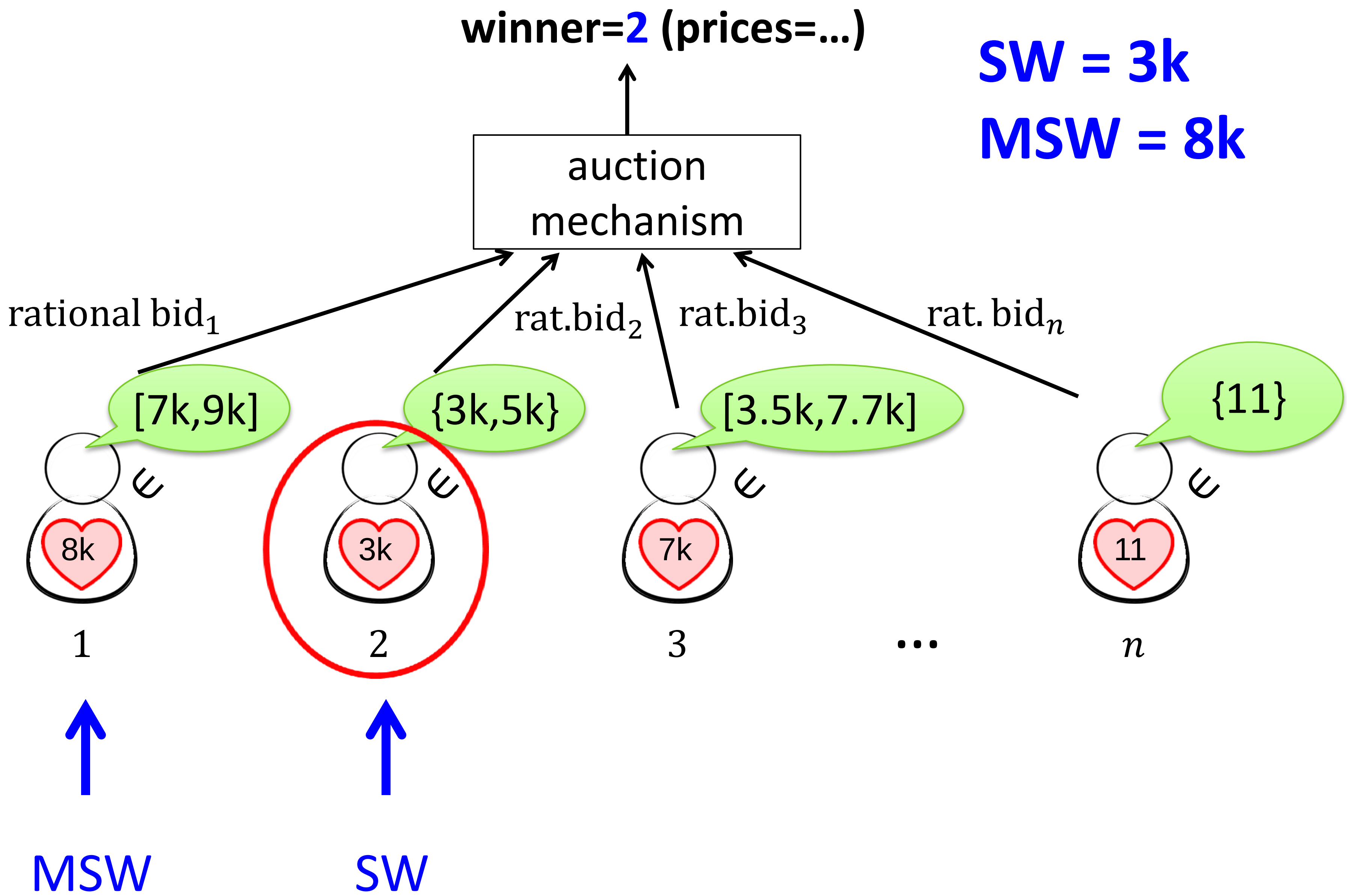
Adversarial Performance Measure



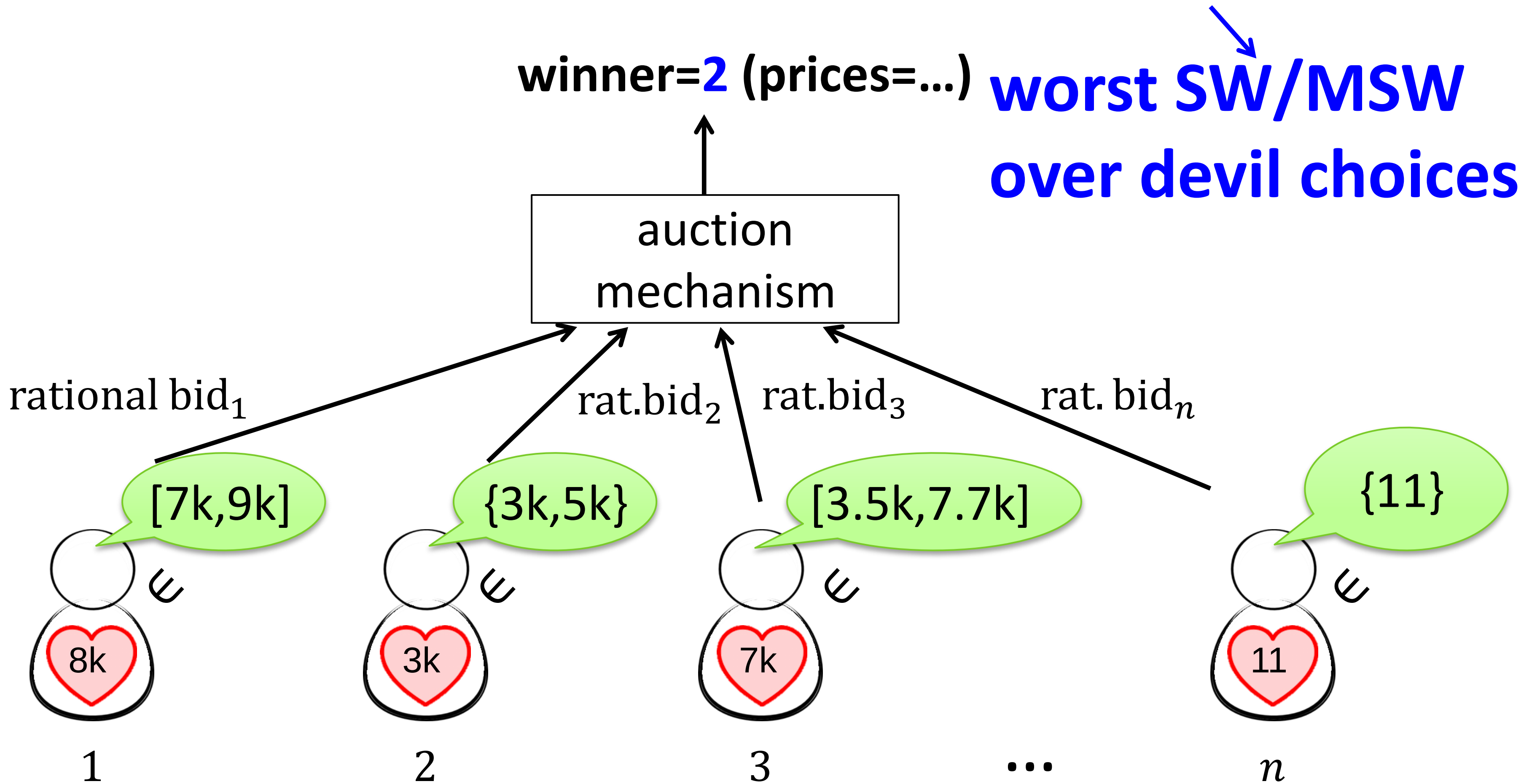
Adversarial Performance Measure



Adversarial Performance Measure



Adversarial Performance Measure



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Our Results

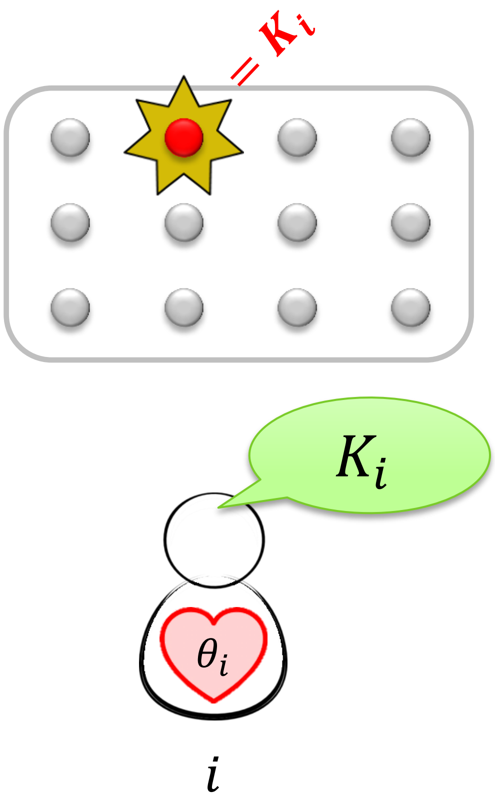
	Dominant Strategies	
Single-good auctions		

A classical solution concept, used also by second-price.

Our Results

Implementation in …

… Dominant Strategies



Nonsense:

If $K_2 = \{3k, 5k\}$, how can player 2 know which 3k and 5k is his best strategy?

Not So!!

each i could bid a set of valuations—in particular K_i

Can a mechanism with such property … a dominant-strategy mechanism.

$$s_i \geq s_i' \text{ iff } \forall s_{-i} \quad \forall \theta_i \in K_i \quad u_i(\theta_i; s_i, s_{-i}) \geq u_i(\theta_i; s_i', s_{-i})$$

(Coincides with Knightian decision theory, i.e., 1-player behavioral analysis.)

Our Results

	Dominant Strategies	
	Negative result	Positive result
Single-good auctions		$f(\delta)$? $(1-\delta)$? $(1-\delta)^2$?

Our Results

	Dominant Strategies	
	Negative result	Positive result
Single-good auctions	$(\forall \delta > 0, n)$ $\leq \frac{1}{n}$	$\geq \frac{1}{n}$

Trivial: assign at random!

Interpretation: dominant strategy is useful iff <u>exact</u> valuation

~~$70(1 \pm 0.1)$~~

~~$70(1 \pm 0.01)$~~

~~$70(1 \pm 0.001)$~~

70

Dominant-Strategy for Single-Good

- Thm': $\forall n\ \forall \delta > 0\ \forall B \geq \frac{1}{\delta}\ \ \forall$dst M, $\exists K_1 \ldots K_n$, $\exists \theta_1 \ldots \theta_n \in K_1 \ldots K_n$
  $\mathbb{E}[SW(\theta, M(K))] \leq$ ... $MSW(\theta)$

Valuation bound.

1. players bid sets of va
2. bidding his true K_i is a dominant strategy.

- Thm': $\forall \delta > 0$, $\forall$dominant-strategy-truthful M,
  it can guarantee no more than $\frac{1}{n} \cdot \mathrm{MSW}$

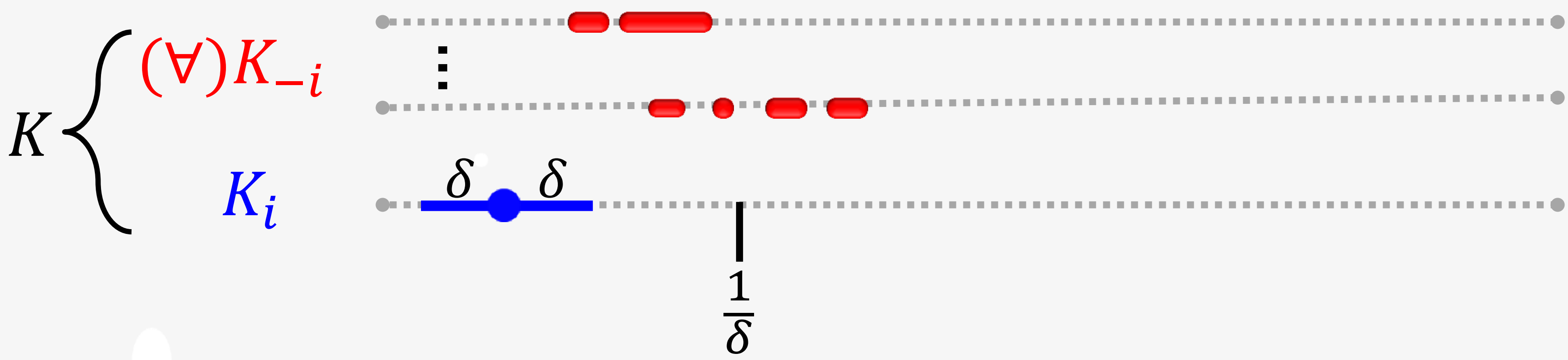
Revelation Principle

- Thm: $\forall \delta > 0$, $\forall$dominant-strategy M,
  it can guarantee no more than $\frac{1}{n} \cdot \mathrm{MSW}$

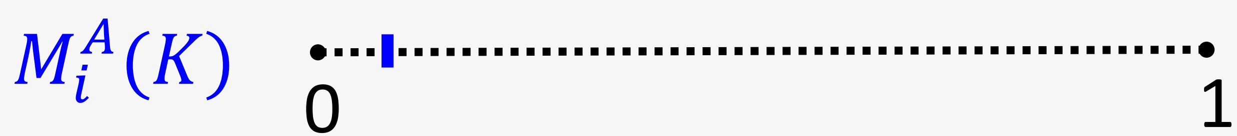
Dominant-Strategy for Single-Good

- Thm’: $\forall n\ \forall\delta\ \forall B \geq \frac{1}{\delta}\ \ \forall$dst $M,\ \ \exists K, \exists\theta \in K$
  $$\mathbb{E}[SW(\theta, M(K))] \leq \left(\frac{1}{n} + \frac{\left\lceil\frac{1}{\delta}\right\rceil + 1}{B}\right) MSW(\theta)$$
- Proof: $\delta[x] \stackrel{\text{def}}{=} (x - \delta x, x + \delta x) \cap \{0, \ldots B\}$

Dominant Freezing Lemma



player i’s allocation probability under M:

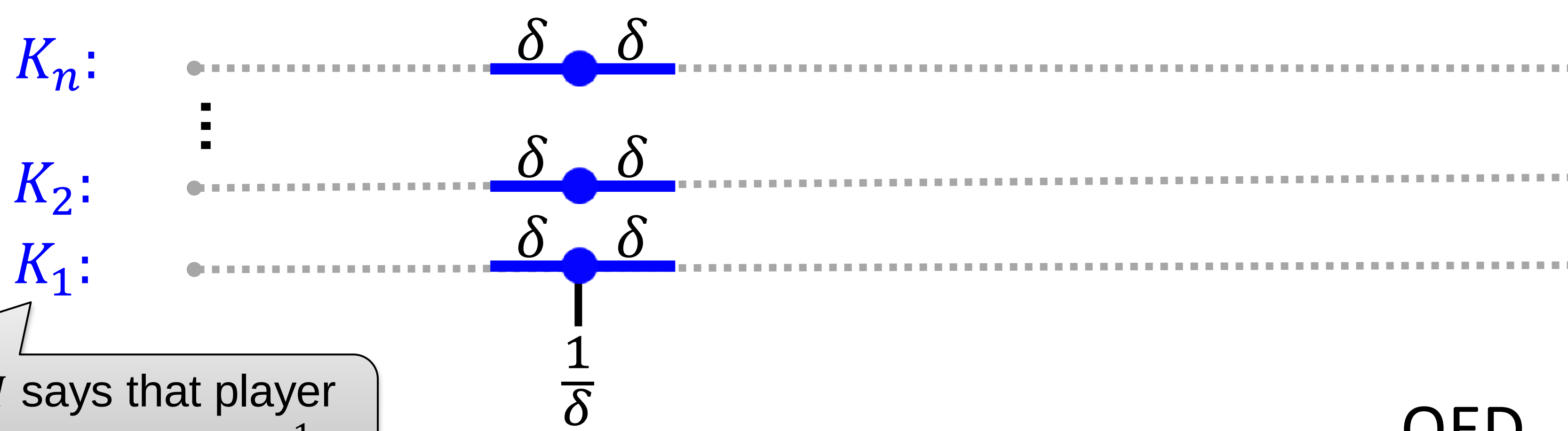


Dominant-Strategy for Single-Good

- Thm’: $\forall n\, \forall \delta\, \forall B \geq \frac{1}{\delta}$ $\forall$dst M, $\exists K, \exists \theta \in K$
  $$\mathbb{E}[SW(\theta, M(K))] \leq \left(\frac{1}{n} + \frac{\lceil \frac{1}{\delta} \rceil + 1}{B} \right) MSW(\theta)$$
- Proof: $\delta[x] \stackrel{\text{def}}{=} (x - \delta x\, , x + \delta x) \cap \{0, \ldots B\}$

Dominant Freezing Lemma

- $\forall i, \forall K_{-i}\, , \forall x \geq \frac{1}{\delta}$ $\quad M_i^A(\delta[x], K_{-i}) = M_i^A(\delta[x+1], K_{-i})$



WLOG, M says that player 1 gets the good w.p. $\leq \frac{1}{n}$

QED

Dominant-Strategy for Single-Good

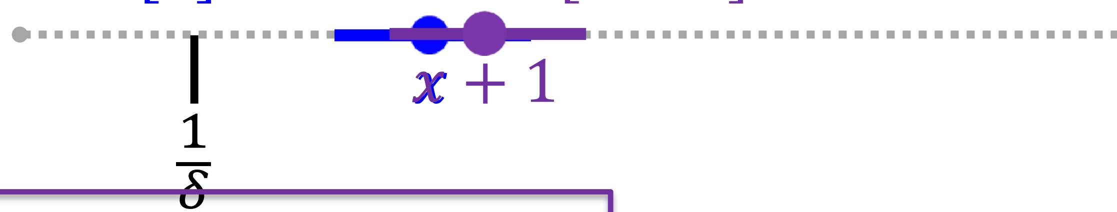
Dominant Freezing Lemma

- $\forall i, \forall K_{-i}, \forall x \geq \frac{1}{\delta} \quad M_i^A(\delta[x], K_{-i}) = M_i^A(\delta[x+1], K_{-i})$

Proof:

($\delta[x]$ dominates $\delta[x+1]$)

$K_i = \delta[x]$

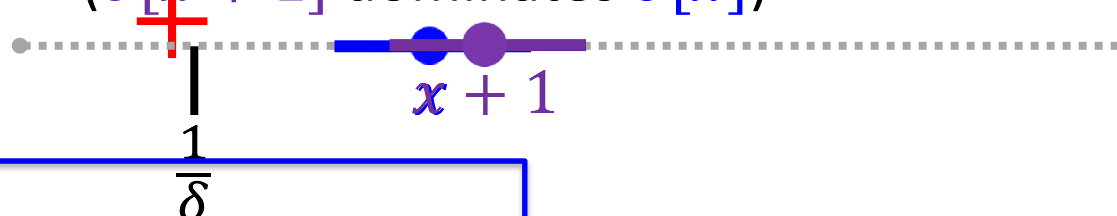


$\forall \theta_i \in \delta[x]$ / $\theta_i = x$

$M_i^A(\delta[x], K_{-i})\theta_i - M_i^P(\delta[x], K_{-i}) \geq M_i^A(\delta[x+1], K_{-i})\theta_i - M_i^P(\delta[x+1], K_{-i})$

($\delta[x+1]$ dominates $\delta[x]$)

$K_i = \delta[x+1]$



$\forall \theta_i \in \delta[x+1]$ / $\theta_i = x+1$

$M_i^A(\delta[x+1], K_{-i})\theta_i - M_i^P(\delta[x+1], K_{-i}) \geq M_i^A(\delta[x], K_{-i})\theta_i - M_i^P(\delta[x], K_{-i})$

$M_i^A(\delta[x+1], K_{-i})$ $\quad$ $M_i^A(\delta[x], K_{-i})$

Dominant-Strategy for Single-Good

Dominant Freezing Lemma

- $\forall i, \forall K_{-i}, \forall x \geq \frac{1}{\delta} \quad M_i^A(\delta[x], K_{-i}) = M_i^A(\delta[x+1], K_{-i})$

Proof:

To claim that $x+1 \in \delta[x]$, we need $x \geq \frac{1}{\delta}$

$\forall \theta_i \in \delta[x]$

$\theta_i = x$

~~$M_i^A(\delta[x], K_{-i})\theta_i$~~ $- M_i^P(\delta[x], K_{-i}) \geq$ ~~$M_i^A(\delta[x+1], K_{-i})\theta_i$~~ $- M_i^P(\delta[x+1], K_{-i})$

$M_i^A(\delta[x], K_{-i}) \quad M_i^A(\delta[x+1], K_{-i})$

+

$\forall \theta_i \in \delta[x+1]$

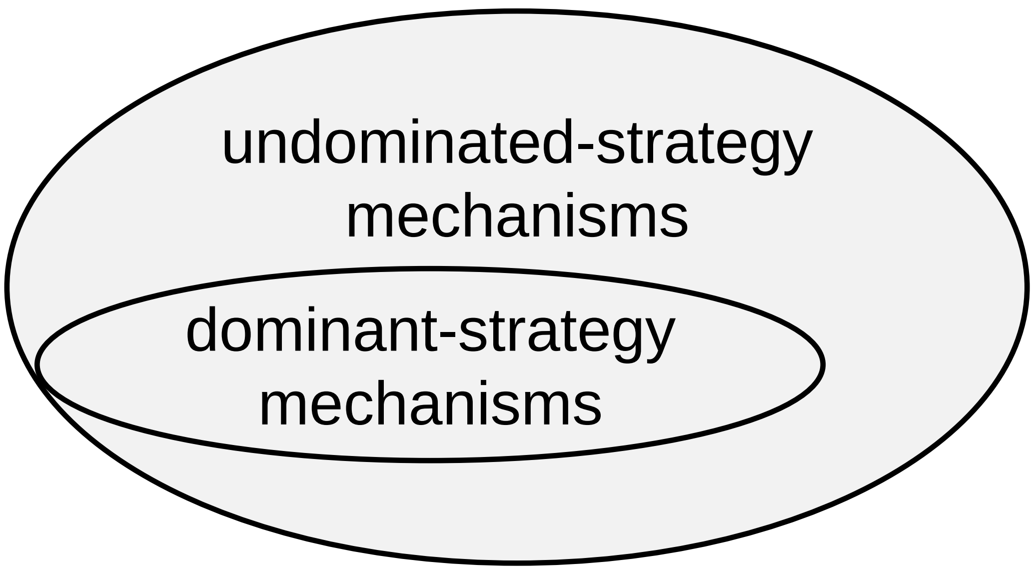
$\theta_i = x$

~~$M_i^A(\delta[x+1], K_{-i})\theta_i$~~ $- M_i^P(\delta[x+1], K_{-i}) \geq$ ~~$M_i^A(\delta[x], K_{-i})\theta_i$~~ $- M_i^P(\delta[x], K_{-i})$

Our Results

	Dominant Strategies		Undominated Strategies	
	Negative result	Positive result	Negativ	Positive result
Single-good auctions	$(\forall \delta > 0, n)$ $\leq \frac{1}{n}$	$\geq \frac{1}{n}$		

A weaker notion than dominant strategies.

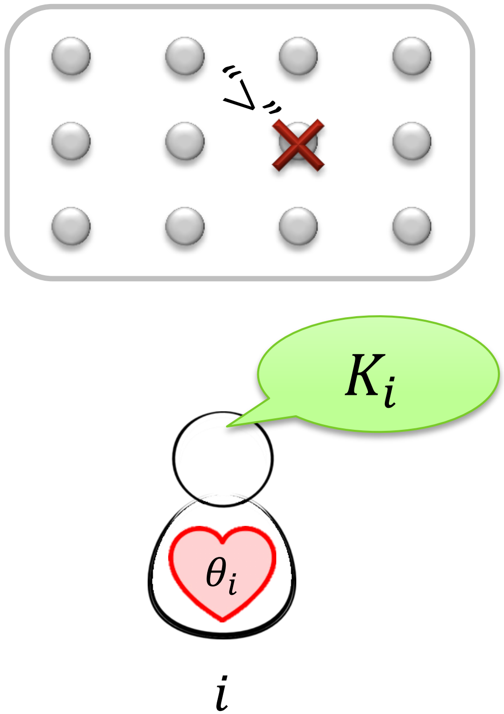


Our Results

Implementation in …

… ~~Dominant Strategies~~

… Undominated Strategies

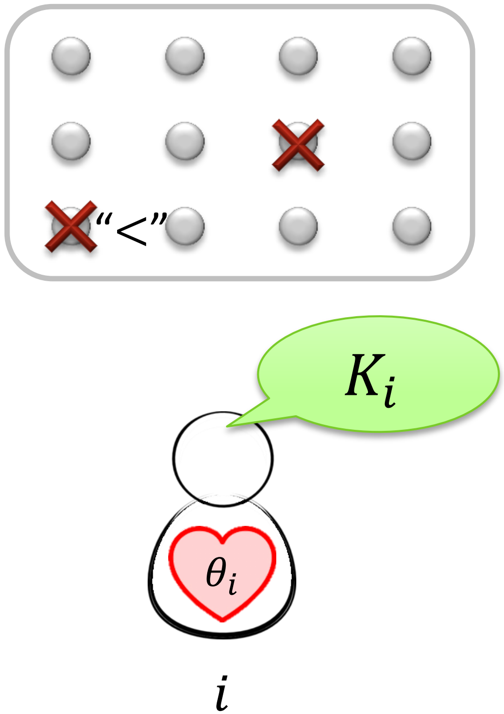


Our Results

Implementation in …

… ~~Dominant Strategies~~

… Undominated Strategies



$s_i > s_i'$ **iff:**

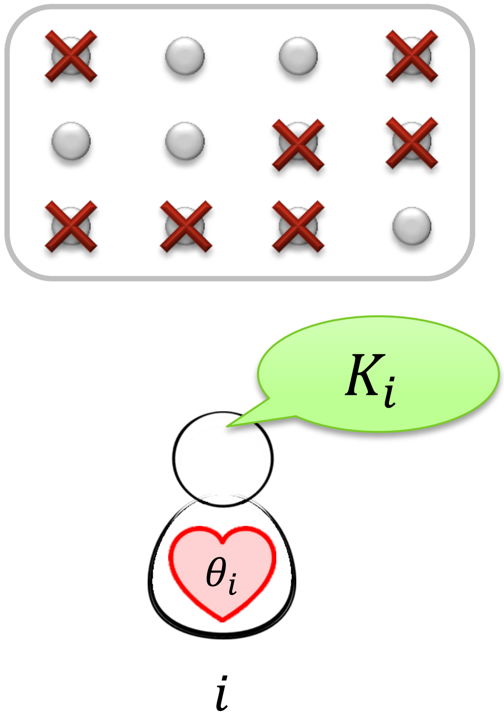
1) $\forall s_{-i} \quad \forall \theta_i \in K_i \quad u_i(\theta_i; s_i, s_{-i}) \geq u_i(\theta_i; s_i', s_{-i})$
2) $\exists s'_{-i} \quad \exists \theta_i \in K_i \quad u_i(\theta_i; s_i, s'_{-i}) > u_i(\theta_i; s_i', s'_{-i})$

Our Results

Implementation in …

… ~~Dominant Strategies~~

… Undominated Strategies



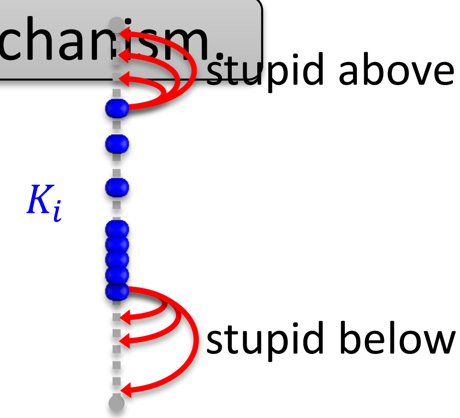
Our Results

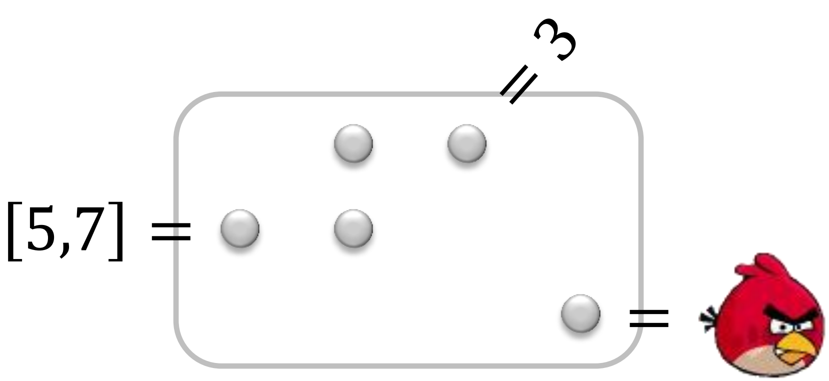
Non-trivial! Need to deal with all mechanisms!
Strategies could be numbers, sets, or even angry birds!

				…Strategies
	Negative result	Positive result	…esult	Positive result
Single-good auctions	$\le \frac{1}{n}$	$\ge \frac{1}{n}$	det $\le \left(\frac{1-\delta}{1+\delta}\right)^2$ prob $\le$ $(1-\delta)^2+4\delta$ …	det $\ge \left(\frac{1-\delta}{1+\delta}\right)^2$ prob $\ge \frac{(1-\delta)^2+\frac{4\delta}{n}}{(1+\delta)^2}$

The classical second-price mechanism.

Our own probabilistic mechanism.





Our Results

	Dominant Strategies		Undominated Strategies	
	Negative result	Positive result	Negative result	Positive result
Single-good auctions	$\leq \frac{1}{n}$	$\geq \frac{1}{n}$	$\det \leq \left(\frac{1-\delta}{1+\delta}\right)^2$ $\text{prob} \leq \frac{(1-\delta)^2+\frac{4\delta}{n}}{(1+\delta)^2}$	$\det \geq \left(\frac{1-\delta}{1+\delta}\right)^2$ $\text{prob} \geq \frac{(1-\delta)^2+\frac{4\delta}{n}}{(1+\delta)^2}$

e.g. $\theta_i(\{1\}) = 7,\ \theta_i(\{2\}) = 10,\ \theta_i(\{1,2\}) = 12$

Combinatorial auctions			$\text{VCG} \overset{?}{\leq} \left(\frac{1-\delta}{1+\delta}\right)^{2^m-2}$	

m goods on sale, players may be interested in arbitrarily subsets. [in submission]

VCG Characterization Lemma

- Characterizing player's entire set of undominated strategies.

Undom. Strat. in Comb. Auctions

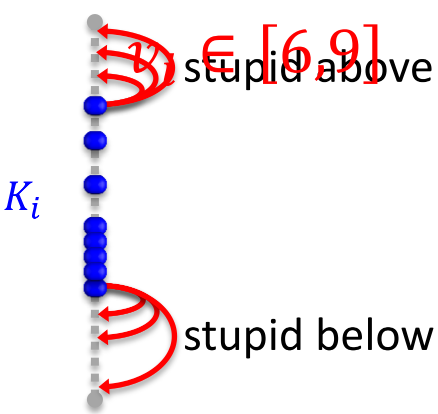
VCG Characterization Lemma

- under the VCG mechanism for combinatorial auctions of m goods, for every player i, his bidding strategy v_i is undominated if and only if…

Single-good (2nd price):

- v_i is a number
  - e.g. $v_i = 7$
- K_i is δ-approximate
  - e.g. $K_i = [6,9]$
- v_i is non-stupid iff:

$v_i \in [6,9]$



Combinatorial auction (VCG):

- v_i is a function $2^{[m]} \setminus \{\emptyset\} \to \mathbb{R}_{\geq 0}$
  - e.g. $v_i(\{1\}) = 7,\ v_i(\{2\}) = 10,\ v_i(\{1,2\}) = 12$
- $K_i(S)$ is δ-approximate
  - e.g. $K_i(\{1\}) = [6,9],\ K_i(\{2\}) = [8,11],\ K_i(\{1,2\}) = [10,13]$
- v_i is non-stupid iff:

~~$v_i \in [6,9] \times [8,11] \times [10,13]$~~

Undom. Strat. in Comb. Auctions

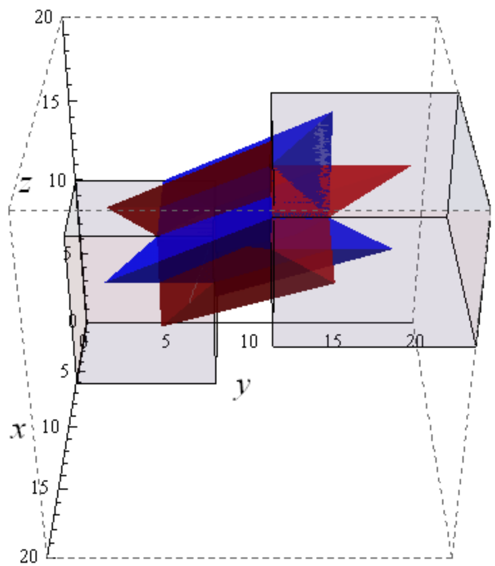
VCG Characterization Lemma

- under the VCG mechanism for combinatorial auctions of m goods, for every player i, his bidding strategy $v_i \in \mathrm{UDed}(K_i)$ if and only if…

“v_i is inside the union of $m!$ triangular cylinders, minus two hypercubes…”

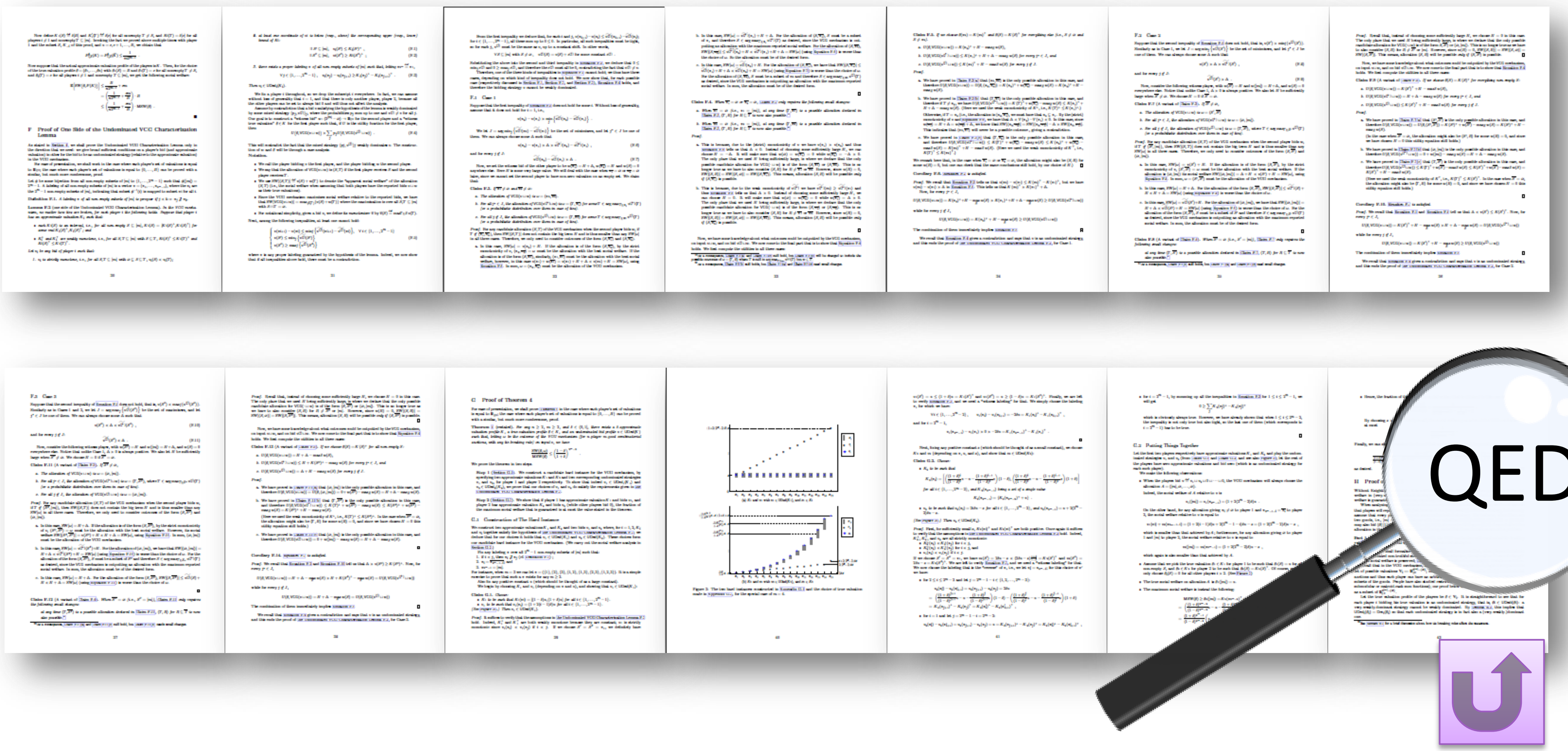
e.g. $K_i(\{1\}) = [6,9],\ v_i(\{2\}) = [8,11],\ v_i(\{1,2\}) = [10,13]$

undominated strategies =



Undom. Strat. in Comb. Auctions

- Thm: $\forall n \geq 2, m \geq 2, \delta > 0$, the VCG mechanism guarantees $\left(\frac{1-\delta}{1+\delta}\right)^{2^m-2} \cdot \mathrm{MSW}$.



Hyperlink

Undominated Intersection Lemma

	Dominant Strategies		Undominated Strategies	
	Negative result	Positive result	Negative result	Positive result
Single-good auctions	$\leq \frac{1}{n}$	$\geq \frac{1}{n}$	det $\leq \left(\frac{1-\delta}{1+\delta}\right)^2$ prob $\leq \frac{(1-\delta)^2+\frac{4\delta}{n}}{(1+\delta)^2}$	det $\geq \left(\frac{1-\delta}{1+\delta}\right)^2$ prob $\geq \frac{(1-\delta)^2+\frac{4\delta}{n}}{(1+\delta)^2}$
Multi-unit auctions	$\leq \frac{1}{n}$	$\geq \frac{1}{n}$	$\leq \frac{(1-\delta)^2+\frac{4\delta}{n}}{(1+\delta)^2}$	$\geq \left(\frac{1-\delta}{1+\delta}\right)^2$
Combinatorial auctions	$\leq \frac{1}{n2^{m-1}}$	$\geq \frac{1}{n(2^m-1)}$	VCG $\leq \left(\frac{1-\delta}{1+\delta}\right)^{2^m-2}$	

Revelation Principle

Dominant Freezing Lemma

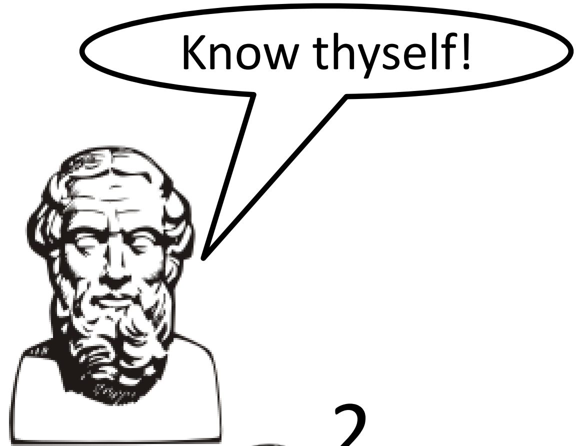
Distinguishable Monotonicity Lemma

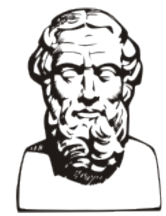
VCG Characterization Lemma

Today’s Agenda

- Motivation
- Our Valuation Model
- Our Performance Measure
- Our Results
- Conclusions

Conclusion



mechanism design =  ^{2} $+ \, n \times$ 

GOAL: want to learn about others,

who may not know themselves very well.

Today's positive results:

The GOAL is desirable and doable! (But more work.)

Today's negative results:

More exciting work to be done!

Thank you!